Proceedings of
the second Asian Pacific Plasma Theory Conference

APPTC '97

Combined with

Japan-Australia fusion theory workshop, and

US-Japan JIFT workshop on Theoretical Study for Helical Plasmas

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Abstract

This issue is the proceedings of the second Asian Pacific Plasma Theory Conference (APPTC '97), which was held on September 24-26, 1997 at National Institute for Fusion Science (Toki, Japan) under the auspices of the Japan Society of Plasma Science and Nuclear Fusion Research and the National Institute for Fusion Science. A part of APPTC '97 was a joint session with Japan-Australia fusion theory workshop and US-Japan JIFT workshop on Theoretical Study for Helical Plasmas. The conference covers all plasma theory areas including magnetic confinement, inertial fusion, space plasmas, astrophysical plasmas, industrial processing plasmas, and dusty plasmas, etc.

Keywords: plasma theory, magnetic confinement, inertial fusion, space plasma, astrophysical plasma, industrial processing plasma, dusty plasma

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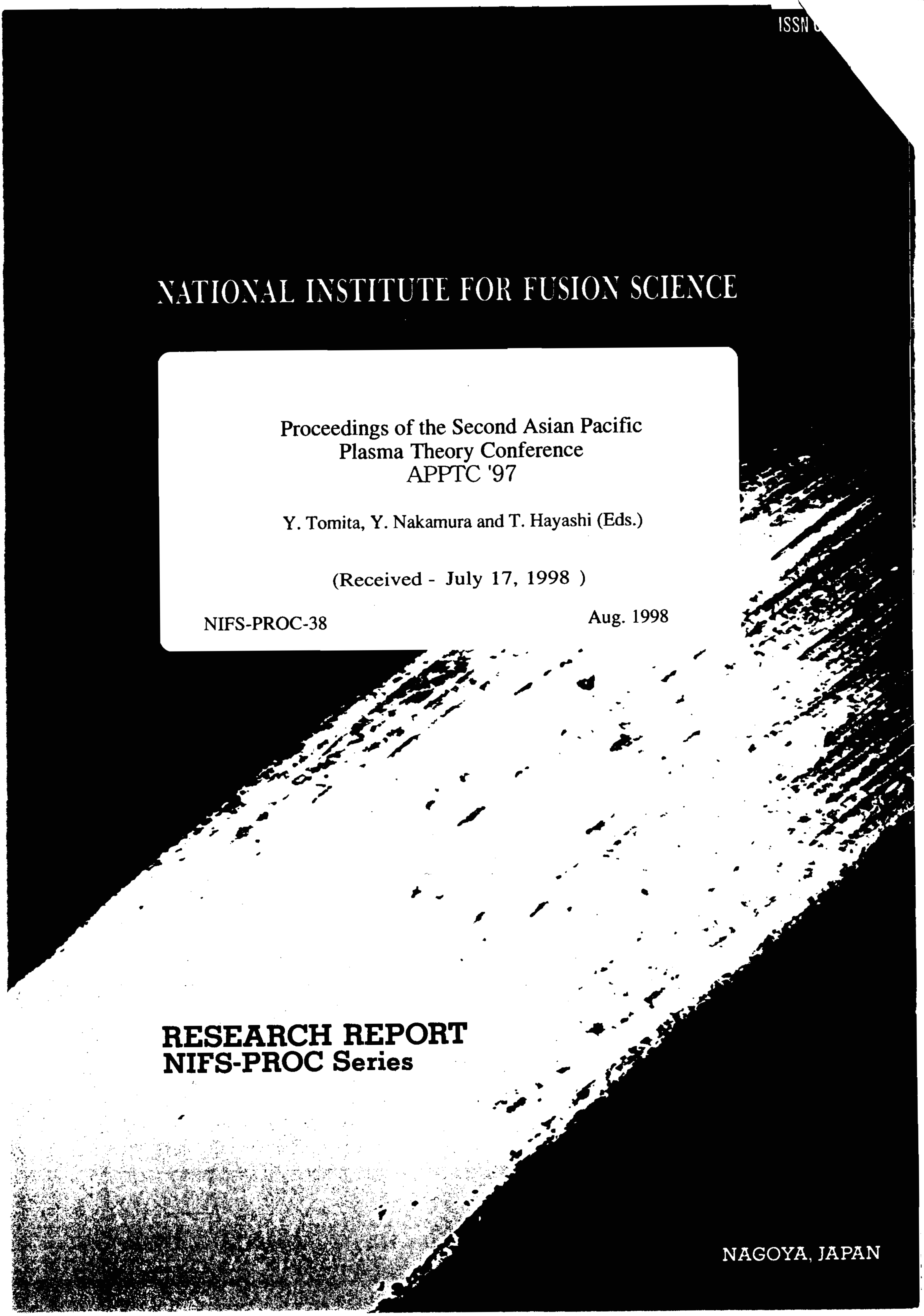

