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Some properties of magnetized pair plasmas

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Abstract. Three aspects of the physics of pair plasmas are discussed. The main emphasis is on the properties of waves in cold, magnetized pair plasmas, where it is shown that the inclusion of an admixture of positrons introduces an additional 'cyclotron mode' into the magneto-ionic theory. Thermal effects on these waves are described. The quadratic nonlinear response and the fluid description of a pair plasma are discussed briefly.

1. Introduction

Pair plasmas have been of interest in astrophysics for several decades. Highly relativistic pairs are created in the polar cap regions and populate the magnetospheres of pulsars (e.g., Michel 1991, Mészáros 1992, Beskin *et al* 1993), ultimately escaping to form a pulsar wind. Pairs also play a central role in the high-energy emission from active galactic nuclei (AGN) (e.g., Lightman and Band 1981, Zdziarski 1985) and from x-ray transients (e.g., Levinson and Blandford 1996). Recently pair plasmas have become of interest in the laboratory (e.g., Surko *et al* 1989, Greaves *et al* 1994). A wide variety of different aspects of the physics of pair plasmas are of relevance in these various contexts. The properties of waves in pair plasmas (e.g., Melrose and Stoneham 1977, Allen and Melrose 1982, Beskin *et al* 1993, Kazbegi *et al* 1991, Volokitin *et al* 1985) and of three-wave interactions (e.g., Istomin 1988, Luo and Melrose 1996) are of interest in pulsar magnetospheres, where the pairs flow outward at highly relativistic speed along superstrong ($B \gtrsim 10^7$ T) curved magnetic field lines. The pair-plasma counterpart of MHD theory, and of MHD shocks (e.g., Gallant *et al* 1992), are relevant to pulsar winds and to jets in AGN and x-ray transients. In connection with laboratory pair plasmas, a major theme in theoretical work has been the properties of waves in pair plasmas.

In this paper, three aspects of pair plasmas are discussed, with emphasis on the properties of waves in non-relativistic, magnetized, pair plasmas. The ratio of positrons to electrons is described by the average charge number $\xi = (n_+ - n_-)/(n_+ + n_-)$, where $n_\pm$ are the number densities of positrons and electrons, respectively. The case $\xi = -1$ corresponds to an electron gas and $\xi = 0$ corresponds to a pure pair plasma (PPP). The properties of waves in a cold, magnetized pair plasma are discussed in section 2, and thermal modifications to these properties are considered in section 3. Some brief remarks are made about the quadratic nonlinear response of a PPP in section 4. The fluid equations for a relativistic, cold pair plasma are written down and discussed briefly in section 5.

2. Waves in a cold magnetized pair plasma

The properties of waves in cold, magnetized, PPP have been discussed by Stewart and Liang (1992) and Iwamoto (1993). Zank and Greaves (1995) used a fluid theory to treat

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waves in magnetized PPP. How the properties change from the magneto-ionic modes (for $\xi = -1$) to the PPP ($\xi = 0$) is described here. Note that the ions play no role at frequencies well above their natural frequencies, and are ignored here, and that charge neutrality is not assumed.

2.1. The dispersion equation

In a coordinate system in which the ambient magnetic field, $\boldsymbol{B}$, is along the z-axis and the wavevector, $\boldsymbol{k}$, is in the x–z plane at an angle θ to $\boldsymbol{B}$, the cold plasma dispersion equation is (cf Stix 1962)

$$\begin{vmatrix} S - n^2\cos^2\theta & -\mathrm{i}D & -n^2\sin\theta\cos\theta \\ \mathrm{i}D & S - n^2 & 0 \\ -n^2\sin\theta\cos\theta & 0 & P - n^2\sin^2\theta \end{vmatrix} = 0 \tag{1}$$

$$S = 1 - \frac{X}{1-Y^2} \qquad D = -\xi\left(\frac{XY}{1-Y^2}\right) \qquad P = 1 - X \tag{2}$$

with $\omega_\mathrm{p}^2 = \omega_{\mathrm{p}+}^2 + \omega_{\mathrm{p}-}^2$, $\Omega_\mathrm{e} = eB/m$, $X = \omega_\mathrm{p}^2/\omega^2$ and $Y = \Omega_\mathrm{e}/\omega$. The solutions of the dispersion equation (1) are the two magneto-ionic modes in the familiar case of an electron gas ($\xi = -1$). There are four branches of the magneto-ionic modes (cf the full curves in figure 1). Starting at high frequency, the extraordinary mode includes the x-mode which extends from $\omega = \infty$ to a cut-off at $\omega_{\mathrm{x}+}$,

