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Production of Pairs by Linear Acceleration Emission

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Abstract: The possibility is considered that core emission in pulsars results from secondary $e^{\pm}$ pairs generated through linear acceleration emission (LAE), rather than the conventional curvature emission (CE). It is found that this is possible in principle, but only under conditions that appear not to be satisfied in practice.

Keywords: pulsars — gamma rays – pairs

1 Introduction

Polar-cap models for pulsars (e.g. the reviews by Arons 1979; Michel 1982, 1991; Beskin, Gurevich & Istomin 1993) involve $e^{\pm}$ pairs being produced on field lines that are open in the sense that they cross the light cylinder. The conventional picture of the pair creation involves primary particles accelerated to very high Lorentz factors, $\Gamma \sim 10^7$ (e.g. Ruderman & Sutherland 1975), by an electric field, $E_{\parallel}$, parallel to the magnetic field lines, $\mathbf{B}$, producing curvature emission (CE) as they propagate along the curved magnetic field lines (radius of curvature R_c). This $E_{\parallel}$ arises if the charge density, ρ, is not equal to the Goldreich–Julian density, ρ_{GJ}, required for corotation of the polar-cap plasma with the angular velocity, $\mathbf{\Omega}$, of the star. The photons generated by CE are of high energy and subsequently decay into free pairs. Provided that the probability of a photon being converted into a pair before escaping from the pulsar magnetosphere is of order unity, this model can account for the creation of a pair plasma that is dense enough to provide a partial screening of $E_{\parallel}$.

This standard model invokes the curvature of the field lines in two separate roles. First, the photons are attributed to CE, which requires finite R_c. In particular, one has $\varepsilon_\gamma \propto 1/R_c$ and the power in curvature radiation $\propto 1/R_c^2$. Hence, both the energy of the curvature photons and the power in CE are maximised on the most strongly curved field lines. For CE to be effective in producing pairs, R_c must be much smaller than the dipolar value, and of order the radius R_* of the star (e.g. Ruderman & Sutherland 1975). Second, the photons generated by the primary particles propagate at such small angles to the magnetic field, $\theta \lesssim 1/\Gamma$, that initially they do not satisfy the threshold condition required for decay into pairs [relation (3) below]. The curvature of the field lines is invoked to account for the necessary increase in θ as the photons propagate away from their point of emission. In this paper an alternative (to CE) source of photons is considered, but this second requirement still needs to be satisfied in order for pairs to be generated. For this purpose the requirement on R_c is not severe, and the dipolar value usually suffices.

The radio emission from pulsars can be separated on phenomenological grounds into 'core' and 'cone' emission (e.g. Rankin 1983, 1990; cf. also Lyne & Manchester 1988). Core emission, which dominates in young and millisecond pulsars, appears to peak at the centre of the pulse window. Assuming that the radio emission correlates with the generation of pairs, the simplest interpretation is that the pair production is at a maximum in the centre of the polar cap. However, the central field line in a dipolar model has no curvature. More generally, provided that the field lines are not too far from dipolar, one would expect the centre of the polar cap to correspond to the least curved field lines. This presents a difficulty for the standard model in that one would not expect the production of pairs due to CE to peak where the field lines are least curved.

In this paper, the suggestion is explored that the secondary pairs that lead to the screening of $E_{\parallel}$ and to the radio emission are produced by linear acceleration emission (LAE). LAE requires spatial (or temporal) oscillations in $E_{\parallel}$ (e.g. Melrose 1978). The primary electrons or positrons radiate due to the acceleration by this oscillating field. The idea explored here is that the generation of photons by LAE replaces the generation by CE.

2 Production of Pairs due to Curvature Emission

In order to compare the production of pairs by CE with that by LAE, it is appropriate to review the former process briefly.

2.1 Curvature Emission

The power radiated by a highly relativistic particle due to CE is

$$|\dot{\varepsilon}| = \frac{e^2 c}{6\pi\varepsilon_0 R_c^2}\Gamma^4 . \quad (1)$$

The energy spectrum of the curvature photons increases with the one-third power of the photon energy to a maximum at

$$\bar{\varepsilon}_\gamma \approx \frac{3\hbar c}{2R_c}\Gamma^3 , \quad (2)$$

above which the spectrum falls off exponentially.