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Preface

The second Asian Pacific Plasma Theory Conference (APPTC '97) was held on September 24-26, 1997 at National Institute for Fusion Science (Toki, Japan) under the auspices of the Japan Society of Plasma Science and Nuclear Fusion Research and the National Institute for Fusion Science. Apart of APPTC '97 was a joint session with Japan-Australia fusion theory workshop and US-Japan JIFT workshop on Theoretical Study for Helical Plasmas. The Conference was attended by about 115 participants from 14 countries. The program included 43 oral presentations and 40 poster presentations.

APPTC '97 was a continuation of APPTC '96 held in Korea, which covered all plasma theory areas including magnetic confinement, inertial fusion, space plasmas, astrophysical plasmas, industrial processing plasmas, and dusty plasmas, etc. The purposes of the conference are to facilitate cooperation among plasma theorists and also to induce and educate young and bright scientists in the Asia-Pacific countries where rapidly increasing research activities in plasma science and technologies have been taking place. This conference was the first international conference held at the new site of the National Institute for Fusion Science, which had moved to the Toki site from Nagoya in April, 1997. In this conference particular emphasis was placed on the pure scientific aspects of plasma physics, hoping that plasma physics would be able to emanate attractive fragrance not only to plasma scientists but also to scientists in other fields.

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Conference Chair

Pulsar Electrodynamics – Some Outstanding Problems

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October 10, 1997

Abstract

The "standard model" for pulsar electrodynamics is summarized. The properties of natural wave modes in the plasma rest frame in the region above the pair production front are discussed. This plasma is one dimensional and extremely strongly magnetized with a relativistic spread in energies. The break down of MHD in the pulsar wind is also discussed.

1 Introduction

The electrodynamics of pulsars in the "standard model" incorporates a number of different physical processes (see the reviews by Michel 1982; Beskin, Gurevich & Istomin 1986; Mestel 1993; Shibata 1996): large parallel electric fields ($E_{||}$) in the polar cap regions and in "outer gaps"; acceleration of "primary" particles to extremely relativistic energies (Lorentz factors $\Gamma \sim 10^7$) by these $E_{||}$ fields: emission of gamma rays by the primary particles; decay of the gamma rays into relativistic "secondary" pairs in a pair production front (PPF);the loss of perpendicular momentum due to gyromagnetic emission resulting in a one-dimensional pair plasma in the PPF and beyond; the screening of $E_{||}$ by these pairs in the PPF; and, the outflow of the pairs beyond the light cylinder to form a pulsar wind which has a relativistic flow speed and a relativistic spread of particle energies. Most of these processes are well understood in principle but their application to pulsars is subject to various uncertainties. For most pulsars the only observational data available are from the radio emission, which involves only a tiny fraction of the inferred power output in the relativistic particles, and hence it may be a relatively poor diagnostic of the electrodynamics processes involved.

In this paper, after a review of the important features of the electrodynamics in the standard model, I discuss two specific plasma problems which arise in any variant of the standard model. The first problem concerns the properties of the natural wave modes of the one-dimensional relativistic pair plasma. Generation of these waves somewhere above the PPF is required to account for the observed radio emission, and the wave properties are needed to discuss both the generation of the radio emission and its propagation through the overlying plasma regions (e.g., Melrose 1995, 1996). The second problem concerns the properties of the wind. All theories of the wind rely on some form of MHD theory, but simple arguments imply that MHD theory must break down within the wind zone.

The implications of this break down cannot be neglected, but it may be that the changes required can be incorporated by slightly modifying the MHD wind models.