$$\omega_{\mathrm{x}\pm} = \pm\tfrac{1}{2}\xi\Omega_\mathrm{e} + \tfrac{1}{2}(4\omega_\mathrm{p}^2 + \Omega_\mathrm{e}^2)^{1/2} \qquad \text{for } |\xi| = 1 \tag{3}$$

and the z-mode which extends from a resonance at $\omega_+(\theta)$,

$$\omega_\pm^2(\theta) = \tfrac{1}{2}\omega_\mathrm{UH}^2 \pm \tfrac{1}{2}[\omega_\mathrm{UH}^4 - 4\omega_\mathrm{p}^2\Omega_\mathrm{e}^2\cos^2\theta]^{1/2} \tag{4}$$

where $\omega_\mathrm{UH} = (\omega_\mathrm{p}^2 + \Omega_\mathrm{e}^2)^{1/2}$ is the upper hybrid frequency, to a cut-off at $\omega_{\mathrm{x}-}$. The ordinary mode includes the o-mode which extends from $\omega = \infty$ to a cut-off at ω_p and the whistler mode, which extends from a resonance at $\omega_-(\theta)$ to zero frequency (provided the motion of ions is neglected).

For a PPP, the dispersion equation (1) with $\xi = 0$ is of the same form as that for a uniaxial crystal, with $K_\perp = S$ and $K_\parallel = P$. The dispersion relations are then given by the well known formulae for the 'ordinary' (O) and 'extraordinary' (X) modes of a uniaxial crystal. The PPP modes have dispersion relations

$$n_\mathrm{O}^2 = 1 - \frac{X}{1-Y^2} \qquad n_\mathrm{X}^2 = \frac{(1-X-Y^2)(1-X)}{1-X-Y^2+XY^2\cos^2\theta} \tag{5}$$

respectively. The O-mode propagates for $\omega > \omega_\mathrm{UH}$ and $\omega < \Omega_\mathrm{e}$, and is evanescent for $\Omega_\mathrm{e} < \omega < \omega_\mathrm{UH}$. The high-frequency branches of the PPP O-mode and X-mode transform into the magneto-ionic x-mode and o-mode, respectively, as $|\xi|$ is increased from $0 \to 1$.

The longitudinal part of the linear response tensor is unaffected by the gyrotropy and hence by the value of ξ. Thus, the properties of Langmuir waves in a cold, magnetized, pair plasma are identical to those is a cold, magnetized, electron gas. For propagation along the magnetic field, Langmuir waves have frequency $\omega = \omega_\mathrm{p}$ (plus the usual thermal correction $3k^2V_\mathrm{e}^2/2\omega_\mathrm{p}$ in practice). At arbitrary angle θ there are two solutions with frequencies given by (4), with the upper frequency solution corresponding to Langmuir waves for $\Omega_\mathrm{e} < \omega_\mathrm{p}$. For perpendicular propagation ($\theta = \pi/2$) one has $\omega_+(\pi/2) = \omega_\mathrm{UH}$ and $\omega_-(\pi/2) = 0$. For parallel propagation, $\omega_\pm(0)$ reduce to the maximum and minimum of ω_p and Ω_e, respectively. Both these resonances occur in the PPP X-mode.

