

## LANGMUIR WAVE GENERATION THROUGH A NEUTRINO BEAM INSTABILITY

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### ABSTRACT

A standard version of a kinetic instability for the generation of Langmuir waves by a beam of electrons is adapted to describe the analogous instability due to a beam of neutrinos. The interaction between a Langmuir wave and a neutrino is treated in the one-loop approximation to lowest order in an expansion in  $1/M_W^2$  in the standard electroweak model.

It is shown that this kinetic instability is far too weak to occur in a suggested application to the reheating of the plasma behind a stalled shock in a Type II supernova. This theory is also used to test the validity of a previous analysis of a reactive neutrino beam instability, and various shortcomings of this theory are noted. In particular, relativistic plasma effects have a significant effect on the calculated growth rates, and any theoretical description of neutrino-plasma interactions must be based directly on the electroweak theory. The basic scalings discussed in this paper suggest that a more complete investigation of neutrino-plasma processes should be undertaken to look for an efficient process capable of driving the stalled shock of a Type II supernova.

*Subject headings:* elementary particles — instabilities — plasmas — supernovae: general — waves

### 1. INTRODUCTION

Processes involving neutrinos are important in a variety of astrophysical contexts, usually involving highly degenerate, relativistic plasmas. For example, neutrinos provide a cooling mechanism for red giants in the later stages of their evolution through the generation of neutrino-antineutrino pairs from photons (Adams, Ruderman, & Woo 1963; Braaten & Segel 1993). Of particular interest here is the intense flux of neutrinos generated in the first few seconds of a Type II supernova explosion (Bethe 1990). There is a difficulty in understanding how the explosion occurs. The supernova (SN) shock, which is needed to eject the stellar envelope, stalls as a result of energy losses through dissociation of nuclei. Heating by neutrinos is thought to be required to drive this shock. Wilson (1985) considered heating through direct neutrino scattering: highly beamed neutrinos are scattered by particles in the postshock plasma and deposit energy and momentum into this material, thereby driving the shock expansion.

An alternative mechanism for the deposition of the neutrino energy behind the SN shock is the neutrino beam instability (Bingham et al. 1994), where the neutrinos emitted from the SN core drive a plasma instability which produces Langmuir turbulence. These Langmuir waves are then collisionally damped, thereby heating the plasma and driving the SN shock. This instability may be understood by analogy with a more familiar electron beam instability: neutrinos propagating through a plasma emit longitudinal waves (Langmuir waves in an unmagnetized plasma) just as electrons do. This implies that for any plasma instability which relies on Cerenkov emission of Langmuir waves by electrons as the driving microscopic process, there is an analogous neutrino-driven plasma instability. The simplest plasma instability is the beam instability, where a beam of electrons (e.g., Briggs 1964), photons (Gedalin & Eichler 1993; Melrose 1994), or neutrinos (Bingham et al. 1994), causes Langmuir waves to grow through induced Cerenkov emission.

In this paper a systematic development of the neutrino beam instability of Langmuir waves is developed, based on the analogy with the electron beam and photon beam cases. A kinetic version of the neutrino beam instability is described, and the growth rate for this instability is calculated. The results are applied to the SN shock problem, and it is argued, contrary to the conclusions of Bingham et al. (1994), that the instability is ineffective and unimportant.

It is known that a neutrino propagating through a medium acquires an induced charge (Oraevsky & Semikoz 1987; Nieves & Pal 1994) through resonant interactions with the electrons in the medium. It is through this induced charge that neutrinos couple to plasma waves. Emission by a neutrino has been considered both for longitudinal waves (Tsytovich 1964; Oraevsky & Semikoz 1987) and for transverse waves (D’Olivo, Nieves, & Pal 1990; Giunti, Kim, & Lam 1991). (We note that the emission of a transverse wave by a neutrino is kinematically allowed only if the refractive index of the waves is greater than unity.) The interaction between a neutrino and a Langmuir wave is treated here using the standard electroweak model in the one-loop approximation. The derivation of the probability for Cerenkov emission by a neutrino of a Langmuir wave to lowest order in an expansion in  $1/M_W^2$ , where  $M_W$  is the mass of the  $W$  boson, is summarized in Appendix A. We do not consider the effects of extensions to the standard model, such as allowing neutrinos to have inherent electric and magnetic moments or a nonzero intrinsic mass.

Bingham et al. (1994, hereafter BDSB) calculated the growth rate of a reactive version of the neutrino beam instability and predicted that the neutrino flux from the SN core would be sufficient to drive the instability. The theory used by BDSB was overly restrictive, in that it was assumed that the neutrino beam was monoenergetic and that it propagated in a cold plasma, two assumptions which increase the calculated growth rate. We also show that the “effective potential” neutrino-plasmon coupling used by BDSB does not incorporate all of the necessary electroweak physics,

which may also have a dramatic effect on the growth rate of the reactive instability. We develop a kinetic version of the reactive instability proposed by BDSB, using methods explicitly based on the electroweak theory and drawing on techniques used in analysis of electron and photon beam instabilities.

In § 2 we discuss the emission of longitudinal plasmons by neutrinos and relate this process to the theory of plasma instabilities. In § 3 the emission and absorption coefficients are evaluated for some specific neutrino distributions. It is shown that for a beamed distribution the absorption coefficient is negative over a range of values, implying that wave growth occurs for a sufficiently intense neutrino beam. In § 4 the absorption coefficient is evaluated for parameters relevant to Type II supernovae, and it is shown that the kinetic instability is far too weak to operate. A comparison with the work of BDSB is then made, identifying important concerns for an explicit electroweak calculation of the reactive version of the neutrino beam instability. We also suggest that collisional damping completely swamps any tendency to wave growth. Natural units ( $c = \hbar = 1$ ) are used throughout except where otherwise specified.

## 2. EMISSION AND ABSORPTION OF LANGMUIR WAVES BY NEUTRINOS

The treatment here of a neutrino beam instability is closely analogous to the earlier treatments of an electron beam instability (e.g., Melrose 1986, p. 53) and a photon beam instability (Melrose 1994). The time evolution of the occupation number of the Langmuir waves is described by a kinetic equation. In general, the kinetic equation is written in a semiclassical form involving the probability per unit time,  $w_M(\mathbf{k}, \mathbf{p})$ , that a particle (electron, photon, or neutrino) emits a wave quantum in an arbitrary wave mode  $M$ . For present purposes we restrict attention to the emission of the simplest longitudinal electromagnetic quantum in a plasma, a Langmuir wave. After a description of the mechanism through which a neutrino in a plasma may emit a Langmuir wave, the probability of this process, which is derived in Appendix A, is written down for massless neutrinos and is simplified.