2 The "standard model"

Pulsars (PSRs) are rotating neutron stars (NSs). They fall into two categories: "ordinary" PSRs which are created in supernova explosions and "millisecond" PSRs (MSPs) which are very old NSs spun-up in binary systems. The parameters of NSs are radius R, $\sim$ 10 km and mass strongly concentrated around M $\sim 1.4 M_{\odot}$. For ordinary PSRs, the range of rotational frequencies, $\Omega = 2\pi/P$ is typically $\sim$ 1–10s–I, and the range of magnetic fields is B $\sim 10^{8\pm1}$ T. For MSPs the ranges the frequencies are higher, $\Omega \sim 10^3\,\mathrm{s}^{-1}$, and the magnetic fields lower, $B \sim 10^{5\pm1}$ T. An important parameter is the radius of the light cylinder, $r_{\mathrm{lc}} = c/\Omega = 5\cdot10^3\, R, (T/1\,\mathrm{s})$.

A "vacuum" model for a pulsar, that is, a magnet rotating in vacuo, is clearly wrong in detail, but nevertheless is the basis for the estimate of the magnetic field. The power radiated in magnetic dipole radiation is

$$\mathcal{P} = \frac{2\pi\Omega^4 B_*^2 R_*^6 \sin^2\chi}{3\mu_0 c^3}, \tag{1}$$

where B, is the magnetic field at the north magnetic pole and χ is the angle between the rotation and magnetic axes. The power (1) is equated to the rate of rotational energy loss,

$$\mathcal{P} = -I\Omega\dot{\Omega} = \frac{I(2\pi)^2 P}{P^3} \tag{2}$$

to find $B_* \propto (P\dot{P})^{1/2}$, which is the basis for observational estimates of B,. The vacuum model also implies a quadrupolar surface charge on the NS, with potential

$$\Phi = -\frac{\Omega B_* R_*^5}{3r^3} P_2(\cos\theta). \tag{3}$$

The associated electric field has a component

$$E_{\|} = \frac{\mathbf{E}\cdot\mathbf{B}}{B} = \frac{2\Omega B_* R_*^5}{r^4} \frac{\cos^3\theta}{(3\cos^2\theta - 1)^{1/2}} \tag{4}$$

along the magnetic field, and this field is thought strong enough to rip charges off the surface of the star, undermining the vacuum assumption.

Before proceeding to discuss the role of a charge magnetosphere, it is relevant to define the "polar caps" on the surface of the NS as the regions magnetically connected to the light cylinder and beyond. The equation $r = r_0 \sin^2\theta$ for a dipolar field line in polar coordinates implies that the polar cap extends to angles 8 < θ_{pc} from the poles, with

$$\theta_{\mathrm{pc}} = \left(\frac{\Omega R_*}{c}\right)^{1/2}. \tag{5}$$

The potential drop available across a polar cap is

$$\Delta\Phi \sim \frac{\Omega^2 R_*^3 B_*}{c} \sim 10^{13} \left(\frac{B_*}{10^8\,\mathrm{T}}\right) \left(\frac{P}{1\,\mathrm{s}}\right)^{-2} \mathrm{V}. \tag{6}$$

