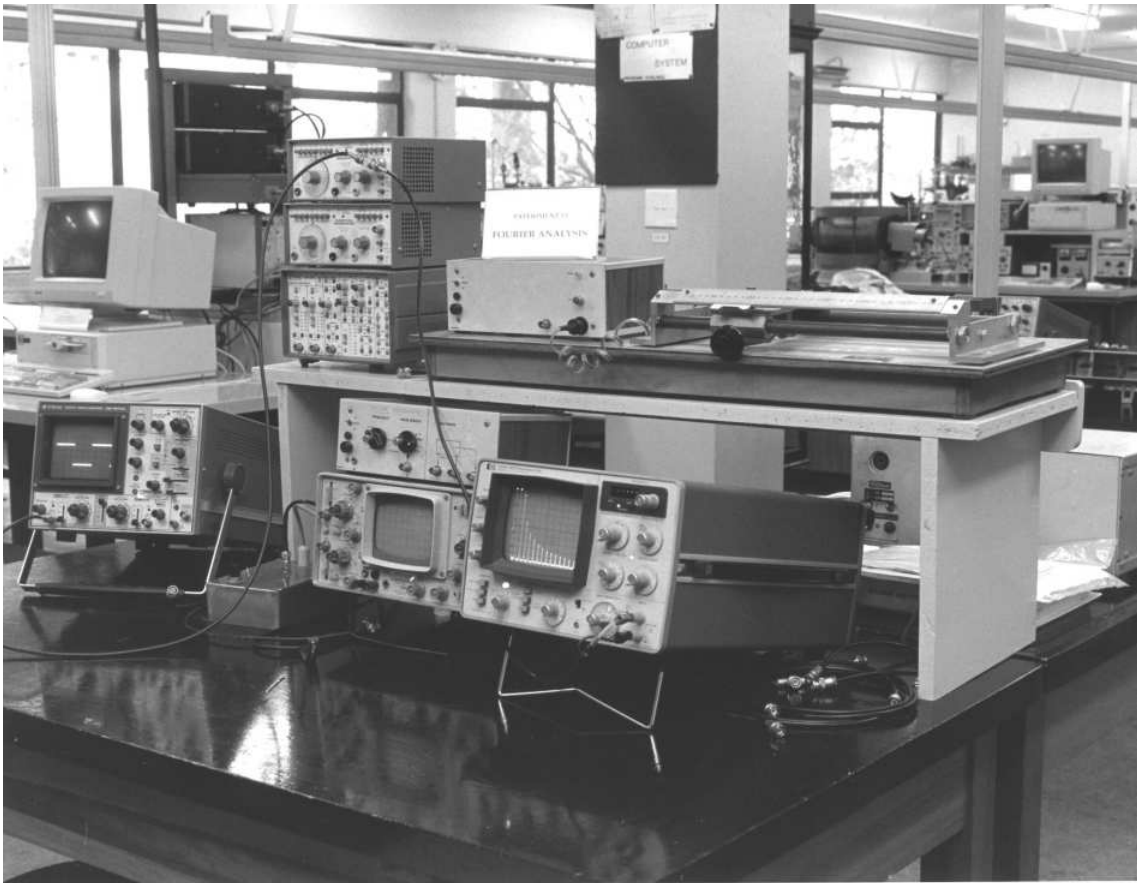
EXPERIMENT 3

FOURIER ANALYSIS



1 Equipment list

- AM radio station pickup box and its two cables. This is located about 2 m above the floor just near the north wall of the lab.
- The control box for the pickup box.
- BWD 530A oscilloscope.
- Cable to connect the TB OUTPUT at the rear of the 'scope to the TB 530A input of the control box.
- HP 3580A spectrum analyser.
- Two BWD 160A function generators.
- Multiplier and Pulse Generator.
- Cables to interconnect the analyser, function generator, and the pulse generator.
- Thurlby DSA524 Digital Storage Adaptor.
- Tuning fork assembly.

Reference: E.O. Brigham, "The Fast Fourier Transform".

2 Aim

An investigation is carried out into analog and digital Fourier Transforms of time dependent signals. The two techniques are applied to signals from AM radio stations, which enable us to identify their frequencies and therefore their names. We investigate the limits of digital Fourier transform in the context of the signal sampling rate and show that under specific conditions, a false signal (alias) can be obtained. This is particularly useful since most Fourier analysis in experimental labs is digital and one must be wary of the limitations of the technique.

3 Introduction

In this experiment you will become familiar with the two manifestations that any physical signal can have: a voltage signal dependent on time $s(t)$ can be also viewed as a voltage which is a function of the frequency $S(f)$. The transformation from $s(t)$ to $S(f)$ is a mathematical process called a **Fourier transform**. The fact that $f = 1/t$ is important; any pair of variables with this inverse relationship could serve as the argument of S. The variable x (of position) and its inverse (spatial frequency) serve as such a pair in the Fourier Optics experiment.

In the present experiment we use the pair: t and $1/t$. Although the function $s(t)$ under study is a voltage $V(t)$ one can use the Fourier technique on any physical variable after all it is usually possible to transduce the variable to a voltage (if not in fact, then certainly in principle).

Whilst ordinary oscilloscopes can record the signal $V(t)$, we need a rather more special instrument to record a signal plotted against the frequency: the spectrum analyser. There are two general types of spectrum analysers, sweeping analysers and digital Fourier analysers. The first uses analog electronic circuits to do its work while the second converts $V(t)$ into digital form and then changes it into $V(f)$ using a computer.

A simple electrical signal (or an electrical representation of any signal) $V = V_s \sin(2\pi f_s t + \phi_s)$ can be pictured as though it was viewed by an oscilloscope (with V on the vertical and t on the horizontal axes) as shown below in Fig. 1(a). In Fig. 1(b) the horizontal axis is frequency, f, and the signal is now represented as a "spike" at $f = f_s$. (Note that phase information is not shown in Fig. 1(b), an omission which could be avoided, to be discussed later).

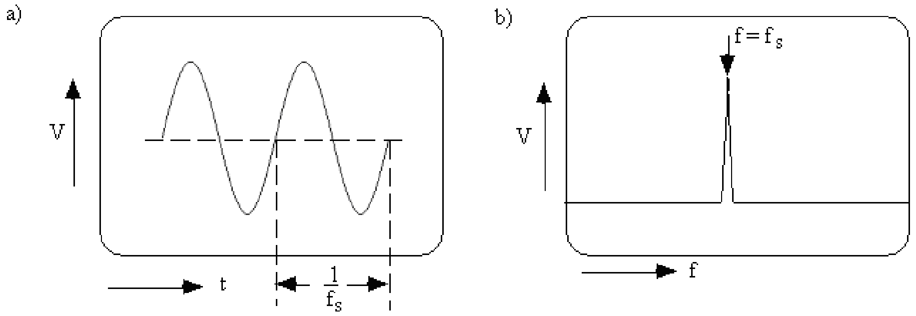


Figure 1: Frequency spectrum for a pure sine wave

Both representations of the signal have V as the dependent variable whilst the independent variable is t in the first version and its **inverse** $f = 1/t$ in the second. We say that the second version is the **Fourier transform** of the first (and the first is the inverse Fourier transform of the second). We can also say that Figure 1a shows the signal in the **time domain** and Figure 1b shows it in the **frequency domain**.