2.2 Threshold for Pair Production

In order for a photon to decay into a pair, the threshold condition

$$\varepsilon_\gamma \sin\theta > 2mc^2 \quad (3)$$

must be satisfied, where m is the rest mass of the electron. The photon trajectory, which is a straight line, deviates from the curved magnetic field by an angle θ after the photon has propagated a distance

$$\tilde{s} = R_c\theta . \quad (4)$$

Hence the photon must propagate a distance

$$\tilde{s} = \frac{2mc^2 R_c}{\varepsilon_\gamma} = \frac{4}{3\alpha}\frac{R_c^2}{a}\frac{1}{\Gamma^3} , \quad (5)$$

before it can decay into a pair. For example, for a dipolar field line with $R_c \sim 10^6$ m, one has $\tilde{s} = 3{\times}10^3 (R_c/10^6\ \mathrm{m})^2 (\Gamma/10^7)^{-3}$ m. In the final expression in equation (5) the energy of a curvature photon [eq. (2)] is inserted, and the Compton wavelength, $\hbar/mc$ is expressed in terms of the Bohr radius $a = r_e/\alpha^2$, where $\alpha \approx 1/137$ is the fine-structure constant:

$$r_e = \frac{e^2}{4\pi\varepsilon_0 mc^2} = \alpha^2 a, \qquad \frac{\hbar}{mc} = \alpha a . \quad (6)$$

2·3 Opacity for Pair Production

The absorption per unit length leading to one-photon pair production is

$$\kappa = \frac{\alpha^2}{4r_e}\frac{B\sin\theta}{B_{\mathrm{crit}}}\exp(-4/3\chi) , \quad (7)$$

where $B_{\mathrm{crit}} = 4{\cdot}4 \times 10^9$ T is the critical magnetic field, and

$$\chi = \frac{\varepsilon_\gamma}{2mc^2}\frac{B\sin\theta}{B_{\mathrm{crit}}} . \quad (8)$$

The exponential dependence on χ in equation (7) implies that the condition for the optical depth to exceed unity is only logarithmically dependent on other parameters. Estimates for the condition for the optical depth to be unity lead to

$$\chi \sim 1/N , \quad (9)$$

with different authors estimating the number N to be in the range 14–20.

3 Linear Acceleration Emission

In treating the production of pairs due to linear acceleration emission (LAE), it is assumed that electrons or positrons with $\Gamma \gg 1$ propagate along field lines where there is an oscillating parallel electric field, $\Delta E_\parallel$. It is unimportant whether the oscillations are in time, with frequency ω_0, or in space, with wavenumber k_0, as the latter case is related to the former simply by replacing ω_0 by $k_0 c$.

3.1 Linear Acceleration Emission

The emissivity in LAE due to a relativistic particle is (e.g. Melrose 1978)

$$\eta(\omega,\theta) = 16 r_e^2 c\,\varepsilon_0 |\Delta E_\parallel|^2 \frac{\Gamma^4\theta^2}{(1+\Gamma^2\theta^2)^5} \times \delta[\omega - 2\omega_0\Gamma^2/(1+\Gamma^2\theta^2)] . \quad (10)$$

The frequency of emission, as determined by the δ-function, arises from the frequency, ω_0, of an assumed harmonic variation in the electric field being identified with the Doppler-shifted frequency of emission, $\omega - \mathbf{k}\cdot\mathbf{v} \approx \omega(1+\Gamma^2\theta^2)/2\Gamma^2$, by the particle with velocity $\mathbf{v}$.