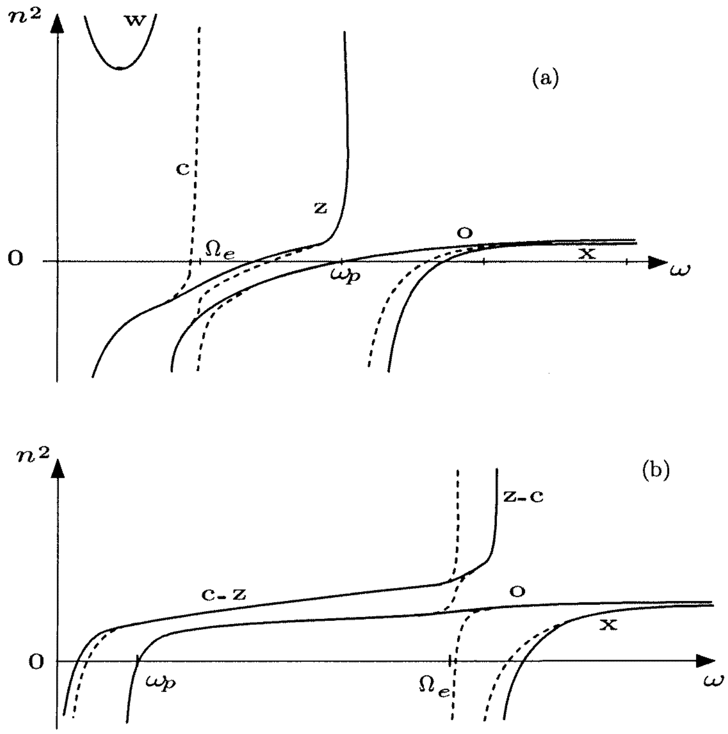

Figure 1. (a) A schematic illustration of the refractive index curves for $\omega_{\rm p} \gg \Omega_{\rm e}$ in a cold electron gas (full curve) and the modifications introduced by a small admixture of positrons (dashed curves). (b) As for (a) but for $\omega_{\rm p} \ll \Omega_{\rm e}$ and omitting the whistler mode.

2.2. *Waves for arbitrary ξ*

The case of an electron gas, $\xi = -1$, and a PPP, $\xi = 0$, are both special. The terms proportional to $1/(1 - Y^2)^2$ cancel in (1) for $\xi = -1$. This cancellation does not occur for $\xi \neq -1$. The term proportional to $1/(1 - Y^2)^2$ for $\xi \neq -1$ leads to an additional cut-off and an additional resonance, and hence to an additional branch of the cold plasma modes. The special nature of the PPP is apparent when one considers how the cut-offs change as a function of $\xi \to 0$.

The two cut-offs at (3) for $\xi = -1$ are replaced by three cut-offs for $\xi \neq -1$. (The o-mode cut-off at $\omega_{\rm p}$ is unaffected.) These are the positive-frequency solutions of the cubic equations

$$\omega^3 - \omega(\omega_{\rm p}^2 + \Omega_{\rm e}^2) \pm \xi \omega_{\rm p}^2 \Omega_{\rm e} = 0. \tag{6}$$

The cubic functions are plotted schematically in figure 2 for $\Omega_{\rm e} < \omega_{\rm p}$. Consider first the case $|\xi| = 1$. The two cubic equations, corresponding to the $\pm$ signs in (6), give the two dashed curves. They give zeros at $\omega = \omega_{\rm x\pm}$, $\omega = \Omega_{\rm e}$, corresponding to the points 1, 2 and 3 in figure 2. Now decreasing $|\xi|$ the dashed curves move together, and approach the full curve, which corresponds to $\xi = 0$. The upper two cut-offs approach each other, and coincide at $\omega = \omega_{\rm UH}$ for $\xi = 0$. The third cut-off, which is at the cyclotron frequency for $|\xi| = 1$, moves to lower frequency with decreasing $|\xi|$, and approaches zero as ξ approaches

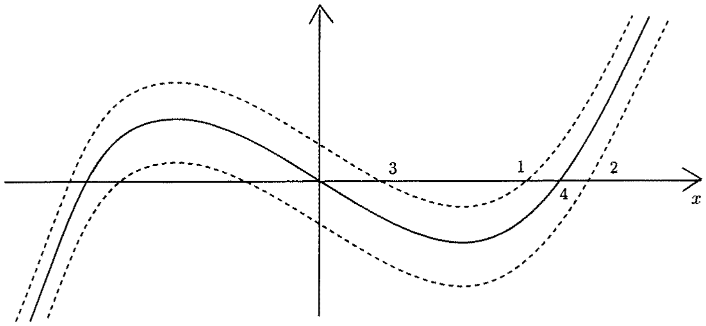

Figure 2. The cubic equations (6) are plotted schematically for $\omega_p > \Omega_e$ as a function of $x = \omega/\Omega_e$ for $\xi = -1$ (dashed curves) and $\xi = 0$ (full curve).

zero. Thus the PPP case is special in that the number of cut-off reduces by two for $\xi = 0$ compared with $\xi \neq 0$.

