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Induced Three-wave Interactions in Eclipsing Pulsars

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Abstract: Three-wave interactions involving two high-frequency waves (in the same mode) and a low-frequency wave are discussed and applied to pulsar eclipses. When the magnetic field is taken into account, the low-frequency waves can be the w-mode (the low-frequency branch of the ordinary mode) or the z-mode (the low-frequency branch of the extraordinary mode). It is shown that in the cold plasma approximation, effective growth of the low-frequency waves due to an anisotropic photon beam can occur only for z-mode waves near the resonance frequency. In the application to pulsar eclipses, the cold plasma approximation may not be adequate and we suggest that when thermal effects are included, three-wave interaction involving low-frequency cyclotron waves (e.g. Bernstein modes) is a plausible candidate for pulsar eclipses

Keywords: radio continuum: binaries: eclipsing pulsars

1. Introduction

The observations of some eclipsing pulsars, e.g. PSR 1957 + 20 (Fruchter et al. 1988a), PSR 1744–24A (Lyne et al. 1990), show that intense radio emission is periodically eclipsed by the plasma in the companion wind (Fruchter et al. 1990; Gedalin & Eichler 1993). Several mechanisms have been proposed to explain these eclipses. These include (i) refractive and reflective mechanisms (Phinney et al. 1988; Emmering & London 1990), (ii) absorption models, e.g. free–free absorption (Wasserman & Cordes 1988; Rasio et al. 1989, 1991) and cyclotron absorption (Thompson et al. 1994), and (iii) induced scattering models, e.g. induced Raman scattering (Gedalin & Eichler 1993; Melrose 1994). The plasma density derived from the measurements of the excess time delay due to plasma dispersion in the companion wind is too low to produce substantial refraction and deflection (Fruchter et al. 1990). Moreover, the refraction and deflection model (Phinney et al. 1988) cannot predict a correct frequency dependence of eclipse duration. Among the absorption models, free–free absorption requires a rather cold wind with a plasma temperature in the eclipse region as low as $T_e \approx 300$ K. The cold wind assumption appears inconsistent with the observational evidence that the companion wind may be driven and undergo substantial heating by the strong pulsar wind (Fruchter et al. 1988b; Phinney et al. 1988; Cheng 1989; Harding & Gaisser 1990). Cyclotron absorption is effective only for a relatively strong magnetic field, i.e. much higher than the upper limit set by the measurements of Faraday rotation. The inferred upper limit on the parallel magnetic field at the entrance and exit of eclipse is $\sim$few$\times 10^{-4}$ T (Fruchter et al. 1990); however, to produce cyclotron absorption the magnetic field must exceed 5×10^{-3} T and at the same time the temperature must be $\gtrsim 10^8$ K (Thompson et al. 1994).

Among these proposed mechanisms, induced scattering seems the most plausible for pulsar eclipses on the following grounds. (i) Pulsar radio emission has a high brightness temperature and induced scattering processes can be important when the intense radio waves traverse a nonrelativistic wind plasma (from the companion). (ii) Pulsar radio emission is beamed and it can be highly anisotropic. An anisotropic photon beam may cause instability of low-frequency waves in a way similar to that in which an anisotropic electron beam causes growth of plasma waves. A specific induced scattering model for pulsar eclipses is induced Raman scattering, proposed by a number of authors, e.g. Eichler (1991), Gedalin & Eichler (1993), Melrose (1994) and Thompson et al. (1994). The model is based on an assumption that the magnetic field in the eclipse region is not important (the plasma frequency ω_p is assumed to be much higher than the cyclotron frequency Ω_e). In practice, the magnetic field in the eclipse region (Gedalin & Eichler 1993; Thompson et al. 1994) may contribute a substantial fraction of the pressure

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needed to balance the pulsar winds, since with the plasma density estimated from the dispersion measure one finds that the ram pressure from the companion wind without magnetic field appears not large enough to hold off the pulsar wind, cf. Gedalin & Eichler (1993). One may estimate the magnetic field by measuring the Faraday delay between two (opposite) circularly polarised signals (Manchester & Taylor 1977). For PSR 1957+20, observation gives an upper limit on the parallel magnetic field in the eclipse region, of about 2×10^{-4} T.

