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Quantum plasmadynamics: role of the electron self-energy and the vertex correction

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The linear response 4-tensor for a relativistic quantum electron gas may be calculated by reinterpreting the electron propagator in the expression for the vacuum polarization tensor as statistical averages over the electron gas. We apply a similar procedure to two other radiative corrections: the electron self-energy and the vertex correction. When the photon propagator in these expressions is interpreted as a statistical average over a distribution of waves in the medium, these radiative corrections lead to a relativistic quantum expression for the ponderomotive force and to a new class of 'hybrid' emission processes.

1. Introduction

A synthesis of the kinetic theory of plasmas and quantum electrodynamics (QED) results in a covariant relativistic quantum theory for plasmas, referred to here as quantum plasmadynamics (QPD). The synthesis may be achieved by replacing the electron and photon propagators in QED by statistically averaged propagators. This idea, which is familiar in the context of thermal Green functions (see e.g. Kogan 1959; Bonch-Bruevich 1959; Fradkin 1959; cf. also Bechler 1981; Fetter & Walecka 1971), was introduced by Tsytovich (1961) to calculate the linear response tensor for an arbitrary distribution of electrons ($\epsilon = 1$) and positrons ($\epsilon = -1$), described in terms of their occupation numbers $n^{\epsilon}(\mathbf{p})$. A manifestly covariant and gauge-invariant form of the classical kinetic theory of plasmas, developed by one of the present authors (Melrose 1973, 1974), has been combined with the idea of statistical averages of propagators in QED to calculate covariant relativistic quantum forms of the linear response tensors for an unmagnetized electron gas (Melrose 1982; Melrose & Hayes 1984; cf. also Kowalenko *et al.* 1985) and a magnetized electron gas (Svetozarova & Tsytovich 1962; Arunasalam 1969; Melrose 1980) and has been extended to derive nonlinear response tensors (Melrose & Kuijpers 1984). The important ingredients in the resulting theory for QPD are reviewed in $2.

The linear response tensor is calculated in QPD by interpreting the propagators in the (unregularized) expression for the vacuum polarization tensor as statistically averaged propagators. The divergence in the vacuum polarization, described by the 'bubble' Feynman diagram, is removed by charge renormalization. One may regard renormalization as a procedure for converting the theory of free particles and free electromagnetic field into a theory for a collective medium, where the 'medium' is the QED vacuum. The identification of the vacuum polarization tensor as the linear response tensor for this 'medium' leads naturally to the use of the same procedure to calculate the linear response tensor for an actual medium. This procedure may also be applied to the 'triangle' and 'box' diagrams, leading to expressions for the quadratic and cubic, respectively, nonlinear response tensors of a relativistic quantum electron gas (see e.g. Melrose 1972).

The vacuum polarization is one of three familiar radiative corrections in QED. The other two are the electron mass operator and the vertex correction. The divergences in these are removed by mass renormalization. What are the implications of applying the statistical average to the propagators in the expressions for the electron mass operator and for the vertex correction ? We provide an answer to this question in the present paper. In **93,** after noting that the mass operator may be used to treat macroscopic mass renormalization (Tsytovich 1962), we show that the statistical average (over a distribution of waves in the plasma) of the mass operator may be used to provide a relativistic quantum calculation of the ponderomotive force due to a distribution of waves in the plasma. In 94 we show that the statistical average (again over a distribution of waves in the plasma) of the vertex function leads naturally to a new class of radiative processes that we refer to as hybrid processes. Familiar processes involve the modulus squares of relevant amplitudes: Čerenkov emission is a first-order process that is described by the square of a first-order amplitude, and Compton scattering is a second-order process that is described by the square of a second-order amplitude. The lowest order hybrid process involves the outer product of a first-order amplitude and a third-order amplitude. The resulting process is of second order, in the sense that it is the same order as Compton scattering, but is physically distinct from familiar second-order processes. Such hybrid processes exist in principle in QED, but, owing to kinematic restrictions, the lowest-order hybrid process that is allowed arises from the outer product of a second-order amplitude and a fourth-order amplitude. Thus the process itself is of third order, the same order as double Compton scattering.

2. Review of QPD

The following is summary of the steps involved in generalized QED to QPD. Natural units ($\hbar = 1$, $c = 1$) are used, so that one has $\varepsilon_0 \mu_0 = 1$ in the following. Otherwise the notation is as in Berestetskii *et al.* (1982).