3.1 The sweep analyser

A simple spectrum analyser can be made with an LC tuned circuit provided one (or both) of L and C can be varied. For convenience, one wishes to **sweep** L or C as is done in our AM station pickup illustrated in Fig. 2. The inductance L also acts as a pickup device; it is in effect a magnetic dipole and a voltage is induced in it by the magnetic fields of incoming radiation from radio stations. The capacitor C is a so called varactor diode i.e. a reverse biased junction diode. As this bias is increased the depletion layer thickness increases and the junction's capacitance reduces in accord with the well known formula for a parallel plate capacitor

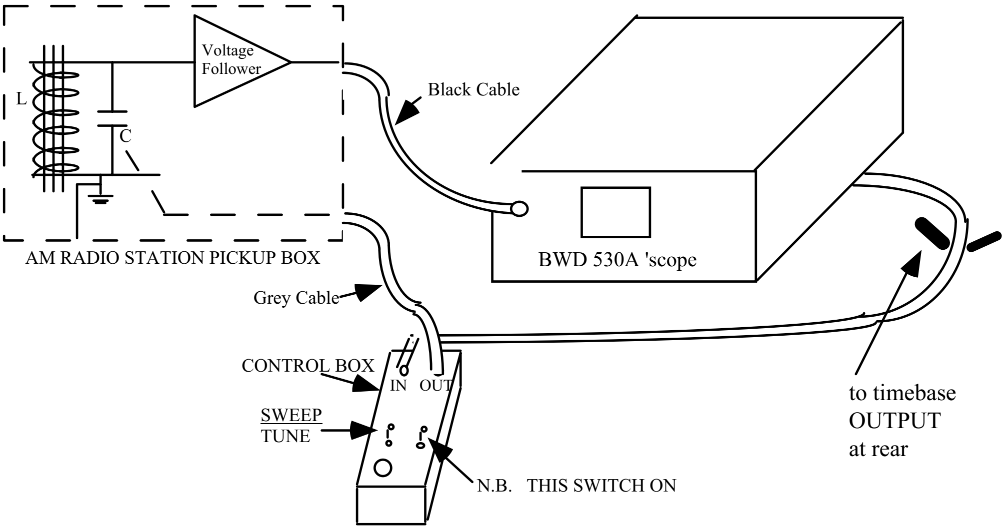


Figure 2: Experimental arrangement for analysing radio signals.

$$C = \frac{\varepsilon A}{d} \tag{1}$$

where A is the area of the plates and d their separation, and ε is the electrical permittivity of the material between the capacitor plates.

As it progresses, the voltage sweep increases d and reduces C (non-linearly). The sweep voltage comes from the Time Base OUTPUT at the rear of the 'scope and goes into the TB 530A INput of the control box.

This simple analyser has the following disadvantages:

(a) The frequency resolution is poor (two signals $f1$ and $f2$ such that $f1$ $f2$ may be recorded as a single signal).

(b) The sweep is non linear because C is inversely proportional to d, which is not proportional to the sweep voltage, and in any case the resonant frequency is not proportional to C.

(c) Signals are shown as “blips” rather than peaks (of minor importance).

(d) The instrument is only sensitive to a given signal at a frequency f at regular intervals equal to the period of the sweep. In the case above this was about 100 ms.

3.2 HP 3580A Analyser

In the Hewlett Packard spectrum analyser shown in block form in Figure 4 there is considerable improvement in (a), (b) and (c) discussed in the previous section. Sweeping spectrum analysers always have the problem (d).

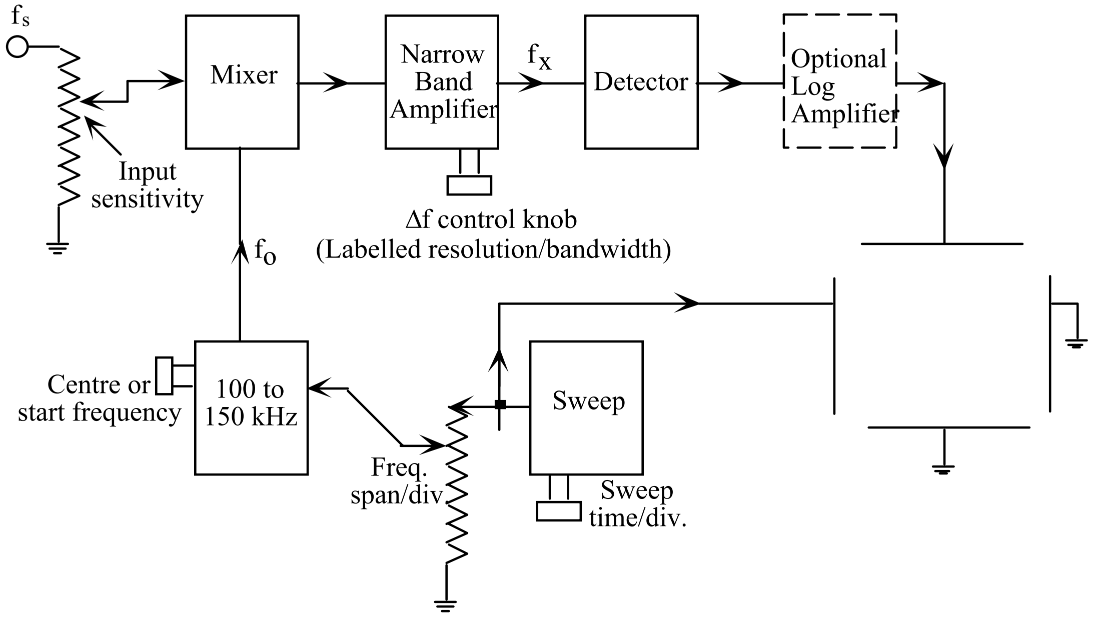


Figure 3: Block diagram for the operation of the 3580A spectrum analyser.

The HP 3580A’s maximum frequency range is 5 Hz to 50 kHz. The input signal at a frequency f_s is shifted in frequency by a process (discussed later) called mixing in which the input is, in effect, multiplied by a signal of frequency f_o. The latter is swept in frequency over a maximum range of 100 to 150 kHz. The difference frequency $f_1 = f_o - f_s$ is amplified by a narrow band amplifier with a centre frequency of 100 kHz and a half width of f. f can be varied, using the RESOLUTION/BANDWITH control, between 1 and 300 Hz.

After amplification, the signal is passed to a detector and thence to the instrument’s cathode ray tube where the Y deflection is proportional to the signal’s amplitude (or to the log of the amplitude if LOG (dB/DIV) is used).

The frequency range displayed on the analyser’s screen is controlled by the FREQUENCY (START or CENTRE) and the FREQ SPAN/DIV controls. The rate of frequency sweeping in Hz/sec is determined by the latter and by the TIME/DIV of the sweep

$$\frac{df}{dt} = \frac{(FREQ/DIV)}{(TIME/DIV)} \tag{2}$$

Note that f is in Hz and t is in seconds.

Sweeping spectrum analysers give erroneous results if df/dt is too high. Its maximum useful value depends on the RESOLUTION/BANDWIDTH setting f. In the 3580A an

ADJUST light comes on if

$$\frac{df}{dt} > 0.05(\Delta f)^2 \tag{3}$$

If t equals the time taken for the sweep to cover f, then the ADJUST light comes on if

$$\Delta t \Delta f < 20 \tag{4}$$

Note that this is similar to the uncertainty principle in quantum mechanics.

