good fits are still obtained, but only when the value for Σ_0 is lowered, so the fits need three parameters again. The reason for this is that the central acceleration in these galaxies is already below A_0, so a fixed value for Σ_0 for all galaxies cannot work.

What can one conclude from the above? Clearly, MOND remains the most 'efficient' way of describing rotation curves, but it is in a way impressive that for normal-sized spirals one can get good one-parameter fits to the rotation curve using a plausible form for the halo. By taking the light profile of the disk and only adjusting its mass-to-light ratio, one describes the rotation curve also in the outer parts where it is completely dark-matter-dominated. But the fact that this does not work for lower-mass spirals makes the scheme, of course, less general. In any case, I think that it should give people food for thought. Does it make sense that dark halos in normal spirals should have the same Σ_0? Is it true for more galaxies than the ones used by Sanders & Begeman? Is there a systematic trend in Σ_0 when going to lower-mass spirals? Perhaps by thinking about these questions, or in general why MOND does so well in fitting rotation curves, we will learn something more about the dark halos in galaxies.

The Magnetic Alternative

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A magnetic field provides either an isotropic pressure $B^2/2\mu_0$ plus a tension B^2/μ_0 along the field lines, or a perpendicular pressure $B^2/2\mu_0$ plus a net tension $B^2/2\mu_0$. Magnetic tension pulls inward on concave field lines, so that hooped field lines can provide a centripetal force. In principle, the centripetal force due to magnetic tension could exceed the gravitational force, and so determine the rotation curve of a galaxy. If this is the case, and one attempts to infer the distribution of matter from the rotation curve assuming that only gravity is present, then one would incorrectly deduce that dark matter is present.

The mathematical model of Battaner et al. (1992, Nature, 360, 652) involves the following assumptions: cylindrical symmetry, azimuthal flow, $v = v_\phi$, and azimuthal magnetic field, $B = B_\phi$. Then with a pressure, P, density, ρ, and gravitational potential, U, the radial component of the equation of motion is

$$\rho\left[-\frac{\partial U}{\partial r}+\frac{v^2}{r}\right]-\frac{\partial P}{\partial r}-\frac{1}{2\mu_0 r^2}\,\frac{\partial(r^2B^2)}{\partial r}=0\,. \qquad (1)$$

Their actual model is obtained by integrating this equation, times r^2, from r_0 to r. This determines B as a function of r.

The parameters used in the model are $r_0 = 5$ kpc, $B_0 = 0{\cdot}1$ nT $= 1\,\mu$G, complemented by observational data on M31. The form of B influences the synchrotron profile, and they use this to constrain the model. They argued that the synchrotron emission requires relativistic electrons with continuous acceleration, diffusion and synchrotron losses taken into account. This model implies an energy density in relativistic electrons $N_0 = \text{constant} \times (\rho/B)$. The constant of proportionality is chosen to fit the observed synchrotron emission at 10 kpc in M31.

A seemingly crucial step in their argument involves taking the asymptotic limit ($r \to \infty$). The authors assumed that the potential U is due to a point mass, and that ρ and P fall off exponentially with r. This implies the asymptotic form $B \propto r^{-1}$, which they claim to be 'the B-profile that renders the outer rotation curve flat.'

James Binney's commentary on Battaner et al. (1992) begins with a remark that in the 1950s, spiral structure was attributed to magnetism, as were intergalactic filaments (Binney 1992, Nature, 360, 624). Both are now attributed to gravity alone. He then commented that a field of 1 nT (correcting an error in his units) is dynamically significant. If so, then magnetic pressure cannot be ignored. The pressure in the direction perpendicular to the hoops should cause the magnetic hoops to fatten into doughnuts, implying a flaring of the disk. There is evidence for galactic disks flaring at their edges but, Binney asks, 'as rapidly as the model of Battaner et al. requires?'

My comment on the conclusions of Battaner et al. is that the mathematical justification for a flat rotation curve is wrong. In fact $B \propto r^{-1}$ is a solenoidal field which is force-free, and there is zero net tension. (The net magnetic force is inward if B decreases with radial distance more slowly than $1/r$, and is outward if B decreases less slowly than $1/r$.) Their solution of (1) follows because the first two terms are assumed negligible for $r \to \infty$, due here to the assumed exponential decrease in $\rho(r)$ and $P(r)$, so that the final term must also vanish. This indeed gives $B \propto 1/r$, but implies nothing at all about $v(r)$. It is incorrect to assert that $B \propto r^{-1}$ is 'the B-profile that renders the outer rotation curve flat.' In fact, $v(r)$ depends on $\rho(r)$ and on a deviation of $B(r)$ from a solenoidal field.

Does the model makes sense in spite of this error? Forget all the assumptions except one: beyond some radius $r = r_1$, suppose that magnetic effects dominate, that is, $B^2/2\mu_0 > \frac{1}{2}\rho v^2$. Then magnetic field lines are 'tied' to the plasma at $r = r_1$. This is analogous to a planetary or stellar magnetosphere which corotates with the planet or star. Such rigid-body rotation implies $v(r) \propto r$, for $r \gg r_1$. Hence, the model does fail to reproduce flat rotation curves.

A mathematically acceptable treatment was given by Nelson (1988, MNRAS, 233, 115), who found that the derived rotation curves are not flat, but have a minimum in r around 30 kpc. My argument suggests that they should tend to $v(r) \propto r$, and Nelson's curves are compatible with this. Nelson also pointed out that the magnetic torque requires $B^2/2\mu_0 \sim \rho v_r v_\phi$, whereas the Battaner et al. analysis ignores v_r.

The existing case for magnetic fields providing an alternative to dark matter is seriously flawed. The mathematical arguments of Battaner et al. (1992) are incorrect, and a mathematically correct treatment (Nelson 1988) does not imply a flat rotation curve as is observed, but rather (as argued here) one with $v(r) \propto r$ at sufficiently large r.