### 2.1. Effective Charge on the Neutrino

Cerenkov emission by a neutrino in a plasma occurs through resonant interactions of the type shown diagrammatically in Figure 1. An electron in the plasma scatters off a neutrino via either  $W$ -boson or  $Z$ -boson exchange and in doing so emits a wave quantum. This process is resonant when the incoming electron momentum is equal to its outgoing momentum. The probability of the emission of a wave quantum by a neutrino in a plasma is then calculated by averaging the probability of this resonant process over all the electrons in the plasma.

Alternatively, one may consider the processes shown in Figure 2 and interpret the internal electron lines as electron propagators averaged over the electrons and positrons in the plasma (this is the approach adopted in Appendix A). There are then two contributions from each of the diagrams of Figure 2. The first contribution is due to the vacuum contribution of virtual electron-positron pairs. This term does not contribute here because the emission of a photon by a massless neutrino in vacuo is kinematically forbidden. The second contribution is due to the real electrons and

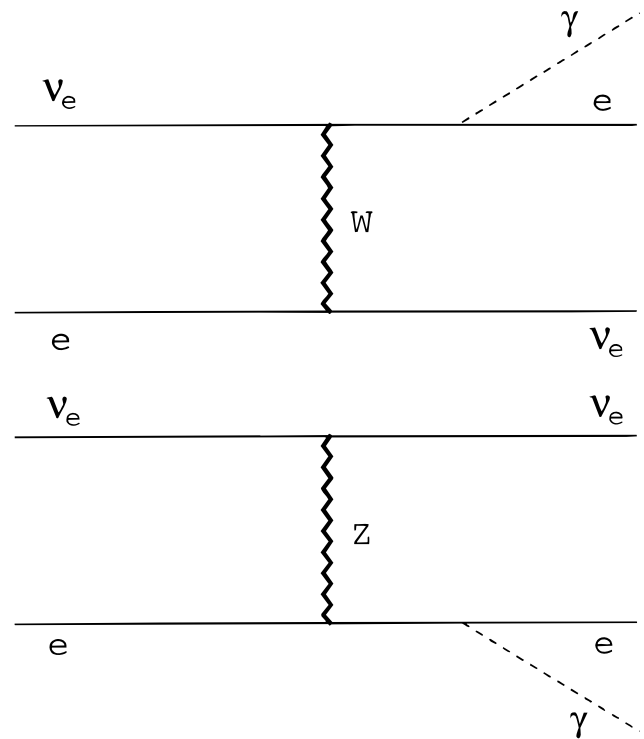


FIG. 1.—Feynman diagrams representing radiative decay of a neutrino mediated by a plasma electron.

positrons in the plasma and is equivalent to the average over the resonant interactions shown in Figure 1. To lowest order in  $1/M_W$  the two diagrams of Figure 2 may be combined to form the 4-point interaction shown in Figure 3.

Nieves & Pal (1994) have calculated the induced charge on a neutrino in a medium. The matrix element for the interaction shown in Figure 3 (cf. Appendix A) may be written in the form of an electromagnetic interaction

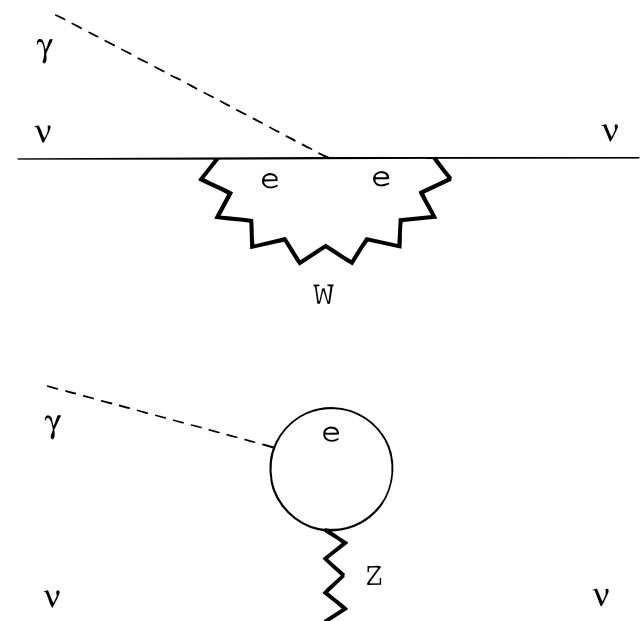


FIG. 2.—Feynman diagrams representing radiative decay of a neutrino through an electron-positron pair.

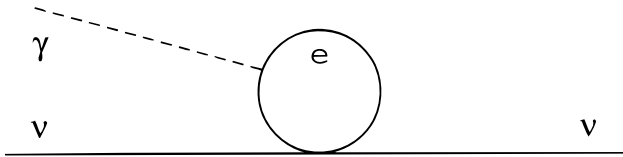


FIG. 3.—Feynman diagram for radiative decay of a neutrino to lowest order in  $1/M_W^2$ .

$J^\mu(k)A_\mu(k)$ , from which the 4-current,  $J^\mu$ , for the neutrino is identified. The actual value of the effective charge on the neutrino, which follows from  $J^0$ , is a fraction of order  $G_F n_e/T_e$  of the charge of the electron, where  $G_F$  is the Fermi constant,  $n_e$  is the electron number density, and  $T_e$  is the electron temperature as an energy. For plasma parameters relevant to the postshock region of a Type II supernova, this fraction is of order  $10^{-13}$ .

The only relevant kinematic restriction to the emission process shown in Figure 2 is conservation of 4-momentum. Denoting the 4-momenta of the neutrinos and the photons by  $(\varepsilon, \mathbf{p})$  and  $(\omega(\mathbf{k}), \mathbf{k})$ , respectively, conservation of momentum is satisfied (for  $|\mathbf{k}| \ll |\mathbf{p}|$ ) provided that the phase speed of the wave is less than the group velocity of the neutrino. The phase speed of the Langmuir wave is  $\omega_L(\mathbf{k})/|\mathbf{k}|$ , where  $\omega_L(\mathbf{k})$  is its frequency, given by the solution of the dispersion relation of the plasma. A general form of the dispersion relation for Langmuir waves in a Fermi-Dirac plasma is written down in Appendix B below. For massless neutrinos the neutrino group velocity (to lowest order in  $1/M_W$ ) is the speed of light. Induced effects lead to an energy-momentum relation for massless neutrinos given by (Nieves 1989)  $\varepsilon = |\mathbf{p}| + \sqrt{2}G_F n_e$ , where  $n_e$  is the number density of electrons. Although this implies an effective mass and a phase speed less than that of light, the resulting corrections are negligible in the applications considered here. For example, for electron densities relevant to the postshock region of a Type-II supernova,  $n_e = 10^{36} \text{ m}^{-3}$ , the induced mass is less than  $10^{-7} \text{ eV}$  for a neutrino with energy  $\sim 1 \text{ eV}$ . Thus, we may assume that the simple vacuum energy-momentum relation for the neutrinos and the kinematic restriction on the emission of Langmuir waves is

$$\frac{\omega_L(\mathbf{k})}{|\mathbf{k}|} < v_g, \quad (1)$$

where  $v_g = 1$  for massless neutrinos.