The purpose of this paper is to investigate magnetised three-wave interactions for plasma conditions relevant to eclipsing pulsars in the cold plasma approximation. The full theory including thermal motion is discussed elsewhere (Luo & Melrose, in preparation). The three-wave processes considered here involve a high-frequency wave being scattered into another high-frequency wave by a low-frequency wave. In the cold plasma approximation, the low-frequency waves can be in the (whistler) w-mode, which is the low-frequency branch of the ordinary mode, or in the z-mode, which is the low-frequency branch of the extraordinary mode. Using the quasilinear approximation in which the frequency and wave number of the high-frequency waves are assumed to be much larger than those of the low-frequency waves (e.g. Tsytovich 1970; Melrose 1986), we show that an anisotropic photon beam can induce instability of low-frequency waves in the w-mode or the z-mode. However, effective growth can occur only for the z-mode near the resonance frequency, where the waves are approximately electrostatic. In the application to pulsar eclipses, since the exact field geometry is not known, we consider three cases: $\omega_p \gg \Omega_e$, $\omega_p \sim \Omega_e$, $\omega_p < \Omega_e$. Since the plasma density and magnetic field may have different radial scalings, the three cases should predict different frequency dependences for the eclipse duration. In principle, it is thus possible to differentiate these three cases by comparing the predicted frequency dependence to that inferred from the observations.

In Section 2, the three-wave interaction theory is discussed. The three-wave interaction model for a pulsar eclipse is discussed in Section 3, and the results are summarised in Section 4.

2. Three-wave Interactions

In the following discussion we use the random phase approximation (e.g. Melrose 1986; Gedalin & Eichler 1993; Thompson et al. 1994), i.e. the waves in an arbitrary mode σ are regarded as a collection of wave quanta described in terms of an occupation number $N^\sigma(\boldsymbol{k})$. The energy and momentum of the wave quanta are given by $\hbar\omega$ and $\hbar\boldsymbol{k}$, respectively (where ω and $\boldsymbol{k}$ are the frequency and wave vector). Let σ, σ' and σ'' represent three waves (modes). Then three-wave processes correspond to $\sigma \leftrightarrow \sigma' + \sigma''$. The three-wave beat conditions may be interpreted as the energy and momentum conservation relations, $\omega = \omega' \pm \omega''$, $\boldsymbol{k} = \boldsymbol{k}' \pm \boldsymbol{k}''$. Any three waves can interact provided that they satisfy these beat conditions. However, the plasma conditions derived from the eclipsing pulsar observations constrain the choice of the three waves. Specifically, the pulsar radiation is in a high-frequency mode, with frequency much greater than the natural frequencies of the plasma. Then the only relevant three-wave interactions involve a high-frequency wave interacting with a low-frequency wave to produce a scattered high-frequency wave.

Under these conditions, that is, when the frequency and wave number of the low-frequency waves are much smaller than those of relevant high-frequency waves, the three-wave interaction is similar to a wave–particle interaction. The high-frequency photons act like particles in that they interact with the low-frequency waves in the following two ways: (i) absorption (or amplification) of low-frequency waves by the photon beam, and (ii) diffusion of the photon beam in $\boldsymbol{k}$-space as a result of interaction with the low-frequency waves. For (i), analogous to electrons interacting with the low-frequency waves, using the three-wave beat conditions with $\omega \approx \omega' \gg \omega''$, $k \approx k' \gg k''$, one may define a Cerenkov cone angle χ_c by $\cos\chi_c = \omega''/k''v_g$, where $\boldsymbol{v}_g$ is the group velocity of the high-frequency photons (Figure 1).