The HP 3580A analyser takes the Fourier transform of the input signal, which we will call $s(t)$. Fourier transforms are generally complex - that is, they contain a real and imaginary components, $R(f)$ and $I(f)$, respectively. $R(f)$ is known called the in-phase component and $I(f)$ the quadrature component. The analyser displays the magnitude of the transform

$$M(f)[=\sqrt{(R^2+I^2)}] \quad \text{versus} \quad f \tag{5}$$

R and I can be thought of as components of a phaser, the phase, ϕ, is given by

$$\tan(I/R) = \phi \tag{6}$$

where ϕ has to be placed into the correct quadrant.

3.3 Frequency shift theorem

Consider two signals, $V_s \cos(2\pi f_s t)$. If we multiply this by $\cos(2\pi f_c t)$ we get

$$V_s \cos(2\pi f_s t)\cos(2\pi f_c t) = \frac{V_s}{2}\left\{\cos[2\pi t(f_c+f_s)] + \cos[2\pi t(f_c-f_s)]\right\} \tag{7}$$

The signal at f_s with amplitude V_s has disappeared and replaced by signals at $f_c - f_s$ and $f_c + f_s$ with equal amplitudes $V_s/2$.

This process is called **balanced modulation** and the two resulting components are called **sidebands**. The process is sometimes called mixing and is illustrated in Fig. 4. Similarly, mixing can be applied to a more complex signal, s(t), shown in Fig 5. AM radio uses a similar principle to generate what goes up the transmitter mast except that the carrier frequency at f_c is not eliminated. If $V_s \cos(2\pi f_s t)$ is a component of the audio (music or speech) signal then this is mixed with the basic carrier signal as follows

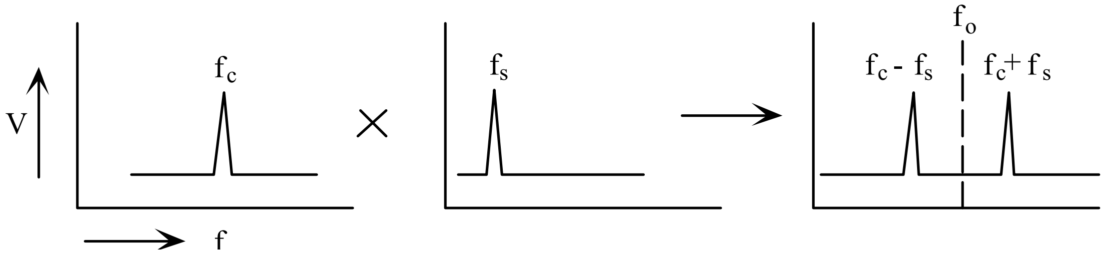


Figure 4: Frequency spectrum for the product of two pure sinewaves.

$$V_s[1+m\cos(2\pi f_s t)] \times V_c \cos(2\pi f_c t) \tag{8}$$

where $0 < m < 1$. In the time domain this may look something like Fig. 6, where $V_{max} = K(1+m)$, $V_{min} = K(1-m)$, and K is a constant. Therefore,

$$m = \frac{(V_{max} - V_{min})}{(V_{max} + V_{min})} \tag{9}$$

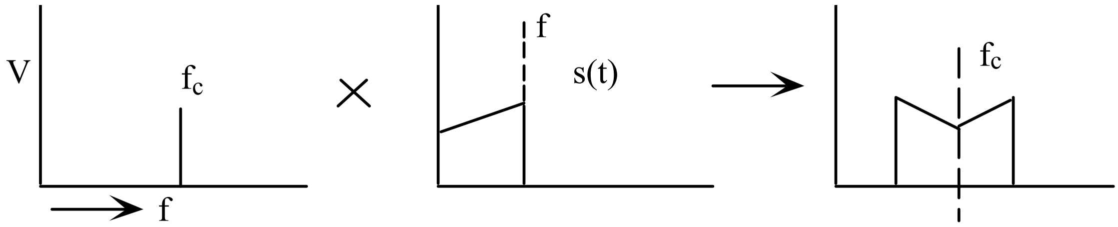


Figure 5: Sideband structure for a complex signal.

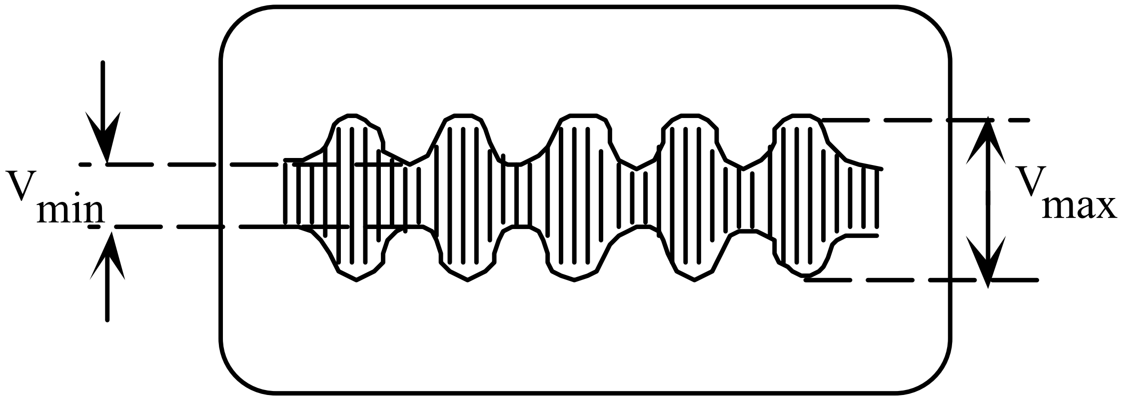


Figure 6: Waveform of an amplitude modulated radio signal in the time domain.

3.4 Fourier spectrum of a pulsed waveform

Consider a pulsed signal, otherwise known as a **pulsed wave-train** as shown in Fig. 7. The ratio of the pulse width to the pulse repetition period, T, is known as the duty cycle, θ. Note the following:

1. Fourier components only occur at integral multiples k of the pulse repetition frequency $F(= 1/T)$. Some components may be missing, in particular for a duty cycle of 0.5, all even k are absent. For $\theta = 1/p$, where p is an integer, all the components with $k = p, 2p, 3p$, etc. are missing (see Appendix 2). This is exactly analogous to the missing orders in the diffraction pattern of a series of slits.

2. For small θ, say less than 0.3, it seems that the amplitudes of the components follow a curve like the dotted one shown in Fig. 8. The envelope has the functional form of $\sin(kx)/(kx)$.

3. As θ decreases (i.e. as the signal approaches a series of spikes) the frequency of the first zero component F_p increases.

4. When θ is very small (i.e. the pulses approach delta functions), then $s(t)$ approaches a waveform we call a **comb**

$$s(t) = \text{comb}(T, t) = \sum_n (t - nT) \tag{10}$$

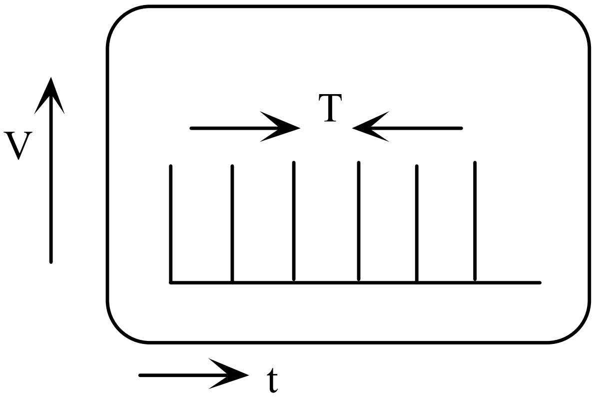


Figure 7: Pulsed wave-train in the time domain.