The power per unit frequency is found by integrating over solid angle, which is equivalent to integrating over $2\pi\theta\, d\theta$, and because the integral is dominated by small θ the range of integration may be extended to $0 < \theta < \infty$ without introducing significant error. This gives

$$P(\omega) = 2\pi r_e^2 c\,\varepsilon_0 |\Delta E_\parallel|^2 \frac{\omega^2}{\omega_0^3\Gamma^6}\left(1 - \frac{\omega}{2\omega_0\Gamma^2}\right) . \quad (11)$$

Then integrating over $0 < \omega < 2\omega_0\Gamma^2$ gives the power per particle radiated in LAE:

$$|\dot{\varepsilon}| = \frac{4\pi r_e^2 c}{3}\varepsilon_0 |\Delta E_\parallel|^2 \Gamma^2 . \quad (12)$$

The mean frequency of LAE is found by noting that $P(\omega)/\hbar\omega$ is the rate per unit time at which photons are emitted. Hence the mean frequency of the photons is given by

$$\bar{\omega} = \frac{\displaystyle\int_0^\infty d\omega\, P(\omega)}{\displaystyle\int_0^\infty d\omega\, P(\omega)/\omega} = \omega_0 \Gamma^2 . \qquad (13)$$

This corresponds to a mean photon energy

$$\bar{\varepsilon}_\gamma = \hbar\omega_0 \Gamma^2 . \qquad (14)$$

Note the close analogy between LAE and inverse Compton emission. According to equation (12) the power in LAE is essentially the same as the power in inverse Compton emission when one expresses the latter in terms of the energy density in the oscillating electric field. Similarly, the typical frequency [eq. (14)] for LAE is analogous to that for inverse Compton emission.

4 Requirement on $\Delta E_\parallel$

The main requirement that needs to be satisfied in order for LAE to be effective in producing pairs is that there be sufficiently strong and sufficiently high-frequency oscillations in $E_\parallel$ in the region where the primary particles with very high Γ are located.

4.1 Natural Frequency of Oscillation

There is a natural frequency of oscillation determined by the plasma frequency, $\omega_{\rm GJ} = (e\rho_{\rm GJ}/\varepsilon_0 m)^{\frac{1}{2}}$, corresponding to the Goldreich–Julian charge density,

$$\rho_{\rm GJ} = -2\varepsilon_0 \mathbf{\Omega} . \mathbf{B} . \qquad (15)$$

It is convenient to write

$$\omega_0 = \omega_{\rm GJ}/\zeta , \qquad (16)$$

that is, to introduce a dimensionless parameter ζ that relates ω_0 to $\omega_{\rm GJ}$. Longitudinal plasma oscillations in a nonrelativistic plasma have $\zeta \sim 1$; for longitudinal plasma oscillations in a highly relativistic plasma have $\zeta \sim \Gamma^{\frac{3}{2}}$ (e.g. Beskin, Gurevich & Istomin 1993).

4.2 Threshold for Pair Production by LAE

The threshold $\hbar\bar{\omega} > 2mc^2$ for pair creation implies

$$\Gamma^4 > \frac{2c}{\alpha a \Omega} \frac{B_{\rm crit}}{B} \zeta^2 . \qquad (17)$$

For typical parameters, equation (17) reduces to $\Gamma^4 \gtrsim 10^{20}\zeta^2$. For $\zeta = 1$, this gives $\Gamma \gtrsim 10^5$, which is satisfied for the primary particles in a typical pulsar. However, for $\zeta = \Gamma^{\frac{3}{2}}$ the condition is not satisfied. Thus LAE can produce pairs only for $\omega_0 \sim \omega_{\rm GJ}$, and but not for $\omega_0 \sim \omega_{\rm GJ}/\Gamma^{\frac{3}{2}}$.

4.3 Source of Oscillations

The model of Mestel & Shibata (1994) implies oscillations at wavenumbers $\omega_{\rm GJ}/c$, which are equivalent to temporal oscillations at $\omega_{\rm GJ}$ for present purposes. However, these oscillations occur near the stellar surface where the plasma is nonrelativistic, that is, well below the region where the primary particles become highly relativistic. This argues against these oscillations being relevant to the pair generation. At greater heights in the gap, where the primary particles are highly relativistic, another possible source of longitudinal waves is an instability due to the return flux of particles flowing through the primary plasma (Lyubarskii 1992). However, one then expects the plasma to be highly relativistic, so that one has $\zeta \sim \Gamma^{\frac{3}{2}}$ in equation (16). Such an instability might be expected to produce large-amplitude longitudinal oscillations in the highly relativistic plasma, as described by Rowe (1992a,b) in discussing the application of LAE to pulsar radio emission.