2.1. *The density matrix*

The effects of a medium may be included through a statistical average over the particles or waves in the medium. This average is based on a density matrix *8,* which may be separated into factors for each species of particle and for each wave mode. For a medium consisting of only electrons and positrons, the density matrix may be written in the form $8 = \hat{w}_P \hat{w}_W$, where *8,* describes the particles and *8,* describes the waves. Assuming the random-phase approximation, in the sense that off-diagonal terms in the density matrix are ignored, one has

$$\hat{w}_P = \sum_q \prod_\epsilon w_{\epsilon q} |\epsilon q\rangle \langle \epsilon q|, \tag{1}$$

where the sum is over a set of quantum numbers $\{q\}$, and

$$\hat{w}_W = V\int\frac{d^3\mathbf{k}}{(2\pi)^3}\prod_M w_M(k)\,|M\mathbf{k}\rangle\langle M\mathbf{k}| \tag{2}$$

where the product is over a set of wave modes $\{M\}$. Explicitly, the statistical averages of an operator K over the particle and wave states are

$$\bar{K} = \mathrm{Tr}\,(\hat{w}_P \hat{K}) = \sum_{eq} (eq\,|\hat{K}|\,eq),$$

$$\bar{K}_M = \mathrm{Tr}(\hat{w}_W \hat{K}) = V\int\frac{d^3\mathbf{k}}{(2\pi)^3} w_M\,\langle M\mathbf{k}\,|\hat{K}|M\mathbf{k}\rangle \tag{3}$$

respectively.

2.2. Wave properties

The wave properties are found by solving the relevant wave equation, which is

$$\Lambda^{\mu\nu}(k)A_\nu(k) = 0, \qquad \Lambda^{\mu\nu}(k) = k^2 g^{\mu\nu} - k^\mu k^\nu + \mu_0\,\alpha^{\mu\nu}(k). \tag{4}$$

The linear response tensor $\alpha^{\mu\nu}(k)$ satisfies the charge continuity and gauge-invariance relations

$$k_\mu \alpha^{\mu\nu}(k) = 0 = k_\nu \alpha^{\mu\nu}(k). \tag{5}$$

It follows that $\Lambda^{\mu\nu}(k)$ also satisfies the relations (5). Thus k^μ is always a solution of (4). It follows that the determinant of $\Lambda^{\mu\nu}(k)$ vanishes identically, and hence the familiar procedure of finding the dispersion equation needs to be modified.

A covariant form of the dispersion equation may be found by noting that because k^μ is an eigenvalue, the matrix of cofactors, $\lambda^{\mu\nu}(k)$, of the matrix $\Lambda^{\mu\nu}(k)$ must be of the form $\lambda^{\mu\nu}(k) = \lambda(k)\,k^\mu k^\nu$, where $\lambda(k)$ is an invariant. Thus $\lambda(k) = 0$ is an invariant form of the dispersion equation. Let $k^\mu = k^\mu_M$ be a solution of the dispersion equation, interpreted as the dispersion relation for waves in a mode M. This dispersion relation may be written $\omega = \omega_M$, with ω_M implicitly a function of k through $\lambda(k_M) = 0$.

The polarization 4-vector e^μ_M for waves in the mode M is defined to be proportional the solution of (4) for $k^\mu = k^\mu_M$. Now, $\lambda(k_M) = 0$ implies that $\Lambda^{\mu\nu}(k_M)$ is of rank two, and thus its second-order cofactors $\lambda^{\mu\nu\alpha\beta}(k_M)$ span a two-dimensional subspace. This subspace is the outer product of k^μ and e^μ_M. This property, along with the fact that all relevant matrices are Hermitian, allows one to write

$$\lambda^{\mu\nu\alpha\beta}(k_M) \propto (e^\mu_M k^\nu - e^\nu_M k^\mu)(e^\alpha_M k^\beta - e^\beta_M k^\alpha)^*. \tag{6}$$

The gauge, phase and normalization of e^μ_M have yet to be specified. All relevant gauge conditions are of the form $G_\alpha e^\alpha_M = 0$, and to construct e^μ_M in this 'G gauge', one writes

$$e^\mu_M e^{*\nu}_M \propto G_\alpha G_\beta \lambda^{\mu\alpha\nu\beta}(k_M). \tag{7}$$