The two resonant frequencies (4) are unaffected by the value of ξ, as is the o-mode cut-off at ω_p. The presence of a term proportional to $1/(1 - Y^2)^2$ in (1) for $|\xi| \neq 1$ leads to an additional resonance at $Y = 1$.

2.3. The cyclotron branch

The additional cut-off and resonance imply the existence of a new wave mode for $\xi \neq -1$, referred to here as the cyclotron (c) mode. The c-mode exists over a frequency range just below Ω_e. The width of the frequency band in which it propagates increases with ξ from zero for $\xi = -1$ to $0 < \omega < \Omega_e$ for $\xi = 0$. The changes to the magneto-ionic modes introduced by including an admixture of positrons are illustrated schematically in figure 1(a) for $\omega_p > \Omega_e$, where the c-mode appears as a new branch, and in figure 1(b) for $\omega_p < \Omega_e$, where the c-mode couples to the z-mode to form z–c and c–z branches.

One physical implication of the appearance of the cyclotron mode in a strongly magnetized plasma, $\Omega_e \gg \omega_p$, is that the o-mode at $\omega \ll \Omega_e$ cannot escape from the plasma. In an electron gas the o-mode can escape without encountering a stop band for all $\omega > \omega_p$. The inclusion of positrons in a cold electron gas implies that the o-mode encounters a resonance and a stop band, just like the z-mode, so that only o-mode waves at $\omega > \Omega_e$ and x-mode waves at $\omega > \omega_{x+}$ can escape.

It might be remarked that the foregoing results do not support some specific interpretations of Iwamoto (1993), who considered the case of parallel and perpendicular propagation in a PPP. Both the PPP and the cases of parallel and perpendicular propagation are special, and cannot be used to infer more general properties of the modes.

3. Thermal effects on the c-mode

Thermal effects modify the wave properties in important ways near the cyclotron resonance. Three thermal effects are relevant: (a) the washing out of the c-mode, and coupling between the c-mode and the known (b) parallel and (c) perpendicular cyclotron resonant modes.

3.1. Washing out of the c-mode

Thermal effects tend to wash out the cyclotron resonance and limit the refractive index, n, to a maximum

$$n < n_{\max} = \frac{1}{\sqrt{2}} \left(\frac{\omega_{\rm p}^2}{\Omega_{\rm e}^2 \beta_{\rm e}} \right)^{1/3}. \tag{7}$$

Thermal effects wash out the cyclotron branch entirely if the frequency range in which the mode exists is less that the thermal Doppler width. This implies that the cyclotron mode is washed out except for

$$1 + \xi \gg \frac{|\omega_{\rm p}^2 - 2\Omega_{\rm e}^2|}{\omega_{\rm p}^2} \left(\frac{\omega_{\rm p}^2 \beta_{\rm e}^2 \cos^2\theta}{\Omega_{\rm e}^2} \right)^{1/3} \tag{8}$$

which applies only for $|\omega_{\rm p}^2 - 2\Omega_{\rm e}^2| \gtrsim \omega_{\rm p}^2$.

3.2. The parallel cyclotron resonance

For $\theta \to 0$ the resonant frequencies (4) approach either $\omega_{\rm p}$ or $\Omega_{\rm e}$, the latter being the parallel cyclotron resonance. For $\xi \neq -1$, the parallel cyclotron resonance coincides in frequency with the resonance in the c-mode. For small θ thermal effects should cause the two dispersion curves to reconnect to form two modified modes. How this occurs in detail has yet to be explored.

3.3. The cyclotron harmonic modes

Cyclotron harmonic wave modes exist for nearly perpendicular propagation in a thermal electron gas. These are often called the Bernstein modes, which name is appropriate only for the class of longitudinal such waves (the GB modes here) discussed by Bernstein (1958) and considered earlier by Gross (1951). Two further classes of (transverse) perpendicular cyclotron wave modes were discussed by Dnestrovskii and Kostomarov (1961, 1962) (cf the review by Bornatici *et al* (1983)). Thus at a given harmonic, $\omega \approx n\Omega_{\rm e}$, for $\theta \approx \pi/2$, there are three cyclotron harmonic waves: referred to here as the GB, DK and ordinary modes (cf Robinson (1988) who discussed relativistic effects on these modes).