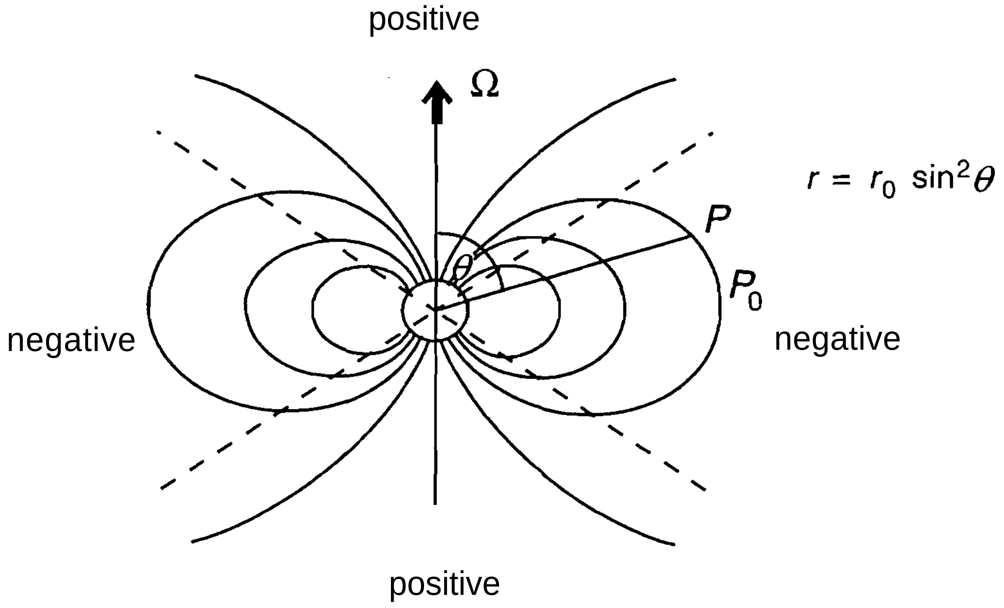

Figure 1: The corotation charge density has the same sign over the polar caps and the opposite sign in the equatorial regions: the separation between the two signs is illustrated by the dashed lines in the case of an aligned rotator.

Now suppose that there is an ample supply of plasma. Then perfect conductivity implies

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0, \quad \mathbf{v} = \boldsymbol{\Omega} \times \mathbf{x}, \tag{7}$$

and the divergence of this corotational electric field implies the Goldreich-Julian (GJ) density

$$n_{\mathrm{GJ}} = \frac{\varepsilon_0 \nabla \cdot \mathbf{E}}{e} = -\frac{\varepsilon_0 \boldsymbol{\Omega} \cdot \mathbf{B}}{e} = (7 \times 10^{17}\,\mathrm{m}^{-3}) \left(\frac{B}{10^8\,\mathrm{T}}\right) \left(\frac{T}{0.1\,\mathrm{s}}\right)^{-1} \cos\theta. \tag{8}$$

Note that n_{GJ} has same sign at both poles, and the opposite signs in the equatorial regions, as illustrated in Figure 1. For field lines originating from the polar cap, the charge density $en_{\mathrm{GJ}} \propto \mathrm{B}\cos\theta$ and the current density, $\mathbf{J} = nec$ (once the particles become relativistic) are incompatible with n $= n_{\mathrm{GJ}}$ in the absence of other sources of charge than the stellar surface. As a result an $E_{||}$ builds up, and leads to acceleration of "primary" particles to very high energies. The details of this acceleration are model dependent, but the acceleration of particles to very high energies occurs in all models.

Plasma escapes freely from polar cap. Assuming a current density $J \sim n_{\mathrm{GJ}}ec$, the total current is $\mathrm{I} \sim J\pi(R\theta_{\mathrm{pc}})^2 \sim \varepsilon_0 \Omega^2 R_*^3 B_*$. The power may be estimated from $\mathcal{P} = I\Delta\Phi$, which gives $\mathcal{P} \sim \Omega^4 B_*^2 R_*^6/\mu_0 c^3$. Apart from a factor of order unity, this is the same as the magnetic dipole value given by (1), thereby justifying its in estimating $B_* \propto (T\dot{T})^{1/2}$.

The large $E_{||}$ accelerates primary particles to Lorentz factor $\Gamma \sim$ e@/Mc^2. A primary particle radiates γ rays due to curvature emission, with a power per particle

$$\mathcal{P}_{\mathrm{c}} = \frac{2r_0 mc^3}{3R_{\mathrm{c}}^2}\Gamma^4, \qquad \varepsilon_\gamma \sim \frac{2\hbar c}{R_{\mathrm{c}}}\Gamma^3 \tag{9}$$

where ε_γ corresponds to the peak of what is a relatively broad spectrum of γ rays. The primaries emit γ rays nearly parallel to B field lines, and as they propagate, the γ rays de-

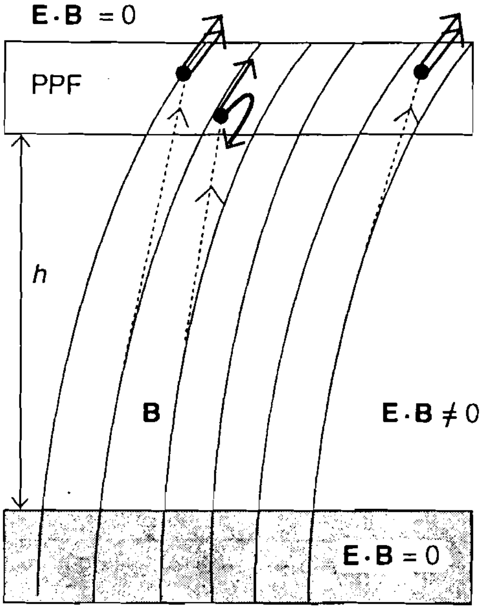

Figure 2: In the region below the pair production front (PPF) primary particles are accelerated and emit gamma rays which decay into secondary pairs in the PPF.

viate from B-lines (the angle ψ between them increases). The γ rays decay into secondary pairs, due to the process $\gamma + \mathrm{B} \to e^- + e^+ + \mathrm{B}$, provided the threshold $\varepsilon_\gamma \sin\psi > 2mc^2$ is exceeded.