It is convenient to choose the temporal gauge, $G^\mu = [1, \mathbf{0}]$, because this is the only gauge in which one can ensure that $e^\mu_M e^*_{M\mu}$ has a definite signature (negative). In the temporal gauge we choose the normalization condition

$e^{\mu}_{M} e^{*}_{M\mu} = -1$. The constant of proportionality in (6) or (7) may then be incorporated into a factor

$$R_M = \left[\frac{\omega\lambda^{0\sigma}_{0\sigma}(k)}{\partial\lambda(k)/\partial\omega}\right]\Bigg|_{k=k_M}, \tag{8}$$

which may be interpreted as the ratio of electric to total energy in waves in the mode M. The amplitude for waves in the mode M is described by the Fourier transform of the 4-potential

$$A^{\mu}_{M}(k) = a_M(k)\,[e^{\mu}_{M}(k)\,(2\pi)^4\,\delta^4(k-k_M) + e^{*\mu}_{M}(k)\,(2\pi)^4\,\delta^4(k+k_M)], \tag{9}$$

with the normalization determined by

$$a_M(k) = \left[\frac{\mu_0 R_M(k) N_M(k)}{V\omega_M}\right]^{\frac{1}{2}}, \tag{10}$$

where V is the volume of the system and $N_M(k)$ is the occupation number of the wave quanta (it is the classical wave action divided by $\hbar$).

2.3. *Statistically averaged propagators*

The statistical average of a propagator may be evaluated by starting from the propagator expressed as a vacuum expectation value, writing this vacuum expectation value as an average over the density matrix $|0\rangle\langle 0|$ for the vacuum, and then replacing this by the density matrix for the system of particles and waves. Thus, using (3), the statistically averaged electron propagator may be written as

$$\bar{G}(x,x') = -i\,\mathrm{Tr}\,[\hat{w}_P\,T\{\hat{\Psi}(x)\,\hat{\bar{\Psi}}(x')\}], \tag{11}$$

where T denotes the chronological product, and $\hat{\Psi}$ is a second-quantized Dirac wavefunction. An explicit calculation of this propagator was given by Hayes & Melrose (1984), where it was shown that the Feynman propagator in momentum space becomes

$$\bar{G}(P) = \mathscr{P}\frac{\not{P}+m}{P^2-m^2} - \frac{i\pi}{2\varepsilon}\sum_{\epsilon}\delta(P^0-\epsilon\varepsilon)\,(\epsilon\not{P}+m)\,[1-2n^{\epsilon}(\epsilon\mathbf{p})], \tag{12}$$

where the 4-momentum $\mathbf{P} = (P^0, P)$ is related to the physical 4-momentum $p = (\varepsilon, p)$ by $P = \epsilon p$, and $\mathscr{P}$ denotes the Cauchy principal value. **A** concise form is

$$\left.\begin{aligned} \bar{G}(P) &= (\not{P}+m)\left[\frac{1}{P^2-m^2+i0} + i\frac{N(P)}{2m}\right], \\ N(P) &= \sum_{\epsilon=\pm 1} 4nm\&(P^2-m^2)H(\epsilon E)\,n^{\epsilon}(\epsilon\mathbf{p}), \end{aligned}\right\} \tag{13}$$

where $H(x)$ is the step function. Only the resonant part of the propagator is modified by the statistical average.

The photon propagator, which is the Green function corresponding to the wave equation (1), is gauge-dependent, and for an arbitrary gauge may be written as

$$D^{\mu\nu}(k) = -\mu_0\frac{G_\alpha G_\beta}{(Gk)^2}\frac{\lambda^{\mu\alpha\nu\beta}(k)}{\lambda(k)}. \tag{14}$$

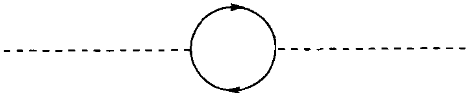

FIGURE 1. Feynman diagram for vacuum polarization

The statistical average again modifies only the resonant part of the photon propagator, which is the anti-Hermitian part. In the temporal gauge the contribution to the resonant part from waves in the mode M is

$$D_M^{A\mu\nu}(k) = i\pi\mu_0 \frac{R_{M(k)}}{\omega_M}\,[e_M^{\mu}(k)\,e_M^{*\nu}(k)\,\delta(\omega-\omega_M)+e_M^{*\mu}(k)\,e_M^{\nu}(k)\,\delta(\omega+\omega_M)]\,[1+2N_M(k)]. \tag{15}$$