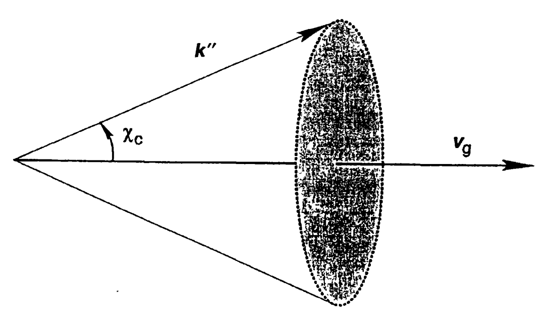

Figure 1—The Cerenkov cone, where the Cerenkov angle χ_c is defined by $\cos\chi_c = \omega''/k''v_g = c/n''v_g$ with $n'' = k''c/\omega''$.

We consider a simple case in which the angular dependence of the high-frequency photon occupation number is separable, with $N(\boldsymbol{k}) = N(k)b(\theta)$, where $N(k)$ depends only on the wave number k and $b(\theta)$ is the angular distribution. The low-frequency wave instability can occur when (i) $N(k)$ has a positive slope in some range of k, e.g. it has a peak at $k_{\max}$, or (ii) the angular distribution is anisotropic. A positive slope of $N(k)$ can give rise to wave growth and a negative slope to damping. When $N(\boldsymbol{k})$ is highly anisotropic, an instability can occur even when $N(k)$ has a negative slope, such as in the case of a power-law distribution (cf. Melrose 1986). In the following discussion, both conditions for wave growth are assumed to be satisfied.

One example of photon-beam induced instability of low-frequency waves is induced Raman scattering, in which the low-frequency waves are Langmuir waves with frequency $\omega'' = \omega_{\rm p}$ (e.g. Gedalin & Eichler 1993; Melrose 1994; Thompson et al. 1994; Luo & Melrose 1994). In the presence of a magnetic field, the low-frequency waves can be w-mode or z-mode. In the cold plasma approximation one may define the resonance frequencies $\omega_{\pm}^2 = (\omega_{\rm p}^2 + \Omega_{\rm e}^2)/2 \pm [(\omega_{\rm p}^2 + \Omega_{\rm e}^2)^2 - 4\omega_{\rm p}^2\Omega_{\rm e}^2 \cos^2\theta]^{\frac{1}{2}}/2$, at which the square of the refractive index is infinite. The frequencies of the w- and z-modes are within the range $\omega'' < \omega_-$ and $\omega_z < \omega'' < \omega_+$, respectively, where $\omega_z = [(4\omega_{\rm p}^2 + \Omega_{\rm e}^2)^{\frac{1}{2}} - \Omega_{\rm e}]/2$ is the cutoff frequency. In the case of eclipsing pulsars, the high-frequency waves (corresponding to pulsar radio emission) have $\omega \gg \omega_{\rm p}$, $\Omega_{\rm e}$. When $k \approx k' \gg k''$, in order to satisfy the beat conditions, the refractive index of the low-frequency waves must not be too large, i.e. $n'' \ll \omega/\omega''$.

3. Application to Pulsar Eclipses

A three-wave interaction model for pulsar eclipses includes (i) a photon beam with high brightness temperature and anisotropic angular distribution that causes growth of the low-frequency waves, and (ii) scattering of the photon beam by the low-frequency waves, provided that these processes occur within a sufficiently short timescale. An eclipse can result from the plasma being translucent due to the enhanced scattering by the low-frequency waves.

Assume that the photon beam is confined to a small angle $\theta_0 \ll 1$, specifically $b(\theta) = \exp(-\theta^2/2\theta_0^2)$. The growth rate (per unit time) of the low-frequency waves is then described by (Luo & Melrose 1994)

$$\Gamma \approx \frac{1}{2\pi}\left(\frac{\omega^2 W}{2\pi c^3}\right) N(k)\,, \qquad (1)$$