The resulting production of "secondary" pairs occurs in a pair production front (PPF). These secondaries shield out $E_\parallel$ above the PPF, as illustrated schematically in Figure 2. The ratio of the number of secondaries to primaries is referred to as the multiplicity factor, M, with $n_{\rm pair} = Mn_{\rm GJ}$. For example,

$$n_{\rm pair} \sim \frac{\mathcal{P}_{\rm c}}{\varepsilon_\gamma}\frac{L}{c}, \tag{10}$$

with L = the height pair creation continues, leads to an estimate of M, but the actual value is model dependent, with M = 10^2–10^6 for the various models.

3 Wave properties in the secondary pair plasma

The pair plasma in the region above the PPF is of particular interest because it is the region from which the radio emission probably originates. The superstrong field $\mathrm{B} \gtrsim 10^8\,\mathrm{T}$ implies a cyclotron frequency $\omega \ll \Omega_e = eB/m$. It also implies that the particles radiate away their perpendicular energy in a very short time, ≈ 2 x $10^{-17}\Gamma$ s, so that the plasma is one dimensional, with $p_\perp = 0$ for all particles.

The resulting plasma is highly relativistic, $\Gamma_0 = \langle\Gamma_\pm\rangle \sim 10^2$–$10^3$, with an intrinsically relativistic spread in energy, corresponding to $\langle(\Gamma_\pm - \langle\Gamma_\pm\rangle)^2\rangle \sim \Gamma_0^2$. The non-zero charge and current densities imply that the plasma is gyrotropic (a pair plasma with identical electron and positron distributions is nongyrotropic), but the gyrotropy is weak, and of order $1/M$, provided the multiplicity is large. As a consequence, the difference in the rest frames of the electrons and positrons may be neglected to a first approximation, provided

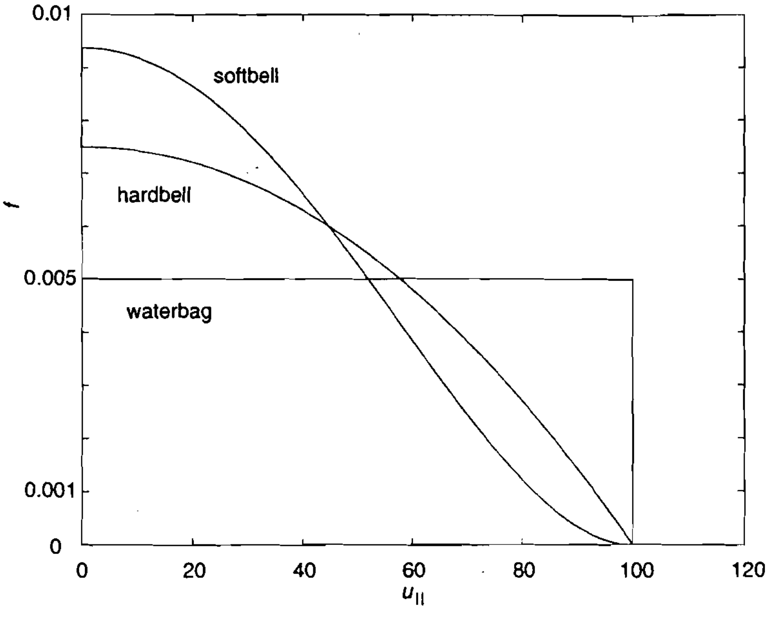

Figure 3: The distribution function, f, is shown for the waterbag, hard bell and soft bell as a function of $u_\parallel = (v_\parallel/c)/[1 + (v_\parallel/c)^2]^{1/2}$, with maximum energy corresponding to $\Gamma = 1/[1 + (v_\parallel/c)^2]^{1/2} = 100$.

that M is much larger than the mean Lorentz factors of the particles. This allows us to define a rest frame, $\langle\Gamma v_\parallel\rangle = 0$, for the plasma, and to ignore the differences between the distributions of electrons and positrons to a first approximation.