## 2.2. Probability of Emission

We turn now to the calculation of the probability of emission of a Langmuir wave by massless neutrinos in a plasma. This expression is then simplified to lowest order in  $\hbar$  in order to calculate the growth rates for the neutrino beam instability.

The calculation of the probability per unit time of emission of a Langmuir wave by a massless neutrino is outlined in Appendix A (cf. Hardy & Melrose 1996). The probability may be written (viz., eq. [A12])

$$w_L(\mathbf{k}, \mathbf{p}) = \frac{G_F^2 c_V^2}{16\pi\alpha} \frac{|\mathbf{k}|^2 \omega}{\varepsilon \varepsilon'} Z(|\mathbf{k}|) 2\pi \delta(\varepsilon' + \omega - \varepsilon) \times [ (2\varepsilon - \omega)^2 - |\mathbf{k}|^2 ] \left( 1 - \frac{\omega^2}{|\mathbf{k}|^2} \right)^2, \quad (2)$$

where  $G_F$  is the Fermi constant,  $\alpha$  is the fine-structure constant,  $Z(|\mathbf{k}|)$  is a plasmon normalization factor (defined in Appendix B), with

$$c_V = \begin{cases} 2 \sin^2 \theta_W + \frac{1}{2} & \text{for } \nu_e, \\ 2 \sin^2 \theta_W - \frac{1}{2} & \text{for } \nu_\mu, \nu_\tau, \end{cases} \quad (3)$$

and where  $\omega = \omega_L(\mathbf{k})$  is understood.

A convenient simplifying assumption in the calculation of the emission and absorption coefficients is to neglect the photon recoil. The recoil is the change in momentum,  $\mathbf{p}$ , of the neutrino due to emission of a wave quantum, with momentum  $\mathbf{k}$ , and the recoil is small for  $|\mathbf{k}| \ll |\mathbf{p}|$ , which also corresponds to  $\omega \ll \varepsilon$ . Expanding the argument of the  $\delta$ -function of equation (2) in  $|\mathbf{k}|/|\mathbf{p}|$  and retaining only the lowest order term gives

$$w_L(\mathbf{k}, \mathbf{p}) = \frac{G_F^2 c_V^2}{2\alpha} |\mathbf{k}|^2 \omega Z(|\mathbf{k}|) \left( 1 - \frac{\omega^2}{|\mathbf{k}|^2} \right)^2 \delta(\omega - \mathbf{k} \cdot \mathbf{v}), \quad (4)$$

where  $\mathbf{v} = \mathbf{p}/\varepsilon$ . Ignoring the small induced mass of the neutrino implies  $|\mathbf{v}| = 1$ . Equation (4) may also be written

$$w_L(\mathbf{k}, \mathbf{p}) = C(\mathbf{k}) \delta(\cos \chi_0 - \cos \chi), \quad (5)$$

with

$$C(\mathbf{k}) = \frac{G_F^2 c_V^2}{2\alpha} |\mathbf{k}|^2 Z(|\mathbf{k}|) \cos \chi_0 \sin^4 \chi_0, \quad (6)$$

with  $\cos \chi_0 = \omega/|\mathbf{k}|$  and where  $\chi$  is the angle between  $\mathbf{k}$  and  $\mathbf{v}$ .

Note that, with the neglect of the photon recoil, the probability per unit time of emission is independent of the initial energy of the neutrino.

## 2.3. Kinetic Equations

In the semiclassical formalism used here, the distribution of plasmons is described by their occupation number,  $N_L(\mathbf{k})$ , and the neutrinos are assumed nondegenerate, with distribution function normalized according to

$$n_\nu = \int \frac{d^3 \mathbf{p}}{(2\pi)^3} f(\mathbf{p}), \quad (7)$$

where  $n_\nu$  is the number density of neutrinos. The time evolution of the plasmon distribution is governed by a kinetic equation, which includes those processes which change the number of plasmons present at a given energy through spontaneous and stimulated emission and by absorption. Thus, the kinetic equation may be written

$$\frac{dN_L(\mathbf{k})}{dt} = \int d^3 \mathbf{p} w_L(\mathbf{k}, \mathbf{p}) \{ [1 + N(\mathbf{k})] f(\mathbf{p}) - N(\mathbf{k}) f(\mathbf{p} - \mathbf{k}) \}, \quad (8)$$

where the first term within the braces represents spontaneous and stimulated emission, and the second term represents the inverse process, absorption of a plasmon (microscopic reversibility requires that the probability of emission and absorption be equal).

For sufficiently smooth distribution functions,  $f(\mathbf{p} - \mathbf{k})$  may be Taylor-expanded, and then equation (8) gives

$$\frac{dN_L(\mathbf{k})}{dt} = \alpha_L(\mathbf{k}) - \gamma_L(\mathbf{k}) N_L(\mathbf{k}), \quad (9)$$

where

$$\alpha_L(\mathbf{k}) = \int d^3p w_L(\mathbf{p}, \mathbf{k}) f(\mathbf{p}) \quad (10)$$

and

$$\gamma_L(\mathbf{k}) = - \int d^3p w_L(\mathbf{p}, \mathbf{k}) \mathbf{k} \cdot \frac{\partial f(\mathbf{p})}{\partial \mathbf{p}} \quad (11)$$

are the emission and absorption coefficients, respectively.

These coefficients appear in explicit solutions to equation (9). In the long time limit, the occupation number of the plasmons goes as

$$N_L(\mathbf{k}) \propto e^{-\gamma_L(\mathbf{k})t}. \quad (12)$$

For stable distributions, such as a thermal isotropic distribution,  $\gamma_L(\mathbf{k})$  is positive, and the plasmon distribution tends to a constant level. If  $\gamma_L(\mathbf{k})$  is negative, wave growth may occur. However,  $\gamma_L(\mathbf{k}) < 0$  is not a sufficient condition for wave growth, as there are other competing plasma processes which remove Langmuir waves from the plasma. Two of these are Landau damping by thermal electrons and collisional damping, with damping coefficients denoted by  $\gamma_{\text{Landau}}$  and  $\gamma_{\text{coll}}$ , respectively. For growth to occur,  $\gamma_L < 0$  must satisfy

$$|\gamma_L| > \gamma_{\text{coll}} + \gamma_{\text{Landau}}. \quad (13)$$

This condition sets the threshold on the neutrino intensity required for the instability to produce Langmuir turbulence. A particularly severe restriction is imposed by collisional damping which is of order  $(\omega_p/\Lambda) \ln \Lambda$ , where  $\Lambda$  is the number of thermal electrons per Debye sphere. For the dense plasma around the SN core  $\Lambda$  is not a particularly large number ( $\Lambda \sim 10^2$ ), so that collisional damping is a strong effect to be overcome in order for the Langmuir waves to grow.