For $\theta = \pi/2$ the dispersion equation factorizes and the effect of $\xi \neq -1$ can be identified relatively simply. The GB mode and ordinary modes are unaffected by $\xi \neq -1$. For harmonics $n > 1$, the modifications to the DK mode are minor. For $n = 1$, there are four perpendicular cyclotron harmonic modes: the GB and ordinary modes and two modified modes that arise from coupling between the DK mode and the c-mode. The resonances in these two coupled modes are unaffected by the value of ξ, but the cut-offs are affected, as in the cold plasma limit. In the special case of a PPP the dispersion curves for the coupled modes separate very slowly with increasing n^2 away from their common cut-off at $\omega = \Omega_{\rm e}$. These qualitative properties have yet to be explored quantitatively.

4. Quadratic nonlinear response

The quadratic nonlinear response tensor, $\alpha_{ijs}(k, k_1, k_2)$, with $k = (\omega, \boldsymbol{k})$ and so on, determines the rate of three-wave interactions with $k = k_1 + k_2$. In an unmagnetized PPP with identical electron and positron distributions, the quadratic nonlinear response tensor is identically zero, and three-wave interactions are forbidden. In a cold magnetized PPP the non-zero components of α_{ijs} arise only from the off-diagonal components analogous to those proportional to ξ in (1):

$$\alpha_{ijs} = -\sum_{\pm} \frac{\pm e^3 n_\pm}{2m_e^2}\bigg[\frac{k_r}{\omega_1}\tau^\pm_{rj}(\omega_1)\tau^\pm_{is}(\omega_2) + \frac{k_r}{\omega_2}\tau^\pm_{rs}(\omega_2)\tau^\pm_{ij}(\omega_1) + \frac{k_{1r}}{\omega}\tau^\pm_{ir}(\omega)\tau^\pm_{js}(\omega_2)$$
$$+ \frac{k_{2r}}{\omega}\tau^\pm_{ir}(\omega)\tau^\pm_{sj}(\omega_1) - \frac{k_{2r}}{\omega_1}\tau^\pm_{rj}(\omega_1)\tau^\pm_{is}(\omega) - \frac{k_{1r}}{\omega_2}\tau^\pm_{rs}(\omega_2)\tau^\pm_{ij}(\omega)\bigg] \qquad (9)$$

$$\tau^\pm_{ij}(\omega) = \frac{\omega^2}{\omega^2 - \Omega_e^2}\left(\delta_{ij} - \frac{\Omega_e^2}{\omega^2} b_i b_j \pm \mathrm{i}\frac{\Omega_e}{\omega}\varepsilon_{ijl} b_l\right) \qquad (10)$$

where $\boldsymbol{b}$ is the unit vector along $\boldsymbol{B}$. Thus, whereas the gyrotropic terms cancel for a PPP in the linear response, cf (1) with $\xi = 0$, the corresponding terms are the only ones that survive in (9) for a PPP ($n_+ = n_-$). The resulting nonlinear response is relatively weak (e.g., Mikhailovskii 1980, Gedalin and Machabeli 1984), except near the cyclotron frequency where $\alpha_{ijs} \propto 1/(1 - Y^2)^2$ has a double resonance (e.g., Luo and Melrose 1996). Thermal effects on the nonlinear response near the cyclotron resonance tend to wash out this resonance, and need to be included in any detailed theory.

5. Relativistic fluid description of a pair plasma

In deriving a fluid description for an electron–ion plasma, an important simplification follows by assuming that the electron–ion mass ratio is small and using it as an expansion parameter. This simplification is not available in a pair plasma. The fluid equations for a relativistically streaming pair plasma were derived and applied to pulsar winds by Melatos and Melrose (1996). The procedure used is to start from the fluid equations for cold, streaming electron ($-$) and positron ($+$) gases, described by their number densities, n_+, n_-, and flow velocities, $\boldsymbol{v}_+$, $\boldsymbol{v}_-$, in the laboratory frame. These parameters are rewritten in terms of the mean number density, n, fluid velocity, $\boldsymbol{U}$, charge density, ρ, and current density, $\boldsymbol{J}$:

$$n = n_+ + n_- \qquad \rho = e(n_+ - n_-) \qquad \boldsymbol{U} = \frac{n_+\boldsymbol{v}_+ + n_-\boldsymbol{v}_-}{n_+ + n_-}$$
$$\boldsymbol{J} = e(n_+\boldsymbol{v}_+ - n_-\boldsymbol{v}_-). \qquad (11)$$