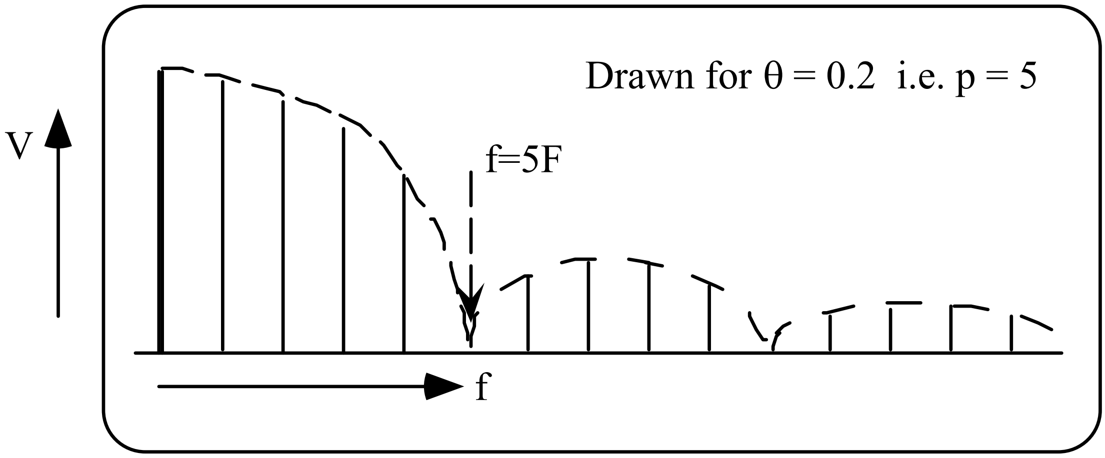


Figure 8: Frequency spectrum for a rectangular pulse with duty cycle 0.2

In the frequency domain the spectrum is also a set of delta functions $S(f) = \mathrm{comb}(F, f)$ as shown in Fig. 9.

3.5 Sampling

A waveform $s(t)$ can be sampled by multiplying it by $\mathrm{comb}(T, t)$ as shown in Fig. 10. In practice this is carried out by using an electronic switch which connects $s(t)$ to the output for a short period every T seconds).

Figure 11 shows examples of the the spectrum of sampled signals. Note that the resultant spectrum below $F/2$ is identical to $s(t)$'s spectrum. This means that no information has been lost in sampling and one could recover the amplitude and phase of $s(t)$ using this part of the spectrum. However, this is only true if $s(t)$'s spectrum does not extend beyond $F/2$. The following section shows what happens if this is not true.

3.6 Aliasing

Now consider the case where $s(t)$'s spectrum does extend beyond $F/2$. We illustrate this for $s(t) = V_s \cos(2\pi f_s t)$ with $f_s \geq F/2$ as shown in Fig. 12. If f_s exceeds $F/2$, then $F - f_s < F/2$

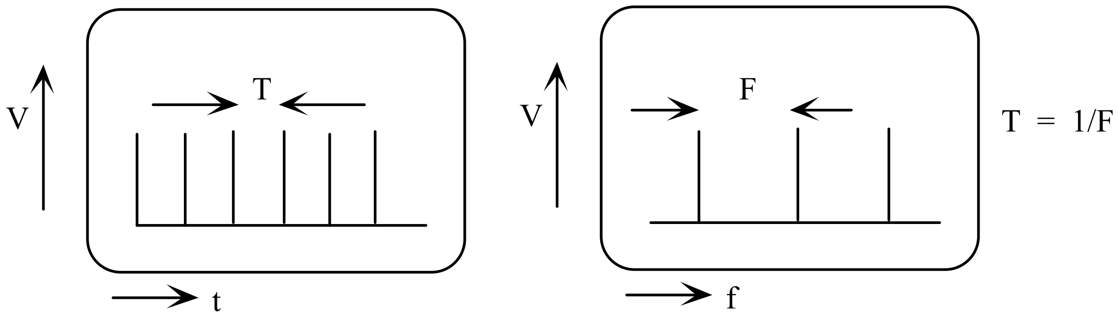


Figure 9: The frequency spectrum of a comb is also a comb.

and the "spike" at this frequency is taken as being the signal because it is within the range 0 to $F/2$. This is called **aliassing** because it gives a false signal. That is, a frequency is generated that does not actually exist. If f_s approaches F the spike approaches a frequency of zero and we have the well known stroboscopic effect. The lowest frequency component of $s(t)$ after sampling is a very low frequency indeed. Such a low frequency was not present in $s(t)$. If $f_s = F/2$ and the phase relation between these two signals is such that $s(t)$ is sampled at its zeros, then we may have no output at all. The left and right sidebands of each tooth of the comb destructively interfere in pairs!

To avoid aliassing with a complicated $s(t)$ we must ensure that the $s(t)$'s highest frequency component is less than $F/2$, the so called **Nyquist frequency**. Hence the Nyquist Sampling Theorem: $s(t)$ is uniquely determined by its sampled values by $\mathrm{comb}(T, t)$ provided that f_{max} of $s(t) < 1/(2T)$.

3.7 Fast Fourier Transform

As stated earlier, sweeping spectral analysers have a fundamental disadvantage: they are only sensitive to a given signal at a frequency f at regular intervals equal to the period of the sweep. This makes them unsuitable for a study of a rapidly changing spectrum (such as that from a vibration transducer on a piece of machinery starting up. Many thousands of dollars are saved each year by detecting incipient breakdowns of machines using this technique).

This problem is solved, at least for low frequencies, by the use of a digital Fourier analyser: $s(t)$ in voltage form (tranduced from the original physical variable if necessary) is turned into a series of numbers using an Analog to Digital Converter (ADC). These are then stored (and often displayed, i.e. "plotted" against time as on a conventional oscilloscope). In out case this is done using the **Thurlby digital storage adaptor** in conjunction with the **Trio oscilloscope**. The stored waveform is then subject to Fourier transformation, in the instrument or, in our case, in a separate computer.

We now write the input signal as $s(k)$ rather than $s(t)$ in order to stress that it is sampled at regular intervals T, at times $t = kT$, with $k = 0, 1, 2 \ldots N - 1$. For continuous data the transform multiplies the signal by sine/cosine (or complex exponentials) and then integrates. A summation replaces the integral for discrete data and the transform can be written in matrix notation as a set of N equations

$$S(n) = W_{nk} s(k) \tag{11}$$

where n is an integer and ranges from 0 to $N - 1$. These are not simultaneous equations: $S(n)$, the vector representing the transform, is on the left hand side and all the known