We turn now to evaluating the emission and absorption coefficients for a distribution of massless neutrinos.

#### 2.4. Emission Coefficient

We assume that the neutrino distribution is axisymmetric and separable in spherical polar coordinates,  $(p, \alpha, \phi)$ , where  $\alpha = 0$  is the streaming direction:

$$f(\mathbf{p}) = f(p)\Phi(\alpha). \quad (14)$$

The plasmon momentum is written in terms of spherical coordinates  $(k, \theta, \phi')$ .

To perform the integrations in equation (10) explicitly, we need the following identity (Melrose & Stenhouse 1977):

$$\int_{-1}^1 d(\cos \alpha) \int_0^{2\pi} d\phi \delta(\cos \chi_0 - \cos \chi) = \int_{\cos \alpha_-}^{\cos \alpha_+} \frac{2d(\cos \alpha)}{F(\alpha, \theta, \chi_0)}, \quad (15)$$

with

$$F(\alpha, \theta, \chi_0) = (1 + 2 \cos \alpha \cos \theta \cos \chi_0 - \cos^2 \theta - \cos^2 \alpha - \cos^2 \chi_0)^{1/2} \quad (16)$$

and  $\cos \alpha_{\pm} = \cos(\chi_0 \mp \theta)$ . Using equation (16), we find

$$\alpha_L(\mathbf{k}) = 2C(\mathbf{k})g(\theta, \chi_0) \int_0^{\infty} p^2 f(p) dp, \quad (17)$$

with

$$g(\theta, \chi_0) = \int_{\cos \alpha_-}^{\cos \alpha_+} d \cos \alpha \frac{\Phi(\alpha)}{F(\alpha, \theta, \chi_0)}. \quad (18)$$

#### 2.5. Absorption Coefficient

To evaluate the absorption coefficient, equation (11), we first write

$$\mathbf{k} \cdot \frac{\partial}{\partial \mathbf{p}} = k \cos \chi \frac{\partial}{\partial p} + \frac{k (\cos \alpha \cos \chi - \cos \theta)}{p \sin \alpha} \frac{\partial}{\partial \alpha} - \frac{k}{p \sin^2 \alpha} \left( \frac{\partial}{\partial \phi} \cos \chi \right) \frac{\partial}{\partial \phi}. \quad (19)$$

Our system is axisymmetric, so  $\partial/\partial\phi = 0$ . Thus equation (11) gives

$$\gamma_L(\mathbf{k}) = -C(\mathbf{k}) \int_0^{\infty} p^2 dp \int_{-1}^1 d \cos \alpha \times \int_0^{2\pi} d\phi \delta(\cos \chi_0 - \cos \chi) \left( \Delta p \frac{\partial}{\partial p} + \Delta \theta \frac{\partial}{\partial \theta} \right) f(p, \alpha). \quad (20)$$

The quantities  $\Delta p = k \cos \chi$  and  $\Delta \theta = k(\cos \alpha \cos \chi - \cos \theta)/p \sin \alpha$  are the coefficients of the relevant derivatives in equation (19). Further evaluation of the absorption coefficient involves either performing the integrations directly or expanding the particle distribution in Legendre polynomials (Melrose & Stenhouse 1977). One finds

$$\gamma_L(\mathbf{k}) = 2|\mathbf{k}| C(\mathbf{k}) \left( 2 \cos \chi_0 - \sin \chi_0 \frac{\partial}{\partial \chi_0} \right) \times g(\theta, \chi_0) \int_0^{\infty} p f(p) dp. \quad (21)$$

Neutrinos of all energies interact with the waves satisfying  $\chi = \chi_0$ . For very low energy neutrinos, the condition  $|\mathbf{p}| \gg |\mathbf{k}|$  is not satisfied. The contribution from these low-energy neutrinos is unimportant in the present discussion, and the integral is extended to  $|\mathbf{p}| = 0$ .

### 3. SPECIFIC NEUTRINO DISTRIBUTIONS

Equations (17) and (21) may be evaluated for any separable axisymmetric particle distribution. Our main application of this work is to the neutrinos emitted from the core of Type II supernovae, where the neutrinos passing through a point in the postshock region are beamed due to their distance from the SN core. However, we first show that no isotropic distribution of massless neutrinos can cause kinetic wave growth.

#### 3.1. Isotropic Distributions

An isotropic distribution of neutrinos with arbitrary distribution function  $f(p)$  has  $\Phi(\alpha) = 1$ . Substituting this in equation (18), we find that

$$g(\theta, \chi_0) = \pi, \quad (22)$$

and hence

$$\alpha_L(\mathbf{k}) = \frac{1}{2} C(\mathbf{k}) \int_0^{\infty} p^2 f(p) dp \quad (23)$$

and

$$\gamma_L(\mathbf{k}) = C(\mathbf{k}) |\mathbf{k}| \cos \chi_0 \int_0^{\infty} p f(p) dp > 0. \quad (24)$$

It follows that the absorption coefficient is nonnegative, and hence wave growth is impossible for an isotropic distribution of massless neutrinos. This result also holds for electron and photon beam instabilities.

### 3.2. Beamed Distributions

The analysis relevant to Type II supernovae is continued through the introduction of a particular form of beamed neutrino distribution which allows the analytic evaluation of equations (17) and (21). The absorption coefficient is then maximized (in the negative sense) with respect to the angle of emission to determine for which direction maximum growth may occur.

We consider a highly beamed distribution of the form

$$\Phi(\alpha) = \frac{2}{\alpha_0^2} \exp\left(-\frac{\alpha^2}{2\alpha_0^2}\right), \quad (25)$$

where  $\alpha_0 \ll 1$  is the beam opening angle (which is the angle subtended by the neutrinosphere on the sky for Type II supernovae). We need to evaluate  $g(\theta, \chi_0)$  for this distribution. Clearly,  $g(\theta, \chi_0)$  is nonzero only for  $\theta - \chi_0 \approx 0$ , so we expand  $F(\theta, \chi_0, \alpha)$  with  $\alpha \ll 1$  and  $\theta - \chi_0 \ll 1$ . This leads to

$$F(\theta, \chi_0, \alpha) \approx [\alpha^2 - (\theta - \chi_0)^2]^{1/2} \sin \chi_0. \quad (26)$$