The two-fluid equations then give the equation of fluid motion and the generalized Ohm's law,

$$(\boldsymbol{J} - \rho\boldsymbol{U}) \times \boldsymbol{B} = \frac{1}{2}mn\left(1 - \frac{\rho^2}{e^2n^2}\right)\frac{\partial}{\partial t}\left[(\Delta_+ + \Delta_-)\boldsymbol{U} + \frac{(\Delta_+ - \Delta_-)\boldsymbol{J}}{en}\right]$$
$$+ \frac{1}{2}mn\left(\boldsymbol{U} - \frac{\rho\boldsymbol{J}}{e^2n^2}\right)\cdot\nabla\left[(\Delta_+ + \Delta_-)\boldsymbol{U} + \frac{(\Delta_+ - \Delta_-)\boldsymbol{J}}{en}\right]$$
$$+ \frac{m}{2e}(\boldsymbol{J} - \rho\boldsymbol{U})\cdot\nabla\left[(\Delta_+ - \Delta_-)\boldsymbol{U} + \frac{(\Delta_+ + \Delta_-)\boldsymbol{J}}{en}\right] \qquad (12)$$

$$\left(1 - \frac{\rho^2}{e^2n^2}\right)\boldsymbol{E} + \left(\boldsymbol{U} - \frac{\rho\boldsymbol{J}}{e^2n^2}\right)\times\boldsymbol{B}$$
$$= \frac{m}{2e}\left(1 - \frac{\rho^2}{e^2n^2}\right)\frac{\partial}{\partial t}\left[(\Delta_+ - \Delta_-)\boldsymbol{U} + \frac{(\Delta_+ + \Delta_-)\boldsymbol{J}}{en}\right]$$
$$+ \frac{m}{2e}\left(\boldsymbol{U} - \frac{\rho\boldsymbol{J}}{e^2n^2}\right)\cdot\nabla\left[(\Delta_+ - \Delta_-)\boldsymbol{U} + \frac{(\Delta_+ + \Delta_-)\boldsymbol{J}}{en}\right]$$

$$+\frac{m}{2ne^2}(\boldsymbol{J}-\rho\boldsymbol{U})\cdot\nabla\left[(\Delta_++\Delta_-)\boldsymbol{U}+\frac{(\Delta_+-\Delta_-)\boldsymbol{J}}{en}\right] \tag{13}$$

$$\Delta_\pm=\left[\left(1\pm\frac{\rho}{en}\right)^2-\left(\frac{\boldsymbol{U}}{c}\pm\frac{\boldsymbol{J}}{nec}\right)^2\right]^{-1/2}. \tag{14}$$

The proper number densities for the electrons and positrons are $\tilde{n}_\pm = n/2\Delta_\pm$. The kinetic energy flux in the particles is

$$\boldsymbol{T}=\frac{1}{2}nmc^2(\Delta_++\Delta_-)\left(\boldsymbol{U}+\frac{\rho\boldsymbol{J}}{e^2n^2}\right)+\frac{mc^2}{2e}(\Delta_+-\Delta_-)(\boldsymbol{J}+\rho\boldsymbol{U}). \tag{15}$$

There is no $\rho\boldsymbol{E}$ term in the equation of motion (12), unlike the case of an electron–ion plasma where this term arises from the differential acceleration of the particles of different mass. Also, there is no Hall term in (13) for a similar reason.

The generalized Ohm's law (13) does not include resistive effects. Blackman and Field (1993) included dissipation in an unmagnetized PPP through a resistivity, η, and derived an Ohm's law of the form

$$F^{\mu\nu}U_\nu=\eta(g^{\mu\nu}+U^\mu U^\nu)J_\nu. \tag{16}$$

The implications of the fluid description (12)–(16) of a pair plasma have yet to be explored in detail.

6. Discussion and conclusions

It is clear from the foregoing discussion that even a small admixture of positrons can cause substantial modification to the properties of waves near the electron cyclotron resonance. It is also apparent that pure pair plasmas ($\xi = 0$) differ in important ways from electron–ion plasmas, notably due to the cancellation of gyrotropic effects in the linear response and the cancellation of non-gyrotropic effects in the quadratic nonlinear response. A fluid description of a pair plasma also has notable differences from that of an electron–ion plasma. From the brief review of these effects here it is obvious that the physics of pair plasmas is far less well developed than that of an electron gas or an electron–ion plasma.

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