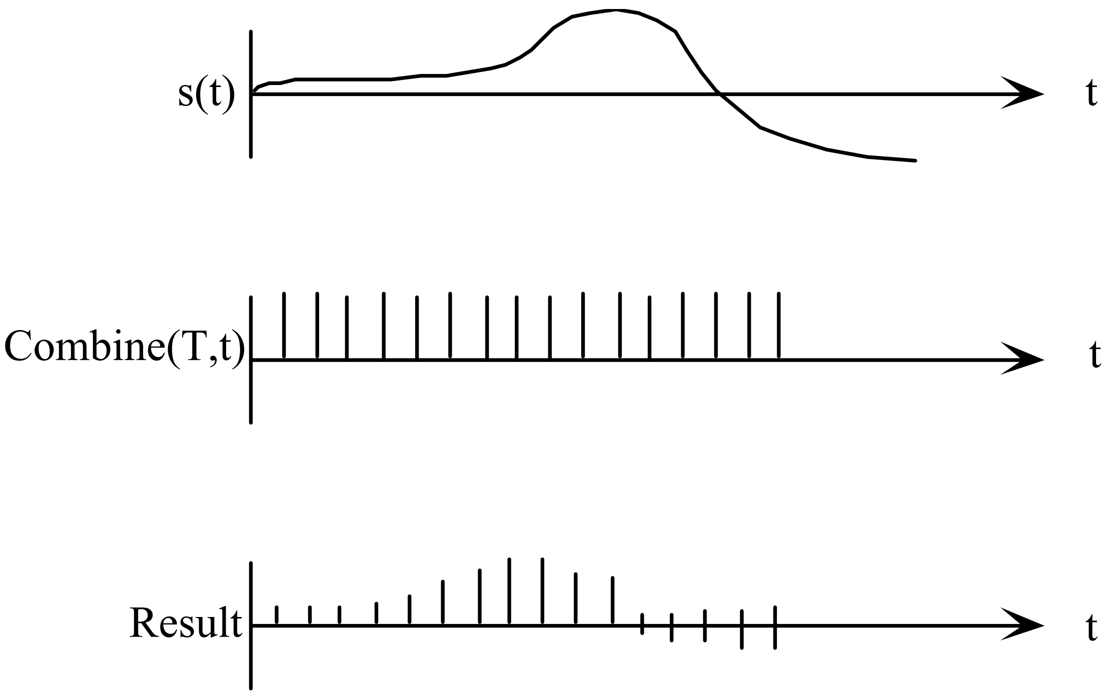


Figure 10: Demonstration of sampling technique.

quantities are on the right hand side. The matrix element W_{nk} is given by

$$W_{nk} = [\exp(-2j/N)]^{nk} \tag{12}$$

The solution of these equations involve N^2 multiplications [imagine a million multiplications for $N = 1000$(!)]. It was realized many years ago that although W_{nk} has N^2 elements, they are drawn from a set containing only N elements:

$$[exp(-2j/N)]^m \tag{13}$$

where $m = 0, 1, 2, 3........N - 1$. This allows W_{nk} to be factorized in a cunning way so as to cut down the number of multiplications needed to $N \log_2(N)$. For $N = 1000$ one needs only 10000 multiplications, certainly no problem for a computer. This sort of transform is called a **Fast Fourier Transform (FFT)**. A minor penalty is that N has to be an integral power of 2, so that $N = 1000$ is not allowed but $N = 1024$ is. Transforms with $N = 65536$ are quite standard now.

The transform $S(n)$ is a complex number $S(n) = R(n) + jI(n)$. The magnitude being given by

$$M(n) = \sqrt{[R(n)^2 + I(n)^2]} \tag{14}$$

In our experiment, the FFT will be carried in by a computer program, which will calculate the M's but before doing so, the program will ask if a Hanning window should be used. Strictly speaking a Fourier transform is supposed to integrate or sum the product of $s(t)$ and the sine/cosine term from minus infinity to plus infinity. Such idealistic situations cannot exist in practice. In the discrete transform above, one sums for a total time of NT. There is no problem if signals are periodic with periods of NT or $NT/2$, $NT/3$ etc. A transform for an extra long time like $10000NT$ is just 10000 repeats of the transform for a time NT.

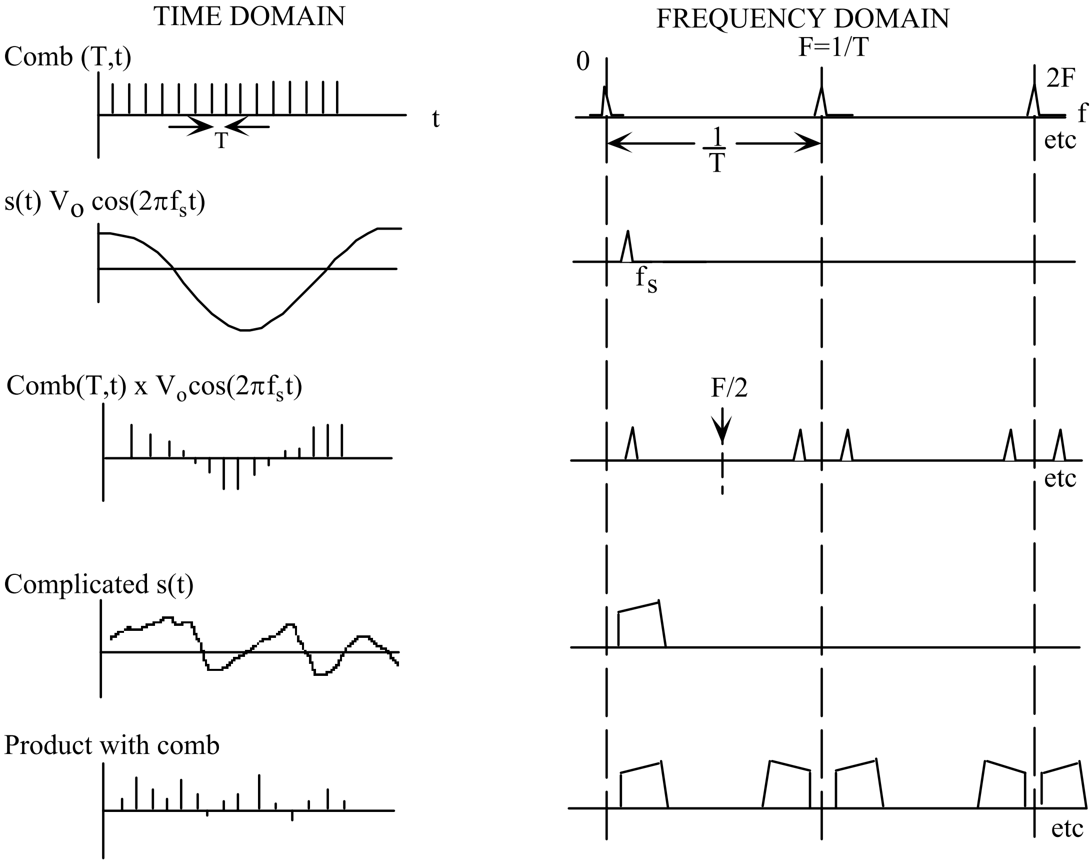


Figure 11: Demonstration of sampling for a complicated spectrum.