Thus,

$$g(\theta, \chi_0) \approx \frac{\sqrt{2\pi}}{\alpha_0 \sin \chi_0} \exp\left[\frac{-(\theta - \chi_0)^2}{2\alpha_0^2}\right], \quad (27)$$

and we also have

$$\frac{\partial}{\partial \cos \chi_0} g(\theta, \chi_0) \approx \frac{-\sqrt{2\pi}}{\alpha_0^3 \sin^2 \chi_0} (\theta - \chi_0) \exp\left[\frac{-(\theta - \chi_0)^2}{2\alpha_0^2}\right], \quad (28)$$

where we neglect the derivative of the slowly varying  $1/\sin \chi_0$  factor. Hence, the emission coefficient is given by equation (17) with equation (27), and the absorption coefficient is given by

$$\gamma_L(k) = 2C(k)|k| \left(2 \cos \chi_0 + \frac{\theta - \chi_0}{\alpha_0^2}\right) g(\theta, \chi_0) \int_0^\infty pf(p)dp. \quad (29)$$

The right-hand side of equation (29) is negative and largest in magnitude for  $\theta - \chi_0 \approx -\alpha_0$ . Thus the maximum growth for Langmuir waves occurs at an angle  $\theta \approx \chi_0 - \alpha_0$  to the beam axis. The maximum growth rate corresponds to

$$\gamma_{L\max}(k) \approx -\frac{2C(k)k}{\alpha_0} \sin \chi_0 g(\chi_0 - \alpha_0, \chi_0) \int_0^\infty pf(p)dp. \quad (30)$$

Equations (30) and (27) imply  $\gamma_{L\max} \propto I_\nu/\alpha_0^2$ , where  $I_\nu$  is the neutrino intensity.

## 4. APPLICATION TO TYPE II SUPERNOVAE

Neutrinos produced in the core of a Type II supernova are scattered many times as they diffuse outward to a region that allows essentially free propagation (for a review of the physics of Type II supernovae see Bethe 1990). Thus, the neutrinos have an approximately thermal energy spectrum and appear to be emitted from a surface enclosing the core known as the neutrinosphere. Hence, we may write the neutrino distribution function as

$$f(p) = \frac{N}{e^{p/T_\nu} + 1}, \quad (31)$$

with

$$N = \frac{120}{7\pi^4} T_\nu^{-4} I_\nu, \quad (32)$$

which is defined by equation (7), and where  $T_\nu$  and  $I_\nu$  are the temperature (in units of energy) and intensity of the neutrinos, respectively. The use of a thermal distribution of neutrinos implies that some neutrinos in the low-energy tail do not satisfy the kinematic condition (eq. [1]), implying that the neglect of the photon recoil is no longer justified for these neutrinos. However, as already remarked, the fraction of neutrinos not satisfying this condition is negligible, provided that the condition  $T_\nu \gg \hbar\omega_p$  is satisfied, as is the case in practice.

At a distance  $r \gg r_\nu$  from the core, where  $r_\nu$  is the radius of the neutrinosphere, the neutrinos are confined to  $\alpha \lesssim r_\nu/r$ . Hence, we take

$$\alpha_0 \approx \frac{r_\nu}{r} \quad (33)$$

in equation (25). Evaluation of the integral in equation (30) gives, in ordinary units,

$$\gamma_{L\max}(|k|) \approx \frac{-5\sqrt{2\pi} e^{-1/2} G_F^2 c_V^2 |k|^2 \omega}{7\pi^3 \alpha \hbar c^2} \times Z(|k|) \left(1 - \frac{\omega^2}{|k|^2 c^2}\right)^2 \frac{1}{\alpha_0^2} \frac{I_\nu}{T_\nu^2}. \quad (34)$$

The Langmuir wave dispersion relation in the postshock region of a Type II supernova requires careful consideration. The relevant plasma is in an awkward parameter regime. Although degeneracy may be neglected, the plasma is neither ultrarelativistic nor nonrelativistic. Furthermore, the kinematic condition on wave emission,  $v_\phi < 1$ , excludes the use of simple, known interpolation formulae which are valid only for  $v_\phi > 1$  (Braaten & Segel 1993). The approach adopted here is to calculate the dispersion relation,  $\omega_L(k)$ , and the normalization factor,  $Z(k)$ , directly from the general expressions for an electron-positron plasma given by Hayes & Melrose (1984). This theory is summarized in Appendix B.

We adopt the parameters used by BDSB for the material in the postshock region of a Type II supernova:  $n_e = 10^{36} \text{ m}^{-3}$ ,  $I_\nu = 3 \times 10^{33} \text{ W m}^{-2}$ ,  $r = 300 \text{ km}$ , and  $\alpha_0 = 3 \times 10^{-2}$ . We also assume a neutrino temperature of  $T_\nu = 3 \text{ MeV}$ . The electron temperature is thought to be around  $2.5 \times 10^9 \text{ K}$  (Miller, Wilson, & Mayle 1993).

### 4.1. Absorption Coefficient for Supernova Plasma

Using the techniques outlined in Appendix B, dispersion relations for a variety of plasma temperatures and a fixed electron density of  $n_e = 10^{36} \text{ m}^{-3}$  are plotted in Figure 4. This is a plot of phase speed,  $v_\phi/c$ , against normalized wavenumber,  $|k|/2mc$ . The cutoff in the dispersion relation at high  $|k|$  occurs when the intrinsic damping of the mode (the Landau damping) becomes comparable to the frequency of the mode. In this event the plasma does not have a normal mode at that wave momentum, as any wave is highly damped within one wave cycle. In a nonrelativistic plasma, Landau damping limits the range of phase velocities to  $v_\phi \geq eV_e = 3(T_e/m)^{1/2}$ . As  $T_e$  increases, the range  $v_{\min} < v_\phi < 1$  shrinks, where  $v_\phi = v_{\min}$  is the limit imposed by Landau damping. In the ultrarelativistic limit the allowed range of  $v_\phi < 1$  becomes very narrow.

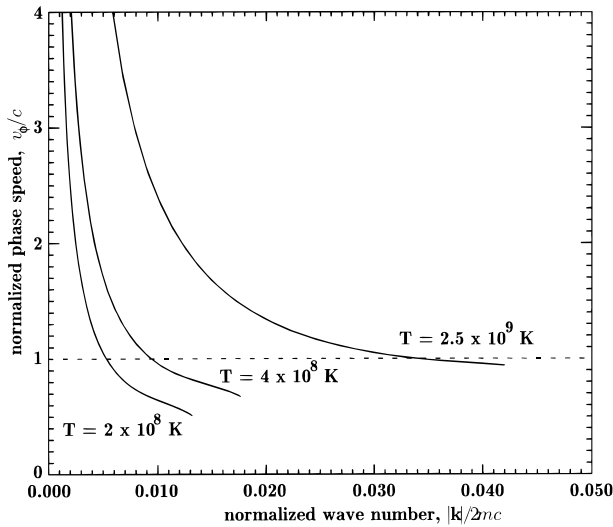


FIG. 4.—Plasmon dispersion relations for  $n_e = 10^{36} \text{ m}^{-3}$ . Note that emission of Langmuir waves by neutrinos is only allowed for  $v_\phi < c$ . The dispersion relation terminates at high wavenumber due to the destruction of the longitudinal mode by Landau damping.