In the model of Beskin, Gurevich & Istomin (1986, 1993), the acceleration of the primary particles is assumed to occur in a double layer at the surface of the star. This suggests that the region should be subject to random break-ups and reformations, as in more familiar double layers. However, no detailed model for possible oscillations of the double layer is available. One might expect an oscillation on the light propagation time across the double layer, implying $\zeta \sim (2R_*/\alpha a)^{\frac{1}{2}}(B/B_{\rm crit})^{\frac{1}{2}}$. Then equation (17) requires $\Gamma^4 > 4R_* c/\Omega\alpha^2 a^2$, which is not satisfied for plausible parameters. Thus relatively high-frequency oscillations are required in order for LAE to lead to pair production.

4.4 Comparison with CE

Let us compare LAE due to given $\Delta E_\parallel$ and $\Gamma = eE_\parallel s/mc^2$ at a height s, with CE from the same particles. From equations (1), (12) and (6) one obtains

$$\frac{P_{\rm LAE}}{P_{\rm CE}} = \tfrac{1}{2}\left(\frac{R_c}{s}\right)^2 \left(\frac{\Delta E_\parallel}{E_\parallel}\right)^2 . \qquad (18)$$

Assuming that $R_c/s \gtrsim 10^3$, the power in LAE exceeds that in CE for $\Delta E_\parallel/E_\parallel \gtrsim 10^{-3}$. This is a relatively modest requirement on the amplitude, $\Delta E_\parallel$, of the oscillations, and suggests that LAE is more relevant than CE.

5 Conclusions

It is possible in principle for LAE to contribute to pair production. LAE by primary particles with $\Gamma \sim 10^7$ moving through large-amplitude oscillations in $E_\parallel$ with a frequency of order $\omega_{\rm GJ}$, which is the nonrelativistic plasma frequency corresponding to the Goldreich–Julian density, would yield photons of sufficiently high frequency to produce pairs. However, although the requirement on the amplitude of the oscillations is relatively modest [cf. eq. (18)], the requirement on the frequency is very restrictive. In effect, one requires oscillations near the natural frequency for a nonrelativistic plasma and yet one also requires very highly relativistic primary particles in the same region. No existing polar-cap model satisfies these requirements.

It is concluded that an LAE model for pair production is not viable within the framework of the existing polar-cap models for pulsars. For the mechanism to work, it is essential that large-amplitude oscillations in $E_{\parallel}$ occur at frequencies much higher than the natural frequency ($\sim \omega_{\rm GJ}/\Gamma^{\frac{3}{2}}$) of longitudinal oscillations in the relativistic plasma formed by the primary particles.

Arons, J. 1979, Space Sci. Rev., 24, 437
Beskin, V. S., Gurevich, A. V., & Istomin, Ya. N. 1986, Soviet Phys. Usp., 29, 946
Beskin, V. S., Gurevich, A. V., & Istomin, Ya. N. 1993, Physics of the Pulsar Magnetosphere (Cambridge Univ. Press)
Lyne, A. G., & Manchester, R. N. 1988, MNRAS, 234, 477
Lyubarskii, Yu. E. 1992, A&A, 261, 544
Melrose, D. B. 1978, ApJ, 225, 557
Mestel, L., & Shibata, S. MNRAS, 271, 621
Michel, F. C. 1982, Rev. Mod. Phys., 54, 1
Michel, F. C. 1991, Theory of Neutron Star Magnetospheres (Chicago: Univ. Chicago Press)
Rankin, J. M. 1983, ApJ, 274, 333
Rankin, J. M. 1990, ApJ, 352, 247
Rowe, E. T. 1992a Aust. J. Phys., 45, 1
Rowe, E. T. 1992b, Aust. J. Phys., 45, 21
Ruderman, M. A., & Sutherland, P. G. 1975, ApJ, 196, 51