with $W = \hbar R |e_i^* e_j' e_s'' \alpha_{ijs}|^2/\varepsilon_0^3 \omega^2 \omega''$. The quantity $2\pi W \delta(\omega'' - k'' v_{\rm g} \cos\chi_{\rm c})$ can be interpreted as the probability of emission of a low-frequency wave by the high-frequency photon [cf. equation (2) in Luo & Melrose 1994]. The tensor α_{ijs} is the quadratic response tensor (Melrose 1987). Since $\omega'' \ll \omega \sim \omega'$, $k'' \ll k \sim k'$, we may use the approximation $\alpha_{ijl} \approx (\varepsilon_0 e\omega''/m_e) k_r''(K_{rl} - \delta_{rl})\delta_{ij}$, where the sum over r is implied and where $K_{rl} = K_{rl}(\omega'')$ is the cold plasma dielectric tensor (where the contribution to the dielectric tensor from ions is neglected). The ratio of the electric to the total energy in the low-frequency waves is denoted by R. The relevant ratios for the high-frequency waves are all assumed to be 1/2 since they can be regarded as the pure electromagnetic waves. The (unimodular) polarisation vectors of the three waves are represented by $\boldsymbol{e}$, $\boldsymbol{e}'$ and $\boldsymbol{e}''$, respectively (the double-primed one corresponds to the low-frequency waves). In deriving (1) we assume that the low-frequency wave vector $\boldsymbol{k}''$ is within the Cerenkov cone and we take $\theta'' - \chi_{\rm c} \approx -\theta_0$ (cf. Melrose 1994; Luo & Melrose 1994). From (1) we have the maximum growth rate for the low-frequency waves,

$$\Gamma_{\max} \approx \eta n''^2 \left(\frac{T_{\rm B}}{m_{\rm e}c^2}\right)\left(\frac{r_{\rm e}\omega^2}{2\pi c}\right)\left(\frac{\omega_{\rm p}}{\omega}\right)^3, \qquad (2)$$

where $T_{\rm B}$ is the brightness temperature of the photon beam, $r_{\rm e}$ is the classical electron radius ($r_{\rm e} \approx 2\cdot 82 \times 10^{-15}$ m), and $\eta = 2R\,|(\mathbf{e}\cdot\mathbf{e}')\hat{k}_i(K_{ij} - \delta_{ij})e_j''|^2(\omega''/\omega_{\rm p})^3$.

The diffusion timescale can be estimated semiquantitatively from the diffusion coefficient (as discussed by Melrose 1994):

$$t_{\rm d} \approx 1/(\Delta\Omega n''^2 \varrho \Gamma_{\max})\,, \qquad (3)$$

where ϱ is the ratio of the energy densities in the low-frequency and the high-frequency waves. The high-frequency photon beam is assumed to be confined to a solid angle $\Delta\Omega \approx \pi\theta_0^2$.

A large growth rate for the low-frequency waves and a strong diffusion of the photon beam occur preferentially for a large refractive index, since $\Gamma_{\max} \propto n''^2$ and $t_{\rm d} \propto 1/n''^4$. This implies that the low-frequency waves propagate approximately perpendicular to the high-frequency photon beam because the Cerenkov angle $\chi_0 = \arccos(c/n'' v_{\rm g}) \sim \pi/2$ for large n''. On the other hand, the refractive index n'' cannot be too large, otherwise damping due to the Cerenkov resonance becomes important. The other constraint on n'' is the condition $n'' \ll \omega/\omega''$ as mentioned above.

In the weak field limit $\omega_{\rm p} \gg \Omega_{\rm e}$, one has $\eta = 1$ and the growth rate (2) reduces to that derived by Melrose (1994). In this case Landau damping is the major constraint on the growth. Landau damping sets an upper limit on n'' given by $n'' < n_0'' = k_0'' c/\omega''$ with $k_0'' \approx 0\cdot 3/\lambda_{\rm D}$, where

$\lambda_D = 0{\cdot}069(T_e/10^6\ \mathrm{K})^{\frac{1}{2}}/(n_e/10^{12}\ \mathrm{m}^{-3})^{\frac{1}{2}}$ m is the Debye length. For $T_e = 10^6$ K and $n_e = 10^{12}\ \mathrm{m}^{-3}$ we have $n_0'' = 21$ (where $\omega'' \approx \omega_p$).