This has an immediate consequence for the strength of the emission process of Langmuir waves by neutrinos. Equation (2) shows that the emission probability is proportional to  $(1 - v_\phi^2)^2$  which is small for  $v_\phi \approx 1$ . This greatly suppresses the absorption coefficient in these high-temperature plasmas. In the ultrarelativistic limit, the region for which  $v_\phi < 1$  is vanishingly small, and all emission is forbidden.

The maximum negative absorption coefficient, equation (34), for  $T = 2.5 \times 10^9 \text{ K}$  is plotted in Figure 5 as a function of normalized wavenumber. It is clear that this growth rate is far too small to produce any Langmuir turbulence over the timescale of the 3 s neutrino burst. Indeed, this instability is too weak to overcome even the Landau damping of the SN plasma. The growth rate calculated here is approximately 11 orders of magnitude smaller than that calculated by BDSB, who obtained a growth rate for a reactive instability of  $\approx 10^6 \text{ s}^{-1}$ . Part of the difference between these results lies in the fact that a kinetic instability has been considered here (with a thermal distribution of neutrinos),

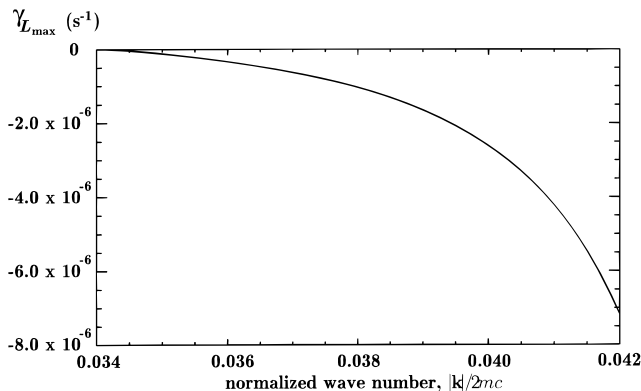


FIG. 5.—Maximum negative absorption for  $n_e = 10^{36} \text{ m}^{-3}$ ,  $I_\nu = 3 \times 10^{33} \text{ W m}^{-2}$ ,  $r = 300 \text{ km}$ ,  $\alpha_0 = 3 \times 10^{-2}$ ,  $T_\nu = 3 \text{ MeV}$ , and  $T_e = 2.5 \times 10^9 \text{ K}$ . The range of wavenumbers is that for which emission of a Langmuir wave is kinematically allowed, and for which a longitudinal mode exists in the plasma (see Fig. 4). The absorption coefficient is far too small to produce Langmuir turbulence in the postshock plasma of a Type II supernova.

rather than the reactive instability considered by BDSB (who assume a monoenergetic distribution of neutrinos). The greatest part of the discrepancy lies in the suppression due to the high phase speed of the longitudinal waves of the SN plasma, which is a factor of approximately  $10^6$ . This does not appear in the work of BDSB because of their artificial assumption concerning the properties of neutrinos, as is discussed below.

One further point regarding the collisional damping of the plasma is required. This damping was not considered by BDSB. For the parameters of the postshock plasma assumed by BDSB the collisional damping is  $\gamma_{\text{coll}} \approx 10^{15} \text{ s}^{-1}$ , which is many orders of magnitude greater than the growth rates calculated either here or in BDSB. It is not clear to the present authors how any wave mode may be generated in a plasma with such overwhelming collisional damping. We conclude that the neutrino beam instability is ineffective in the postshock region of a Type II supernova.

#### 4.2. Comparison with BDSB

The kinetic plasma instability discussed above is different from the reactive instability discussed by BDSB. As with most instabilities, these could be regarded as kinetic and reactive limits of a more general instability (e.g., Briggs 1964; Melrose 1986). The kinetic version applies when the growth rate is less than the bandwidth of the growing waves, when the random-phase approximation applies, and the reactive version applies in the opposite limit when a fixed-phase disturbance grows in association with localized bunching of the particles (neutrinos).

However, this equivalence does not apply to the kinetic instability discussed here or the reactive instability discussed by BDSB because of important differences in the underlying treatment of the neutrinos. The theory of BDSB describes the neutrino-Langmuir wave coupling through an effective potential in a Klein-Gordon equation for the neutrino wave field (Bethe 1986). Thus the assumed neutrino dynamics are those of a spin 0 particle, rather than the left-handed spin  $\frac{1}{2}$  particle of the standard model. The consequence of this assumption is that emission of a Langmuir wave in the forward direction is allowed in the theory of BDSB, while it is forbidden in the electroweak theory due to parity considerations. This is demonstrated in equation (4) above, and in D'Olivo, Nieves, & Pal (1996).

We commence our comparison of the theories from equation (20) of BDSB, which may be written in our notation as

$$(\Omega + i\gamma_{\text{Landau}})^2 - \omega_p^2 = -\frac{G_F^2 \omega_p^4}{2\pi\alpha} \int \frac{\mathbf{k} \cdot \partial f(\mathbf{p})/\partial \mathbf{p}}{\omega - \mathbf{k} \cdot \mathbf{v} + i0} d^3\mathbf{p}, \quad (35)$$

where  $\omega_p$  is the plasma frequency,  $\Omega = \omega - i\gamma$  includes the real wave frequency and the wave damping, and  $i0$  denotes an infinitesimal imaginary part (the Landau prescription). This equation describes the contribution of neutrinos to the dispersive properties of Langmuir waves in the theory of BDSB. Note that in the limit of no neutrino flux, equation (35) gives  $\omega = \omega_p$ , which is the cold plasma dispersion relation, and  $\gamma = \gamma_{\text{Landau}}$ , which is the Landau damping of the finite-temperature plasma.

The growth rate of the reactive instability of BDSB is calculated by considering the real part of the right-hand side of equation (35). The growth rate for the kinetic instability, in the cold plasma limit, is calculated from the imaginary part of the right-hand side of equation (35), because of

the pole in the integrand. This imaginary part may be isolated through the Plemelj formula:

$$\frac{1}{\omega - \omega_0 + i0} = \mathcal{P} \frac{1}{\omega - \omega_0} - i\pi\delta(\omega - \omega_0). \quad (36)$$

This imaginary part picks out the resonant interactions given by the condition  $\omega - \mathbf{k} \cdot \mathbf{v} = 0$ .