2.4. The linear response tensor

The vacuum polarization tensor is calculated from the Feynman amplitude for the 'bubble' diagram in figure 1. The resulting expression for the vacuum polarization tensor is

$$\alpha^{\mu\nu}(k) = -ie^2\,\mathrm{Tr}\int \frac{d^4P}{(2\pi)^4}\frac{\gamma^{\mu}(\not{P}+m)\,\gamma^{\nu}(\not{P}+m)}{[P^2-m^2+i0]\,[(P-k)^2-m^2+i0]}. \tag{16}$$

The only physically relevant contribution comes from the two terms in which one of the factors in the denominator is replaced by its resonant part and the other by its non-resonant part. Replacing the resonant parts by their statistical average, in accord with (13), and discarding the vacuum term, gives the linear response tensor in the form (Hayes & Melrose 1984)

$$\alpha^{\mu\nu}(k) = \frac{e^2}{2m}\int\frac{d^4P}{(2\pi)^4}N(P)\left[\frac{F^{\mu\nu}(P,P')}{P'^2-m^2}+\frac{F^{\mu\nu}(P,P'')}{P''^2-m^2}\right], \tag{17}$$

$$F^{\mu\nu}(P,P') = P^{\mu}P'^{\nu}+P'^{\mu}P'^{\nu}+g^{\mu\nu}(m^2-PP'), \tag{18}$$

with $P' = P-k$ and $P'' = P+k$. The linear response tensor for a thermal distribution has been evaluated both by starting from the form (17) (see e.g. Tsytovich 1961; Melrose and Hayes 1984) and by starting from the propagator averaged over a thermal distribution (see e.g. Weldon 1982; Kowalenko et al. 1985).

3. The electron mass operator and the ponderomotive force

The self-energy of the electron is described by the diagram shown in figure 2. In QED this diagram leads to divergent contributions that must be removed by renormalization. Tsytovich (1962) discussed the use of the amplitude for the diagram in figure 2 to calculate macroscopic mass renormalization (MMR). MMR is independent of the presence of waves in the medium. An idealized example of MMR is for an electron at rest in a thermal plasma with a Debye length λ_D. Compared with the electric field in vacuo, the electric field in the plasma is shielded at distance $> \lambda_D$. Hence the total energy in the electric field is smaller than for an electron in vacuo. This leads to MMR with the mass of the electron in the plasma smaller than the mass in *vacuo* by about $r_e m_e/2\lambda_D$, where r_e is the classical radius of the electron. Tsytovich (1962) considered the

FIGURE 2. Feynman diagram for the self-energy of the electron.

generalization of this effect by replacing the photon propagator *in vacuo* in the expression for the vertex correction by the propagator, cf. (14), in the medium. Here we consider the contribution of this diagram when the statistical average is performed over a distribution of waves using *(15)*. This describes a different effect to the MMR considered by Tsytovich *(1962).*

3.1. The mass operator

The Feynman amplitude for the electron self-energy diagram (figure 2) leads to the identification of the mass operator

$$\Sigma(p) = ie^2 \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^\mu(\not{p}-\not{k}+m)\gamma^\nu}{(p-k)^2-m^2} D_{\mu\nu}(k). \tag{19}$$

In the electron propagator the denominator $1/(\not{p}-m)$ is replaced by $1/[\not{p}+\Sigma(p)-m]$. This leads to a correction $m \to m+\delta m$ to the electron mass given by $[\not{p}+\Sigma(p)]^2 = (m+\delta m)^2$. Assuming $|\delta m| \ll m$, this gives

$$\delta m = -\frac{1}{2m}[\not{p}\Sigma(p)+\Sigma(p)\not{p}]. \tag{20}$$

In evaluating (20), first note that the mass operator is always evaluated for a Dirac wavefunction Ψ, and one may simplify by using the Dirac equation $(\not{p}-m)\Psi = 0$ and its adjoint. Combining this with standard relations for the Dirac matrices allows one to replace the numerator in *(19)* in accordance with

$$\gamma^\mu(\not{p}-\not{k}+m)\gamma^\nu \to (p-k)^\mu\gamma^\nu+(p-k)^\nu\gamma^\mu+g^{\mu\nu}\not{k}+i\varepsilon^{\mu\alpha\nu\rho}(\not{p}\gamma_\rho-\gamma_\rho\not{p})\gamma^5. \tag{21}$$