We examined the growth rates for the w- and z-modes for different field configurations since there are uncertainties on the magnetic field structure and its magnitude in the eclipse region (the Faraday rotation measurements give only partial information). The cyclotron frequency appears in (2) only through η, which we evaluate numerically for different cyclotron frequencies. The results are shown in Figure 2. As an example, the plasma frequency is $\omega_p/2\pi = 8$ MHz, the refractive index is $n'' = 10$, and we assume that the polar axis of the photon beam lies in the $\hat{x}$–$\hat{z}$ plane (the magnetic field is along $\hat{z}$) and has an angle $\pi/4$ to the magnetic field. Since the Cerenkov angle is approximately $\pi/2$ for $n'' = 10$, this means that $\boldsymbol{k}''$ has about a $\pi/4$ angle to the field but with π difference in azimuthal angle (relative to the magnetic field). In Figure 2, we also assume that $\boldsymbol{e}.\boldsymbol{e}' \sim 1$. Compared with the unmagnetised case (Raman scattering), the growth rate for the z-mode increases as the magnetic field increases. The growth rate for the w-mode is much less than for the z-mode (in Figure 2, the solid and dashed curves correspond to the z- and w-modes, respectively). Thus in the application to pulsar eclipses, the three-wave interaction involving low-frequency w-mode waves is unlikely to be important.

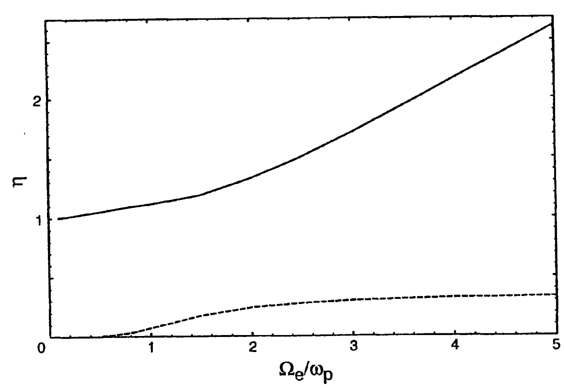

Figure 2—Plots of η, cf. equation (2), against the cyclotron frequency for $\omega_p/2\pi = 8$ MHz and $n'' = 10$. The solid curve and dashed curve correspond to the z- and w-modes, respectively.

4. Discussion and Conclusions

We discuss three-wave interactions involving two high-frequency waves and a low-frequency magnetoionic wave under the plasma conditions relevant to eclipsing pulsars. In the cold plasma approximation, the low-frequency waves can be w-mode or z-mode. Which low-frequency waves are responsible for scattering of high-frequency photons depends on the details of the plasma conditions in the eclipse region. Raman scattering is important only in the weak field limit (except in the case of Langmuir waves propagating along the field lines, when the scattering is similar to that in the unmagnetised case). In reality the magnetic field in the scattering region can be large in the sense that the cyclotron frequency can be comparable to, or even higher than, the plasma frequency. For $\Omega_e \gtrsim \omega_p$, scattering by the low-frequency z-mode near the resonance frequency ω_+ is important (see Figure 2). Since the growth rate for the z-mode waves depends on the plasma density as well as the cyclotron frequency, the three-wave interactions involving low-frequency z-mode waves imply a flatter frequency dependence for eclipse duration than does Raman scattering. For PSR 1957+20, Raman scattering predicts a steeper frequency dependence than is observed, suggesting that including the magnetic field should improve the agreement between theory and observation.

Since the temperature of the eclipsing plasma can be as high as 10^6 K (e.g. Fruchter et al. 1990), the cold plasma description may not be adequate. Including the magnetic field and thermal effects, three-wave interactions involving low-frequency (electron) cyclotron waves, e.g. Bernstein waves (Bernstein 1958; Lominadze 1981), can be important. We discuss this possibility elsewhere (Luo & Melrose, in preparation).

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