To illustrate the effect of the different assumptions made concerning the neutrino properties, we calculate the kinetic counterpart of the reactive instability considered by BDSB. The reactive version is obtained from equation (35) by omitting the factor  $i0$  and partially integrating before setting  $f(\mathbf{p}) \propto \delta^3(\mathbf{p})$ , and solving the resulting algebraic equation for solutions with complex frequency. The analogous kinetic version is obtained by retaining only the resonant part when equation (36) is inserted in equation (35). This gives  $\gamma = \gamma_{\text{Landau}} - \gamma_{\text{L}}^{\text{BDSB}}$ , where

$$\gamma_{\text{L}}^{\text{BDSB}} = -\frac{G_{\text{F}}^2 \omega_p^3}{4\alpha} \int \mathbf{k} \cdot \frac{\partial f(\mathbf{p})}{\partial \mathbf{p}} \delta(\omega - \mathbf{k} \cdot \mathbf{v}) d^3 \mathbf{p}. \quad (37)$$

The result (eq. [37]) is to be compared with equation (11) in the cold plasma limit, where  $Z(\mathbf{k}) = 1$  and  $\omega = \omega_p$ , which gives

$$\gamma_{\text{L}} = -\frac{G_{\text{F}}^2 c_{\text{V}}^2 \omega_p^2 |\mathbf{k}|^2}{4\alpha \omega} \left(1 - \frac{\omega^2}{|\mathbf{k}|^2}\right)^2 \times \int \mathbf{k} \cdot \frac{\partial f(\mathbf{p})}{\partial \mathbf{p}} \delta(\omega - \mathbf{k} \cdot \mathbf{v}) d^3 \mathbf{p}. \quad (38)$$

For  $|\mathbf{k}| \approx \omega_p$ , as assumed by BDSB, equations (37) and (38) differ only in the factor  $(1 - v_\phi^2)^2$  in equation (38). This is precisely the term which appears through the absence of a right-handed component in the neutrino plane-wave state, as discussed above. In particular, in a mildly relativistic plasma where the only relevant Langmuir waves have  $v_\phi < 1$  and  $1 - v_\phi \ll 1$ , the factor  $(1 - v_\phi^2)^2$  in equation (38) implies that the growth rate for left-handed neutrinos is very much smaller than for the boson-like neutrinos assumed by BDSB.

It is clear from the difference between equations (37) and (38) for  $|\mathbf{k}| \approx \omega_p$  that any analysis of neutrino-plasma interactions must be based on an explicit electroweak description of the microscopic processes occurring in the plasma. This has the added advantage of allowing the analysis to be

performed at relativistic temperatures, high densities, and for broadband neutrino distributions, which are the major approximations in the work of BDSB. An electroweak theory description of the reactive neutrino beam instability is in progress.

## 5. CONCLUSION

In principle, a sufficiently intense, highly beamed distribution of neutrinos generates Langmuir turbulence in a manner analogous to an electron or photon beam. The kinetic version of this instability, which is studied here, is found to be too weak an instability to be relevant in any known terrestrial or space plasma. The reactive counterpart of this instability was suggested by BDSB in connection with the reheating of the plasma behind the stalled shock of a Type II supernova. However, for simplicity, BDSB used a boson-like model for the neutrino rather than the electroweak theory which allows only left-handed neutrinos. From a comparison of the results of our work, based on the electroweak theory, with that of BDSB, it is clear that the two theories lead to quantitatively different results for Langmuir waves with phase speeds close to the speed of light; cf. equations (37) and (38). As the case  $v_\phi < 1$  and  $1 - v_\phi \ll 1$  is relevant to the SN problem, an electroweak theory of the reactive instability needs to be constructed, allowing for finite neutrino bandwidth, a finite temperature plasma, and correct inclusion of the neutrino spin effects. Such a theory is presently being developed.

However, given the strong collisional damping of the SN plasma, it is unlikely that even the reactive instability can be sufficiently strong to operate in the way suggested by BDSB. It may be that other plasma instabilities driven by an intense neutrino beam operate in such plasmas. A thorough investigation of the consequences of the large induced current generated by a strong flux of neutrinos in a dense plasma is required. Such an investigation should study the effect of an intrinsic neutrino mass, and other extensions to the standard model of particle physics, to determine whether there are any conditions under which neutrino plasma instabilities can be of astrophysical significance.

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## APPENDIX A

### CALCULATION OF THE PROBABILITY

The theory of the neutrino-photon vertex, as shown in Figure 3, has been derived to first order in  $1/M_{\text{W}}^2$  (D'Olivo, Nieves, & Pal 1989; Braaten & Segel 1993), where it is shown that the scattering matrix for longitudinal plasmon emission may be written

$$M_{\text{fi}} = \frac{G_{\text{F}}}{\sqrt{s}} \frac{ic_{\text{V}}}{\sqrt{4\pi\alpha}} \bar{v} \gamma_\mu (1 - \gamma_5) v \alpha^{\mu\nu}(k) \varepsilon_\nu(k), \quad (\text{A1})$$

where  $\alpha^{\mu\nu}(k)$  is the polarization tensor for the electron gas,  $\varepsilon_\nu(k)$  represents the polarization 4-vector of the longitudinal Langmuir wave,  $\alpha$  is the fine-structure constant, and

$$c_{\text{V}} = \begin{cases} 2 \sin^2 \theta_{\text{W}} + \frac{1}{2} & \text{for } \nu_e, \\ 2 \sin^2 \theta_{\text{W}} - \frac{1}{2} & \text{for } \nu_\mu, \nu_\tau. \end{cases} \quad (\text{A2})$$

As we consider coupling to longitudinal plasmons, only the longitudinal part of the linear response tensor contributes to equation (A1) (Tsytovich 1964; Braaten & Segel 1993). Thus we may write (Melrose 1982; Nieves & Pal 1989)

$$\alpha^{\mu\nu}(k) \approx \alpha^L(k)L^{\mu\nu}(k, u), \quad (\text{A3})$$

where

$$L^{\mu\nu} = -\frac{k^2}{k^2 - (ku)^2} \left[ \frac{k^\nu u^\mu}{ku} + \frac{k^\mu u^\nu}{ku} - \frac{k^2 u^\mu u^\nu}{(ku)^2} - \frac{k^\mu k^\nu}{k^2} \right], \quad (\text{A4})$$

and, from the dispersion relation for the plasmons (Melrose 1986),

$$\alpha^L(k) = -\frac{(ku)^2}{\mu_0} = -\frac{\omega^2}{\mu_0}. \quad (\text{A5})$$

The second form of the above expression is evaluated in the rest frame of the plasma, that is, the frame in which the 4-velocity of the plasma is  $u^\mu = (1, \mathbf{0})^\mu$ .