Then one finds

$$\delta m = -\frac{ie^2}{m}\int \frac{d^4k}{(2\pi)^4} \frac{p^\mu p'^\nu+p^\nu p'^\mu+g^{\mu\nu}(m^2-pp')+i\varepsilon^{\mu\alpha\nu\rho}(\not{p}\gamma_\rho-\gamma_\rho\not{p})\gamma^5}{(p-k)^2-m^2} D_{\mu\nu}(k), \tag{22}$$

with $p' = p-k$.

3.2. The potential energy due to the waves

The additional mass *(22)* for each particle may be integrated over the distribution of particles to find the correction to the proper mass density in the particles. This gives

$$n_{\rm pr}\overline{\delta m} = -\frac{ie^2}{m}\mathrm{Tr}\int \frac{d^4p}{(2\pi)^4}N(p)\int \frac{d^4k}{(2\pi)^4} \frac{p^\mu p'^\nu+p^\nu p'^\mu+g^{\mu\nu}(m^2-pp')}{(p-k)^2-m^2} D_{\mu\nu}(k), \tag{23}$$

where $n_{\rm pr}$ is the proper number density, the overbar denotes a mean value, and we have removed the final term of *(21)* through standard properties of the trace of Dirac gamma matrices. Inserting the expression *(15)* for the photon propagator and retaining only the term proportional to N_M, the resulting quantity may be interpreted as a form of potential energy in the particles associated with the presence of waves in the mode M. Denoting this by U_M, and using the identification (17) with (18) of the linear response tensor, one finds

$$U_M = \frac{n_{\rm pr}\,\overline{\delta m}}{V} = \frac{\mu_0}{2V}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{R_M N_M(k)}{\omega_M} e^*_{M\mu} e_{M\nu} \alpha^{\mu\nu}(k_M). \tag{24}$$

Using the wave equation (4) with the wave amplitude given by (9) with (10), (24) may be written in the form

$$U_M = \frac{\varepsilon_0}{2}\int \frac{d^3\mathbf{k}}{(2\pi)^3}(\omega_M^2 - |\mathbf{k}\times\mathbf{e}_M|^2)\,|a_M(k)|^2. \tag{25}$$

The right-hand side of (25) may be interpreted in terms of the quantity $\frac{1}{2}\varepsilon_0(|\mathbf{E}|^2-|\mathbf{B}|^2)$, which is an invariant constructed from the Maxwell tensor, averaged over space and time and evaluated from waves in the mode M with amplitude given by (9). For longitudinal waves, for which k x $\mathbf{e}_M = \mathbf{0}$, (25) is the electric energy density in the waves.

3.3. The *ponderomotive* 4-force

The potential U_M has a simple interpretation in terms of the ponderomotive force due to the waves (Landau & Lifshitz 1984; Melrose 1986). In a covariant treatment of the ponderomotive force (Manheimer 1985; Achterberg 1986), the energy–momentum tensor $T_b^{\mu\nu}$ for the background plasma experiences a 4-force such that $\partial_\mu T_b^{\mu\nu} = \partial^\nu U_M$. Covariant classical calculations readily reduce to the form (24) or (25) for U_M (Luo & Melrose 1993). Thus the mass operator in quantum plasmadynamics allows one to include the effect of the waves on the background subsystem in an analogous manner to the classical theory of Dewar (1977).

4. Hybrid radiation processes

The third radiative correction that we consider is the vertex correction, corresponding to the Feynman diagram in figure 3.