In order to discuss the properties of waves in such a plasma, it is necessary to assume a specific form for the distribution function of the particles. Three idealized distributions are illustrated in Figure 3: the waterbag, hard bell, soft bell distributions. Only the waterbag distribution has been considered previously in this context (Arons & Barnard 1986), but this distribution can lead to misleading conclusions. In particular it cannot be used to discuss Landau damping, because this depends on the derivative of the distribution function which is artificially made to be zero for the waterbag distribution, except at the cutoff where it is infinite. This also leads to associated misleading features in the dispersion. The hard bell distribution only partly overcomes this difficulty, and the soft bell distribution is the simplest case where the derivative is continuous. Another case which avoids these technical difficulties is the relativistic thermal distribution which has all its derivatives continuous.

The wave properties in plasma rest frame are relatively simple (Gedalin, Melrose & Gruman 1997), as illustrated in Figure 4. There are three distinct modes. The magnetosonic (t or X) mode has a dispersion relation

$$\omega_t^2 = k^2 v_A^2(1 - \Delta\lambda\cos^2\theta), \qquad v_A = \frac{1}{(1+\Delta\langle\Gamma\rangle)^{1/2}}, \quad \Delta = \frac{2\omega_p^2}{\Omega_e^2}, \quad \lambda = \langle\Gamma v_\parallel^2\rangle, \tag{11}$$

with electric vector $\mathbf{E} \parallel \mathbf{k}\times\mathbf{B}$. The L-O mode has a cutoff frequency at $\omega_c = \omega_p(2\langle\Gamma^{-3}\rangle)^{1/2}$, $\omega_p^2 = ne^2/\varepsilon_0 m$. The Alfvén ($A$) mode has a maximum frequency $\omega_{\max}$, which is sensitive to the form of the distribution function. (This maximum is absent in the waterbag and hard bell distributions and evidently requires that the second derivative of the distribution function be continuous.) Both the t and A modes become "firehose unstable" for AX $\gtrsim 1$.

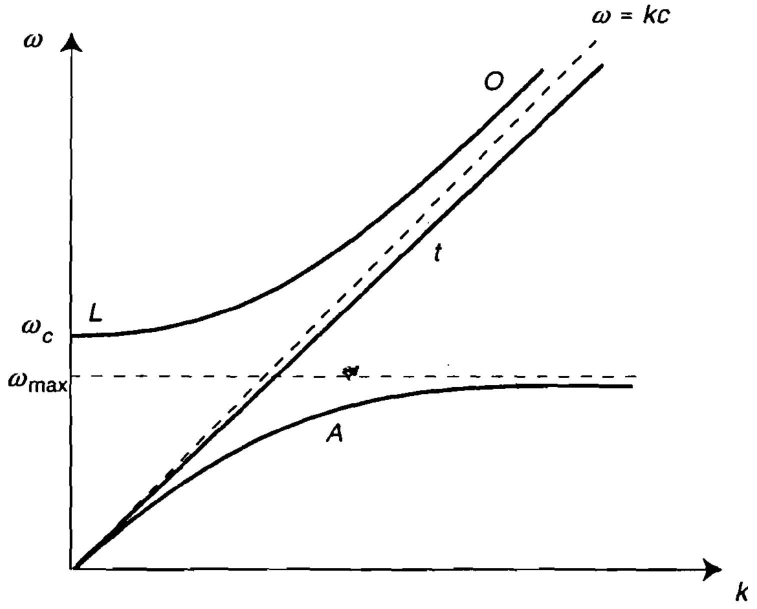

Figure 4: The dispersion relations are illustrated schematically for the three modes in a highly magnetized, intrinsically highly relativistic, one dimensional pair plasma in the rest frame of the plasma..

There is no Landau damping damping or Cerenkov emission of either the t mode (due to $E_{\parallel} = 0$) or of the L-O mode (due to $\omega >$ kc). Hence the only waves that can be generated by a streaming instability are those in the A mode. (The nonzero charge and current densities, that is the gyrotropy in this contest. allow some Landau damping of t mode due to an elliptical component to the polarization, but this is a small effect for $M \gg 1$.) This is a severe restriction on models for the radio emission based on streaming instabilities.