A problem arises if the signal is not periodic or if its period does not equal NT divided by an integer. A succession of $s(t)$'s placed end to end then has a series of discontinuities as shown in Fig. 13. It can be shown that this causes signals which are "spikes" in the frequency domain (const × $\sin(2\pi f_s t)$ in the time domain) to take the form const × $\sin[q(f - f_s)]/[q(f - f_s)]$. The "ripples" associated with this function can be minimised by multiplying $s(t)$ by the Hanning function before transforming

$$s(t) \rightarrow s(t)\{0.5 + 0.5\cos[2\pi/(NT)]\} \tag{15}$$

The program actually does this operation after the transform by replacing $S(n)$ with $S'(n)$, where $S'(n) = S(n) - 0.5S(n-1) - 0.5S(n+1)$. The output of the program is a set of $N/2M(n)$'s extending from $n = 1$ to $N/2$

3.8 Analysis Of The Vibration Of A Cantilevered Beam (Tuning Fork)

Digital Fourier analysis is a particularly powerful technique with waveforms that vary with time. As mentioned earlier, this technique, when used with a vibration transducer on rotating machinery, is capable of detecting malfunctions like lost gear teeth before they cause serious trouble. The method is especially sensitive when it is applied to machinery starting up. The Fourier spectra obtained in these cases are quite complex, even a simple cantilevered beam can vibrate in various modes with a set of possible frequencies, rather like a stretched string except that the frequency of the higher frequency modes, called

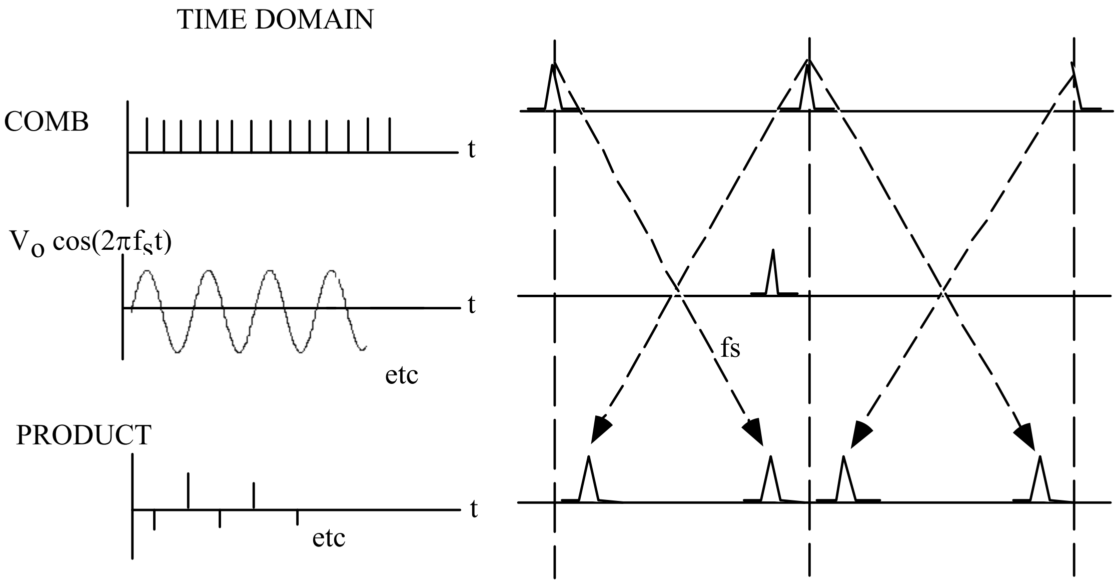


Figure 12: The origin of aliassing.

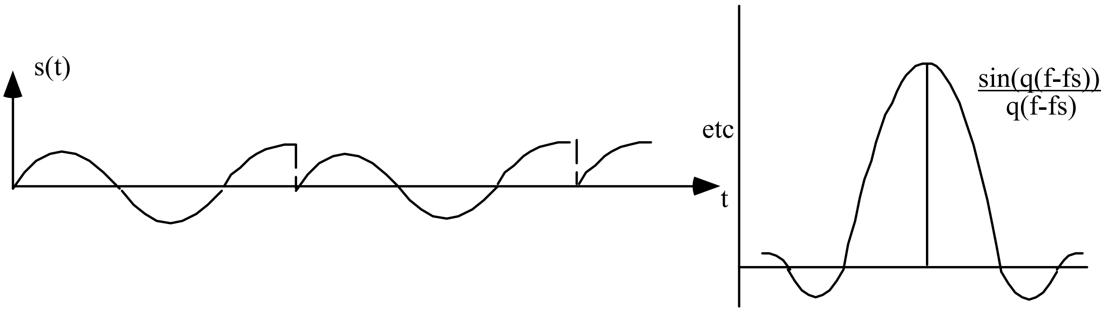


Figure 13: Generation of spurious frequencies due to discontinuities.

overtones, are not integer multiples of the frequency of the fundamental mode (as would be the case for a stretched string).

Consider the cantilever shown in Fig. 14. A rigorous analysis of cantilevers can be used to show that the harmonics, f_i, of the vibrations are given by the following:

$$f_i = \frac{1}{2\pi}[(\lambda L)_i/L]^2[a/(2\sqrt{3})]\sqrt{\frac{E}{\rho}} \tag{16}$$

where E is Young's modulus of the cantilever material, ρ is the density, a is the width of the cantilever along the vibration direction, L is the length, and

$$(\lambda L)_i = 1.875, 4.694, 7.853, 10.996, 14.137 \text{ and } 17.279. \tag{17}$$

are the first six roots of a transcendental equation used in the derivation of Eq. 16.

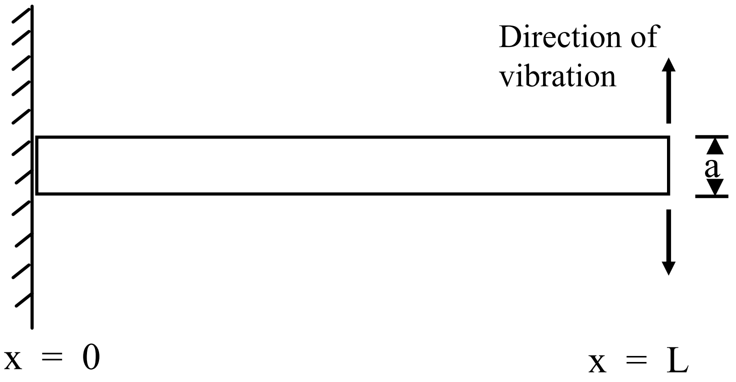


Figure 14: A cantilever is a beam that is fixed only at one end and free to vibrate at the other end.

4 Experimental procedure

4.1 Simple analyser

⊕ **Examine Sydney's radio stations in the time and frequency domains.**

- Put the SWEEP/TUNE switch in the SWEEP position and put the switch to the right of this in the up position (which connects C in parallel with L). The grey cable from the pickup should be plugged into FILTER OUT and the black cable from the pickup plugged into the input of the oscilloscope. Set the sweep to 10ms/cm and the Y sensitivity to 20mV/cm. The trace should look like the schematic drawing in Fig. 15 (though of course not as regular as this!)

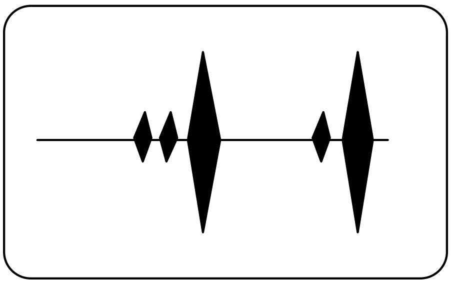


Figure 15: Tuner output for swept radio signals.

You are seeing a piece of the electromagnetic spectrum extending from 230 to 1000 kHz. Some of the AM (amplitude modulated) radio stations of Sydney will be seen.

- Make a simple sketch of the display and then put the SWEEP/TUNE switch on TUNE. The tuning capacitor is then set by the knob.
- Tune in one of the stations. It is quite obvious that amplitude modulation is being used.
- Sketch this then put the timebase speed on 1μs per cm. Adjust the level until a stable pattern is obtained. The so called **carrier wave** will be seen.