4.1. The vertex correction

The amplitude for figure 3 leads to the vertex correction

$$\Gamma^\mu(P,k) = -ie^2\int \frac{d^4k'}{(2\pi)^4}\gamma^\rho G(P-k')\,\gamma^\mu G(P-k-k')\,\gamma^\tau D_{\rho\tau}(k'), \tag{26}$$

which is to be added to γ^μ; that is, γ^μ is to be replaced by $\gamma^\mu + \Gamma^\mu(P,\mathrm{k})$ when treating specific processes. We are interested only in the contribution to (26) from the statistical average over a distribution of waves in the mode M'. (We chose to label the mode M' because we consider the effect of these waves on emission of waves in another mode, for which the label M is reserved.) This leads to a correction $\Gamma^\mu_{M'}(p,k)$, due to the presence of waves in the mode M', given by

$$\Gamma^\mu_{M'}(p,\mathrm{k}) = V\mu_0 \mathrm{e}^2\int \frac{d^4k'}{(2\pi)^4}|a'_M(k)|^2\gamma^\rho[G(p-\mathrm{k'})\,\gamma^\mu G(p'-\mathrm{k'}) + G(p+\mathrm{k'})\,\gamma^\mu G(p'+k')]\,\gamma^\tau e_{M'\mu} e^*_{M'\nu}, \tag{27}$$

with $p' = p - \mathrm{k}$ and $k' = [\omega'_M(\mathbf{k}'), \mathrm{k'}]$.

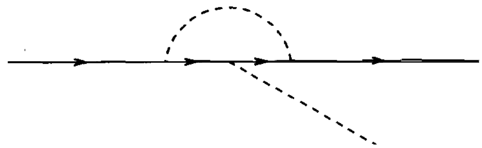

FIGURE 3. The vertex correction.

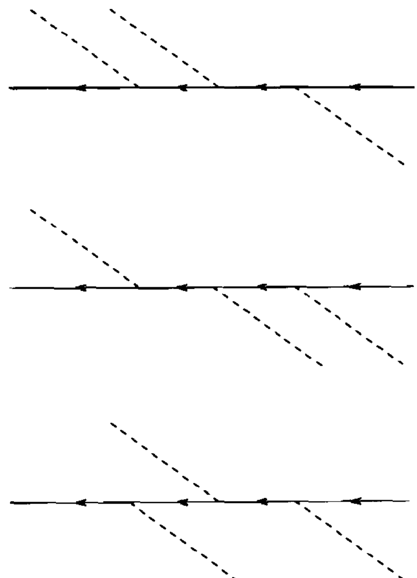

FIGURE 4. The three diagrams that contribute to double Compton scattering. For the hybrid process considered here, two of the photon lines correspond to emission and absorption of identical wave quanta.

There are additional corrections of the same order as the vertex correction that need to be included. These correspond to electron self-energy correction (figure 2) to emission of a wave quantum. To see this, note that the resonant contribution to a propagator corresponds to cutting the relevant line in the Feynman diagram. This follows from the fact that the resonant term corresponds to a real particle or wave quantum, so that the dashed semicircles in figures 2 or **3** are replaced by two photon lines corresponding to emission and absorption of identical wave quanta. Thus there are three distinct diagrams, as illustrated in figure 4, leading to four additional terms that are combined with the two terms of (27). This corresponds to a statistical average over the photon lines in double Compton scattering (DCS). Further simplification may be made using the standard methods applied to DCS (see e.g. Jauch & Rohrlich 1975) or, more sensibly, to its classical counterpart (Melrose 1972). We now argue that this corresponds to a hybrid process to a type not usually considered in QED.

4.2. Modification of Čerenkov emission

The vertex correction (27) corresponds to a third-order diagram that has the same three external moments as the basic (first-order) vertex diagram. Thus the amplitude for this third-order diagram should be added to that of the first-order diagram when considering cerenkov emission and related crossed processes. When determining the rate of such processes, the additional term leads to two cross-terms in the square of the total amplitude. These hybrid first–third-order processes are of the same order as a second-order process such as Compton scattering. From a classical viewpoint, such a process may be interpreted as modifying Čerenkov emission of waves in one model to take account of the perturbations in the motion of the emitting particle due to the presence of waves in another wave mode.