4 The wind zone

The wind zone in a pulsar extends from around the light cylinder to a termination shock. In the case of the Crab Nebula the termination shock is at $r_{\rm shock} \sim 2 \,.\, 10^9 r_{\rm lc} \sim 0.1\,$pc. The energy flux flux in the wind can be separated into electromagnetic and kinetic energy (KE) fluxes, determined by the ratio

$$a = \frac{\text{Poynting flux}}{\text{KE flux}}. \qquad (12)$$

One necessarily has $a \gtrsim 1$ at light cylinder. In the wind zone an MHD model suggests Poynting flux $\propto$ B^2, with $B \sim B_\phi \propto 1/r$, and a KE flus $\propto n\Gamma$, with $n \propto 1/r^2$ and $\Gamma =$ constant. This implies a independent of r in wind. However, a detailed model requires $a \sim 3 \cdot 10^{-3}$ at $r_{\rm shock}$ (Kennel & Coroniti 1984). This raises the important question as to how Poynting flux converted to KE flux (Coroniti 1990).

It is also relevant to ask whether the use of MHD is valid. MHD can break down in two different ways in this context (Melatos & Melrose 1996, hereafter MM). One is the assumption that the Ohm's law may be approximated by $E + U \times B = 0$, where U is the fluid velocity, and the generalized Ohm's law contains various terms on the RHS of this equation that must be smaller than the terms retained in order for MHD to be valid. The other is that MHD implies a conduction current density, J_c, and the maximum current that can be supplied by plasma with a given density is $J_{\max} = nec$; MHD breaks down for $J_c > J_{\max}$.

By starting from a (cold) two-fluid model for the electrons and positrons, and rewriting the fluid densities and velocities in terms of the fluid density and velocity and the charge and current densities, MM estimated the conditions under which the other terms in the resulting Ohm's law may be neglected. They found

$$\mathbf{E} + \mathbf{U} \times \mathbf{B} = \frac{m}{e^2}\frac{\partial}{\partial t}\left(\frac{\Gamma \mathbf{J}}{n}\right) + \frac{m}{e^2}\mathbf{U}\cdot\nabla\left(\frac{\Gamma \mathbf{J}}{n}\right) + \frac{m}{e^2}\mathbf{J}\cdot\nabla(\Gamma \mathbf{U}). \tag{13}$$

Then assuming $d/dt \to 1/T$, V $\to 1/L$, the condition for the terms on the RHS to be negligible reduces to

$$\frac{\Gamma}{\omega_p^2 T^2}\frac{J\,T}{\varepsilon_0 E} \ll \min\left(1, \frac{L}{TU}, \frac{TU}{L}\right). \tag{14}$$

Estimates suggest that MHD breaks down at $r = r_{\mathrm{MHD}} \sim 10^{-4} r_{\mathrm{shock}}$ for the Crab pulsar (MM). The requirement on the current also breaks down at about the same radius. This implies *charge* starvation in the wind. Additional physics is needed to discuss the properties of the wind in the region where MHD breaks down and charge-starvation occurs. MM speculated that the electromagnetic field in the wind becomes similar to transverse waves in underdense plasmas, for which the dispersion relation is

$$n^2 = 1 - \frac{\omega_p^2}{\omega^2(1+\nu^2)^{1/2}}, \qquad \nu = \frac{eE}{mc\omega}, \tag{15}$$

and noted that efficient acceleration of test particles occurs in the fields of such a wave. This model implies a $= 2\omega_p^2/\omega^2$, and a specific model for the Crab pulsar plausibly results in a $\sim 3 \,.\, 10^{-3}$ at $r = r_{\mathrm{shock}}$, as required in the model of Kennel & Coroniti (1984).

5 Conclusions

The main points made is this paper are

1). In the region where the radio emission is thought to be generated, there are three natural wave modes of relevance: the magnetosonic or X mode, the Langmuir-O mode and the Alfvén mode. Only the Alfvén mode can be generated effectively through a streaming instability.

2). MHD theory breaks down in the pulsar wind. It is suggested that the properties of the electromagnetic fields are analogous to transverse waves in an underdense plasma and that the waves are slowly damped through acceleration of particles, leading to a transfer of Poynting flux to kinetic energy flux in the wind.

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