- Measure its period and calculate its frequency. Amplitude modulation can be seen equally well with this timebase speed.

4.2 HP 3580A Analyser

⊕ **Determine the Fourier spectrum of square and triangular signals.**

- Connect the BWD generator OUTPUT both to the INPUT of the spectrum analyser and to one Y channel of a conventional oscilloscope. Set
  - the BWD for a sine wave of 10 kHz
  - the analyser CENTRE FREQ to 10 kHz
  - RESOLUTION/BANDWIDTH to 300 Hz
  - DISPLAY/SMOOTHING to min
  - FREQ SPAN to 5 kHz
  - REPetitive sweep
  - LINear amplitude mode
  - CLEAR WRITE button to out
  - BLANK STORE button to out
  - ADAPTIVE SWEEP to OFF
- Adjust the INPUT SENSITIVITY to obtain a trace with a peak in the centre of the display screen at 10 kHz. Note that the CAL setting of the rotary switch connects a calibration signal instead of the input!
- Try varying the analyser's controls to see their effect. One oddity is that the analyser seems quite "happy" with "negative" frequencies provided they are not too large.
- Now input a 1 kHz sine wave. The peak should move to the correct position to the left of the screen centre.
- Examine and sketch the traces produced by square and triangular waves. (Make sure that these waves are symmetrical about the time axis. If not, then you might have one of the BWD's SYMM. buttons pressed). In both the square and triangular waves you can see odd harmonics present. Because the waves are symmetrical about the time axis, the amplitudes of the even harmonics are zero (See Appendix 1). The frequency of the r^{th} harmonic is r times the frequency of the **fundamental**, the latter being 1 kHz in this case.
- Measure the A_r for both waves and plot A_r against r on a log/log graph. The amplitude Ar of the rth harmonic is proportional to the n^{th} power of r. The slopes, which give the indices of the power laws, should be -1.0 and -2.0 i.e. differ by unity.

QUESTION: Do you expect this relation between the slopes in the two cases? Hint: the triangular wave is the time integral of the square wave?
QUESTION: What dependence of A_r on r would you expect for the time derivative of the square wave? Draw a sketch showing how this wave would appear in the time domain.

C1 ▷**Tutor checkpoint. Obtain tutor's signature before proceeding**.

4.3 Frequency shift

⊕ **Use balanced modulation to producing frequency shifts.**

- Using the multiplier provided set f_c to 10kHz and f_s to 1 kHz.
- Examine the multiplier's output in both the frequency and time domain.
- Measure the separation of the sidebands and compare it with twice f_s.
- Turn on the OFFSET control of the generator of f_s and explain the change that you observe.

⊕ **Now determine the effect of the duty cycle**.

- Remove the multiplier and return to the earlier set-up when you examined the square wave from the BWD generator.
- Check that the display on the analyser is as it was before.
- Now replace the BWD with a pulse generator where the duty cycle θ can be varied. (The duty cycle is the ratio of the pulse width to the pulse repetition period - for a square wave it equals 0.5).
- Vary the pulse width and thus the duty cycle and see the effect on the signal as shown in the two domains.
- Look at what happens as the duty cycle is reduced. (Use a pulse period of 1 ms and set the analyser to a start frequency of zero).

4.4 Sampling and Aliasing

⊕ **Observe and record the effects of aliasing in the time domain on the oscilloscope and frequency domain on the analyser**.

- Set the comb pulse train frequency, F to 10 kHz.
- Start with a sinusoidal f_s at 1 kHz and increase its frequency.
- Make sketches where appropriate and comment.

C2 ▷**Tutor checkpoint. Obtain tutor's signature before proceeding**.

4.5 Spectral analysis using the fast Fourier transform

⊕ **Analyse the electromagnetic spectrum in the AM radio band.**
The experimental set-up used to detect and store the AM radio frequency spectrum is shown in Fig. 16 **Set up the Thurlby data acquisition as follows**

- The SWEEP/TUNE switch on the pickup's control box can be put in either position but the switch to the right of this (which connects C in parallel with L) should be OFF.
- The grey cable from the pickup should be plugged in to FILTER OUT and the black cable from the pickup plugged into channel 1 of the Thurlby digital storage adaptor. This is connected to one of the computers from the connector at its rear using the serial interface. The other two connectors at the rear are connected to the TRIO monitor oscilloscope, COMPOSITE to channel 1 of the oscilloscope and TRIGGER to external trigger of the oscilloscope.

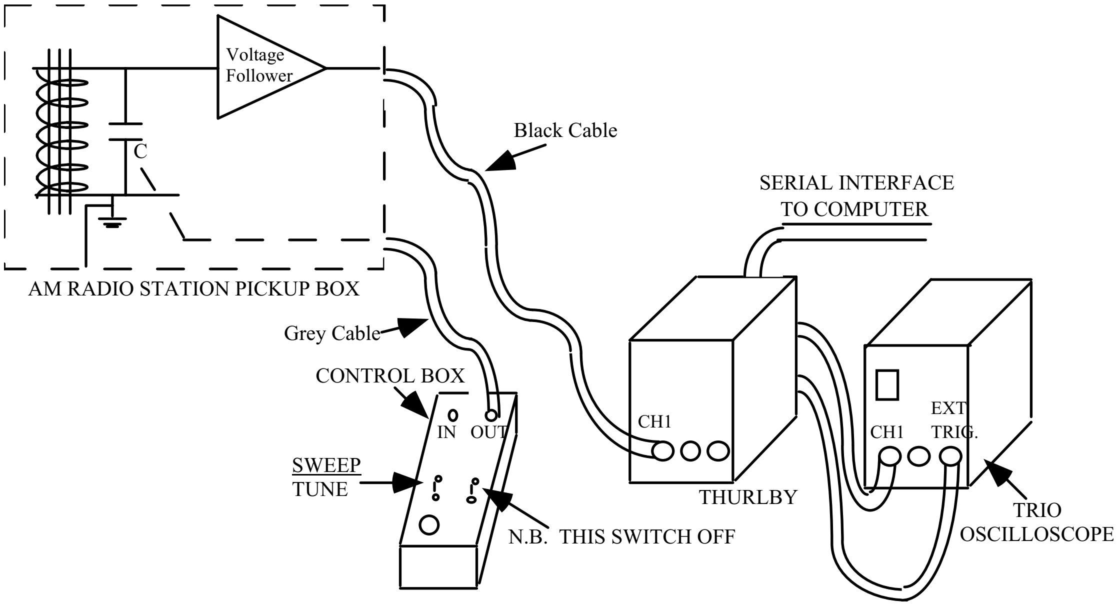


Figure 16: Experimental arrangement for storing radio spectra.

- The following settings should be used for the monitor oscilloscope: 100 mV per divisions, timebase at 50 μs per division, external trigger, normal mode, AC coupling, slope negative and level negative.

To store and display a signal use the following procedure:

1. Switch on the Thurlby. A notice should appear on the CRO screen saying “CENTRE ARROW TIP”. Press ESCAPE.
2. Press SHIFT (on RH side of panel in green); the adjacent ENTER sign will flash red.
3. Press RESET (on top right of panel in green).
4. Set the TIME/DIV to the required value (20 μsec/div for the AM radio signals) and VOLTS/DIV to 2 mV. (**Note these adjustments are made on the Thurlby and not on the monitor oscilloscope**).
5. To simplify the monitor oscilloscope’s display remove the channel 2 trace by pressing CH2 and RCL(NN) together.
6. If RUN/HOLD (top centre) is on HOLD, press once to RUN.
7. When a suitable trace has been acquired press HOLD.