The hybrid process corresponding to cerenkov emission follows from the known formula for Čerenkov emission, calculated using QED (see e.g. Ginzburg **1940**), by replacing γ^μ in the amplitude $\bar{u}(\mathbf{p}')\gamma^\mu u(\mathbf{p})$ by $\gamma^\mu+\Gamma^\mu_{M'}(p,\mathrm{k})$, and keeping only the cross-terms when evaluating the rate. Writing the rate per unit time for this process,

$$w_{i\to f} = V(2\pi)^4\delta^4(p'+\mathrm{k}-\mathrm{p})|M_{fi}|^2\frac{|a_M(k)|^2}{2\varepsilon V 2\varepsilon' V}\frac{V d^3\mathrm{p}'}{(2\pi)^3}\frac{V d^3\mathrm{k}}{(2\pi)^3}, \tag{28}$$

in the form

$$w_{i\to f} = w_M(p,k)\,(2\pi)^3\delta^3(\mathbf{p}'+\mathbf{k}-\mathbf{p})\frac{d^3\mathbf{p}'}{(2\pi)^3}\frac{d^3\mathbf{k}}{(2\pi)^3}, \tag{29}$$

to define the probability $w_M(p,\mathrm{k})$ of cerenkov emission, one has

$$w_M(p,k) = |M_{fi}|^2\frac{V|a_M(k)|^2}{2\varepsilon 2\varepsilon'}2\pi\delta(\varepsilon'+\omega_M-\varepsilon), \tag{30}$$

with, after averaging over the initial states of polarization and summing over the final states of polarization for the electron,

$$|M_{fi}|^2 = e^2 e^*_{M\mu}e_{M\nu}\frac{\mu_0 R_M}{V\omega_M}\tfrac{1}{2}\mathrm{Tr}\,[(\not{p}'+m)\,\gamma^\mu(\not{p}+m)\,\gamma^\nu], \tag{31}$$

with p' $=\mathrm{p}-\mathrm{k}$. For the hybrid process, (27) is modified according to

$$\tfrac{1}{2}\mathrm{Tr}\,[(\not{p}'+m)\,\gamma^\mu(\not{p}+m)\,\gamma^\nu]\to\tfrac{1}{2}\mathrm{Tr}\,[(\not{p}'+m)\,\gamma^\mu(\not{p}+m)\,\Gamma^\nu_{M'}(p,k) + (\not{p}+m)\,\Gamma^\mu_{M'}(p,k)\,(\not{p}+m)\,\gamma^\nu]. \tag{32}$$

On inserting the expression (27) for $\Gamma^\mu_{M'}(p,\mathrm{k})$, (32) gives a correction to the probability for Čerenkov emission that is linear in the occupation number of the waves in the mode M'. Note that the modification to the emission of waves in the mode M by waves in the mode M' has no effect whatsoever on the waves in the mode M'; as implied by figure **3,** these waves are emitted and reabsorbed in identical pairs.

The existence of such hybrid processes was noted (Kuijpers & Melrose 1985; Melrose & Kuijpers 1987) in connection with a controversy concerning the existence of turbulent breamsstrahlung. In a plasma the presence of ion sound waves ($M'=S$) affects the emission of Langmuir waves ($M=\mathrm{L}$). Similarly, the presence of Langmuir waves affects the emission of ion sound waves. Either of these processes may be treated using the foregoing theory (or, more sensibly, a classical version of it). One form of the argument by Kuijpers and Melrose against the original proposal for turbulent bremsstrahlung (Tsytovich et al.

1975) is that it involved an incorrect analysis of the latter process owing to a failure to include the symmetry property that ensures that the Langrnuir waves are emitted and absorbed in identical pairs.

5. Conclusion

Our conclusions are as follows.

(i) QED may be generalized to QPD by replacing the propagators by their statistical averages, in which only the resonant part is modified. The statistical averages of the closed fermion loops in the 'bubble', 'triangle' and 'box ' lead to expressions for the linear, quadratic and cubic response tensors of the plasma respectively.

(ii) The statistical average of the photon propagator applied to the mass operator leads to a mass correction that may be used to calculate a potential associated with the waves. This potential reproduces a known covariant expression for the ponderomotive force.

(iii) The statistical average of the photon propagator applied to the vertex correction leads to a new class of emission processes, referred to here as hybrid processes. In a plasma the simplest example is modification to Čerenkov emission of waves in one mode due to the presence of waves in another mode. Such hybrid processes are not normally considered in QED, but they do exist. The lowest-order such process allowed *in vacuo* is a hybrid second–fourth order, which is of the same net order as double Compton scattering. It involves the effect of waves already present modifying Compton scattering. A classical treatment of this effect, for example one that complements a classical treatment of double Compton scattering (Melrose 1972), is straightforward but is not given here.

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