We choose the temporal gauge and write the polarization 4-vector for the plasmons as

$$\varepsilon^\nu(k) = \frac{k^\nu - (ku)u^\nu}{[(ku)^2 - k^2]^{1/2}} = \left( \frac{0, \mathbf{k}}{|\mathbf{k}|} \right)^\nu. \quad (\text{A6})$$

Explicit evaluation of  $|M_{fi}|$  through equations (A3) and (A6) gives

$$|M_{fi}|^2 = \frac{G_F^2 c_V^2}{2 \cdot 4\pi\alpha} |\mathbf{k}|^2 \omega^2 M_{\mu\nu} N^{\mu\nu}, \quad (\text{A7})$$

with

$$M_{\mu\nu} = 8[2p_\mu p_\nu - k_\nu p_\mu - p_\nu k_\mu + (pk)g_{\mu\nu} + i\varepsilon_{\alpha\nu\beta\mu} k^\alpha p^\beta] \quad (\text{A8})$$

and

$$N^{\mu\nu} = \left( 1, \frac{\omega\mathbf{k}}{|\mathbf{k}|^2} \right)^\mu \left( 1, \frac{\omega\mathbf{k}}{|\mathbf{k}|^2} \right)^\nu, \quad (\text{A9})$$

with  $\omega = \omega_L(\mathbf{k})$  and where  $p^\mu = [\varepsilon(\mathbf{p}), \mathbf{p}]^\mu$  and  $k^\mu = [\omega_L(\mathbf{k}), \mathbf{k}]^\mu$  are the 4-momenta of the neutrino and the plasmon, respectively. Conservation of 4-momentum implies

$$pk = \frac{1}{2}k^2. \quad (\text{A10})$$

The probability per unit time of the emission of a Langmuir wave from a neutrino may be written

$$w_L(\mathbf{p}, \mathbf{k}) = |M_{fi}|^2 \frac{V |a_M(\mathbf{k})|^2}{2\varepsilon(\mathbf{p})2\varepsilon'(\mathbf{p})} 2\pi\delta[\varepsilon'(\mathbf{p}) + \omega_L(\mathbf{k}) - \varepsilon(\mathbf{p})], \quad (\text{A11})$$

where  $\varepsilon(\mathbf{p})$  and  $\omega_L(\mathbf{k})$  denote the neutrino and Langmuir wave dispersion relations, respectively.

Substituting equations (A7) and (A10) in equation (A11), one has

$$w_L(\mathbf{p}, \mathbf{k}) = \frac{G_F^2 c_V^2}{16\pi\alpha} \frac{|\mathbf{k}|^2}{\omega_p^2} \frac{\omega^3}{\varepsilon\varepsilon'} 2\pi\delta(\varepsilon' + \omega - \varepsilon)[(2\varepsilon - \omega)^2 - |\mathbf{k}|^2] \left( 1 - \frac{\omega^2}{|\mathbf{k}|^2} \right)^2, \quad (\text{A12})$$

which is the probability per unit time of the emission of a Langmuir wave by a neutrino propagation in a medium. This expression is consistent with that derived by D'Olivo, Nieves, & Pal (1996), who calculated the spontaneous emission rate of both longitudinal and transverse wave quanta.

## APPENDIX B

### LONGITUDINAL PHOTON DISPERSION RELATIONS IN A FERMI-DIRAC PLASMA

The general dispersion relation for the longitudinal mode of an isotropic Fermi-Dirac plasma may be expressed to first order in the fine-structure constant as an integral equation. This has been done by Hayes & Melrose (1984), and we reproduce the relevant details of their work here.

The frequency of a longitudinal wave at a given wavenumber is given by the solution for  $\omega = \omega_L(\mathbf{k})$  of the dispersion equation

$$\omega^2 = -\mu_0 \alpha^L(k), \quad (\text{B1})$$

where  $\alpha^L(k)$  is the longitudinal response function. This function is given by

$$\alpha^L(k) = \frac{e^2 \bar{n}_e \omega^2}{m |\mathbf{k}|^2} + \frac{e^2 m \omega^2}{2\pi^2 |\mathbf{k}|^3} \left\{ \frac{1}{4} (\omega^2 - |\mathbf{k}|^2) S^{(0)}(k) - m\omega S^{(2)}(k) + m^2 S^{(2)}(k) \right\}, \quad (\text{B2})$$

where

$$\bar{n}_e = 2 \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{m}{\varepsilon} \bar{n}(\varepsilon) \quad (\text{B3})$$

is the proper electron number density, and

$$\bar{n}(\varepsilon) = n^-(\varepsilon) + n^+(\varepsilon) \quad (\text{B4})$$

is the sum of the electron,  $n^-(\varepsilon)$ , and positron,  $n^+(\varepsilon)$ , occupation numbers. The plasma dispersion functions of equation (B2) are given by

$$S^{(0)}(k) = \int \frac{d\varepsilon}{m} \bar{n}(\varepsilon) \ln \Lambda_1, \quad (\text{B5})$$

$$S^{(1)}(k) = \int \frac{d\varepsilon}{m^2} \bar{n}(\varepsilon) \varepsilon \ln \Lambda_2, \quad (\text{B6})$$

$$S^{(2)}(k) = \int \frac{d\varepsilon}{m^3} \bar{n}(\varepsilon) \varepsilon^2 \ln \Lambda_1, \quad (\text{B7})$$

with

$$\Lambda_1 = \frac{(\varepsilon_+ - \varepsilon + \omega)(\varepsilon_+ - \varepsilon - \omega)(\varepsilon_+ + \varepsilon - \omega)(\varepsilon_+ + \varepsilon + \omega)}{(\varepsilon_- - \varepsilon + \omega)(\varepsilon_- - \varepsilon - \omega)(\varepsilon_- + \varepsilon - \omega)(\varepsilon_- + \varepsilon + \omega)}, \quad (\text{B8})$$

$$\Lambda_2 = \frac{(\varepsilon_+ - \varepsilon + \omega)(\varepsilon_+ + \varepsilon - \omega)(\varepsilon_- - \varepsilon - \omega)(\varepsilon_- + \varepsilon + \omega)}{(\varepsilon_- - \varepsilon + \omega)(\varepsilon_- + \varepsilon - \omega)(\varepsilon_+ - \varepsilon - \omega)(\varepsilon_+ + \varepsilon + \omega)}, \quad (\text{B9})$$

and

$$\varepsilon_{\pm} = (\varepsilon \pm 2|\mathbf{p}| |\mathbf{k}| + |\mathbf{k}|^2)^{1/2}. \quad (\text{B10})$$

Various simplifications for the degenerate, nonrelativistic, and ultrarelativistic limits exist for equation (B2), reducing it to either a simple equation or an algebraic transcendental equation (Hayes & Melrose 1984; Braaten & Segel 1993). However, in its full generality, equation (B2) must be solved numerically for specific electron-positron distributions.

One further plasma quantity is required for the analysis of § 4, namely, the wave normalization factor,  $Z(k)$ ; this is given by

$$[Z(k)]^{-1} = \left[ 1 + \frac{\mu_0}{2\omega} \frac{\partial}{\partial \omega} \alpha^L(k) \right] \Big|_{\omega=\omega_L(k)} \quad (\text{B11})$$

and is twice the ratio of electric to total energy in the wave mode (Melrose 1986).

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