The signal, which should appear on the CRO screen for this part of the experiment, is shown in Fig. 17.

The signal is analysed using the computer program FOURIER ANALYSIS. Run the program and examine the spectrum. Note that the displayed spectrum extends up to 2.5 MHz.

There are 1024 samples ([1000 samples) taken during a time of 20 μs /div × 10 div or 200 μs. The sampling interval is thus 0.2 μs (it is indeed exactly 0.2 μs: quartz crystal

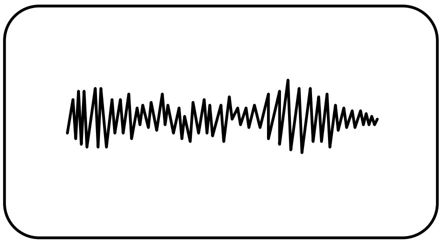


Figure 17: Example of stored radio signal.

accuracy). Since T = 0.2 μs we have that F = 5 MHz and F/2 = 2.5 MHz. Since N/2 = 512, the components are calculated every 5 kHz approx.

- Identify some of Sydney's AM radio stations and comment. Note that there is a facility in the software to calibrate the FFT program using a sine wave generator. This will result in a far more accurate estimate for the frequency of the various radio stations.

4.6 Analysis of the vibration of a cantilevered beam (tuning fork)

⊕ **Determine the frequency spectrum of a vibrating tuning fork**.

In this section you will study the oscillations of a tuning fork, i.e. two parallel cantilevered beams, set into oscillation with an "electromagnetic" hammer applied to one tine of the fork.

- Set the position of the hammer as 70 mm on the scale.

The oscillations are sensed by a pickup at one end of the other tine. The signal from the sensor should be fed into channel 1 of the Thurlby.

- The TIME/DIV of the latter should be set to 20 ms/div for this part of the experiment.
- The procedure to be used for displaying and storing the signal is the same as for the AM radio signals in the previous section.

The signal, however, in this experiment, has to be produced and moreover it is a transient signal. The signal is produced by pressing the EXCITE button of the tuning fork driver. The HOLD on the Thurlby should be pressed immediately after this.

- As before the signal is analysed by the program FOURIT. Run the program and examine the spectrum.
- Note the fundamental at about 60 Hz, the 1st overtone at just under 400 Hz and the second overtone at just above 1000 Hz.
- Obtain accurate frequencies for these components from the frequency spectrum
- Calculate the theoretical frequencies and compare with the experimentally determined values. Note that Young's Modulus for steel is $E = 200$ GN/m^2 , and the density of steel is $\rho = 7873$ kg/m^3. Note, since the Young's modulus E of the fork material has not been measured it may be best to compare the ratios of the three frequencies with the theoretical values. Then calculate a mean value of E from the measured frequencies. A micrometer is available to measure the dimension a.

Now write a conclusion to your experiment that summarizes your results.

C3 ▷**Tutor checkpoint. Obtain tutor's signature.**

5 Appendix 1

Why Even Harmonics are missing in Symmetrical Waves
If a signal, $s(t)$, is periodic then $s(t+T)=s(t)$ then, apart from a scale factor, we can write

$$S(f)=\int_0^T s(t)\exp(-j2\pi ft)dt \tag{18}$$

and $S(f)=0$ when $f\neq 1/T, 2/T,$ etc. This can be rewritten as

$$S(f)=\int_0^{T/2} s(t)\exp(-j2\pi ft)dt+\int_{T/2}^{T} s(t)\exp(-j2\pi ft)dt \tag{19}$$

In a symmetrical waveform we have $s(t-T/2)=-s(t)$. Writing $\tau=t-T/2$ we then have $s(\tau)=-s(t)$, so that we now have

$$S(f)=\int_0^{T/2} s(t)\exp(-j2\pi ft)dt+\int_0^{T/2} -s(\tau)\exp(-j2\pi f(\tau+T/2))d\tau \tag{20}$$

We can now simply change the variable of the second integral above from $\tau\rightarrow t$. Consequently, this becomes

$$S(f)=\int_0^{T/2} s(t)\exp(-j2\pi ft)dt+\int_0^{T/2} -s(t)\exp(-j2\pi f(t+T/2))dt \tag{21}$$

which simplifies to

$$S(f)=\int_0^{T/2} s(t)\exp(-j2\pi ft)\left[\exp(-j2\pi f(t+T/2))\right]dt \tag{22}$$

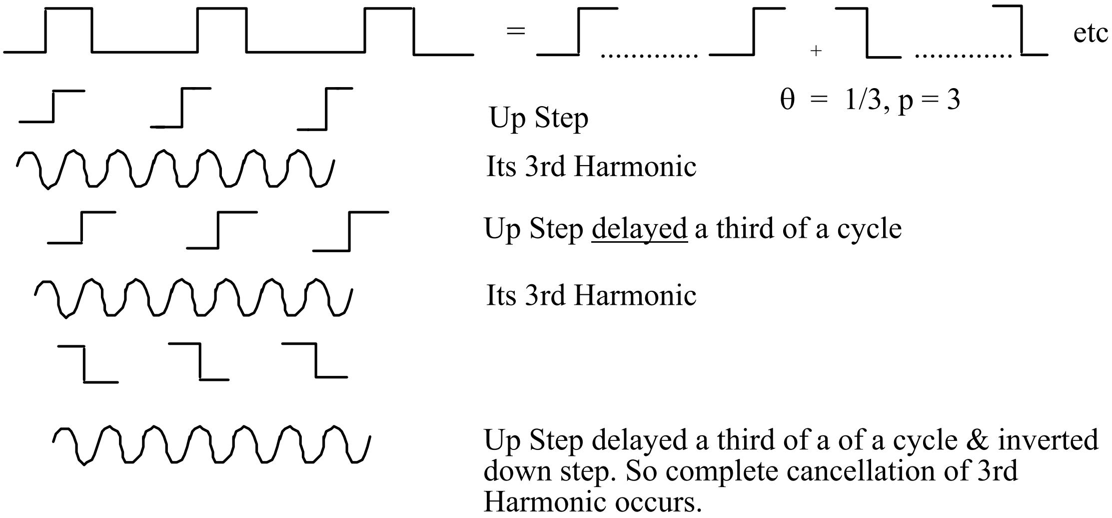
$$=0 \text{ if } f=0,2/T,4/T, \text{ etc.} \tag{23}$$

These are the even harmonics.

6 Appendix 2

Why certain harmonics are missing in a pulsed waveform

The following is a ‘Handwaving” explanation. Consider the pulsed waveform to be the addition of an ”up” step at $t = O$ and a “down” step at $t = T(= T/p)$. We may now use the linearity theorem (which could be called the addition theorem). The amplitude/phase of the k’th harmonic of the pulse waveform is the phasor sum of the amplitude/phase of the “up” step’s k’th harmonic and the “down” step’s k’th harmonic. For a “down” step at $t = T/p$, the two phasors have equal magnitude but exactly opposite directions. If the step at $t = T/p$ had been an up step, then the two phasors would have had the same direction.



Up Step delayed a third of a of a cycle & inverted down step. So complete cancellation of 3rd Harmonic occurs.

Figure 18: Explanation of missing harmonics in a pulsed waveform