EXPERIMENT 27

FOURIER OPTICS

Equipment list:-

a) Optical bench and mounts
b) Beam expander laser diode module (λ = 635 nm)
c) Laser current regulator
d) Adjustable neutral density filter
e) One-dimensional and two-dimensional (mesh) gratings
f) Electrically heated wire as a source of the phase object
g) Lens
h) Adjustable slit
i) Rotating diaphragm
j) Periodic spatial filter
k) Pinhole spatial filter
l) Black dot spatial filter
m) Magnifying glass
n) White card screen
o) Tape measure
p) Travelling microscope and light box

References

Hecht, "Optics", Ch. 11,13.
Ditchburn, "Light", Chapter 8.
Jenkins and White, "Fundamentals of Physical Optics", Chapter 7.

1. INTRODUCTION

The quality of the image produced by an optical system is determined by aberrations in the components of the system and also by diffraction effects. Whether the light is coherent or non-coherent is also important since the diffraction effects for coherent light are not the same as for non-coherent light. This was realized by the physicist Abbe in the latter part of the last century. He developed a theory of image formation with coherent illumination. This was of great importance in the development of high power microscopes. The ideas in the theory have also led to major areas of modern optics. Though he did not express things in this way his theory implies that the image formation is intimately related to the spatial frequencies present in the object. The passage or otherwise by the optical system of these spatial frequencies determines not only its resolution but affects the image sharpness. The failure to pass certain spatial frequencies can result in false structure i.e. false detail can appear. The deliberate suppression of certain spatial frequencies is known as spatial filtering and is of major importance in areas of modern optics such as confocal microscopy and image enhancement. This experiment deals with these ideas.

Though the resolution of an optical system is important in determining the quality of the system, (and indeed until recent times was the only way), it is not the whole story. In terms of spatial frequencies the resolution limit is the high spatial frequency cut-off. It says nothing about how the lower spatial frequencies are passed by the system. A complete description of the performance of the system is to specify its performance as a function of spatial frequency. This information is given by the so-called optical transfer function. In its most general form this function contains the effects of aberrations as well as diffraction. Since the diffraction effects are different, the function for coherent light is different from that for non-coherent light

2. ABBE'S THEORY OF IMAGE FORMATION

With non-coherent light the resolution of an optical system is determined by diffraction effects at the aperture of the system. The essence of Abbe's theory is that when one is treating an object illuminated by coherent light it is necessary to consider the diffraction of the light by the object as well as by the subsequent aperture. The ideas are most easily explained by considering a one dimensional grating as the object as in Fig.1

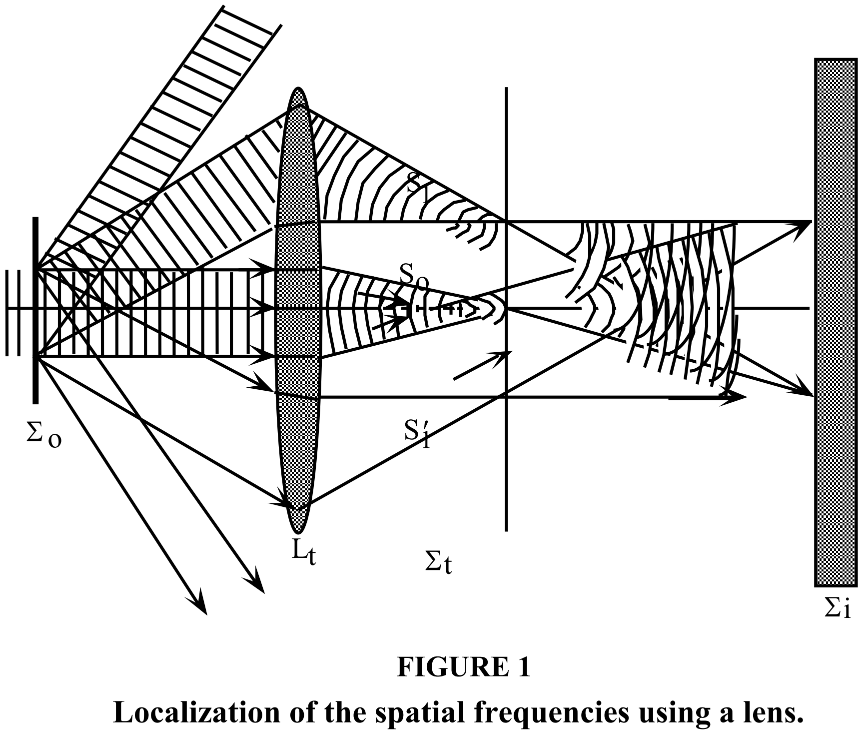

FIGURE 1

Localization of the spatial frequencies using a lens.

The incoming light is diffracted by the grating (situated at Σ_o) and this diffracted light is then diffracted by the lens L_t to form a Fraunhofer diffraction pattern of the grating on the focal plane Σ_t of the lens. The light continues on past this plane to form the image of the grating on the plane Σ_i.

A most important and remarkable thing that can be shown is that the lens acts as a Fourier transformer in that the diffraction pattern on Σ_t is the Fourier transform of the object. This means that the diffraction pattern is the spatial frequency spectrum of the object just as the Fourier transform of a function of time is its frequency spectrum. For the simple one-dimensional grating this means that the point S_0 is the dc component, corresponding to zero spatial frequency, which arises because the light intensity cannot be negative. The point S_1 corresponds to the fundamental spatial frequency present in the object and the subsequent points S_2 etc, further out, correspond to harmonics of this fundamental frequency at higher and higher spatial frequency.

Not only is the diffraction pattern the Fourier transform of the object but the inverse Fourier transform of the diffraction pattern is the image. The importance of the Abbe theory is now clearly evident. Any modification of the diffraction pattern will change the image to some extent. We thus have spatial filtering and the possibility of the introduction of false structure. We see also that every lens must act as a low pass filter since to pass all orders of the light diffracted by the object the lens would have to have an infinite aperture. The loss in high frequency information leads to a loss in sharpness of the image. Finally there is the limit of resolution of the system. For the image to be resolved it is clearly necessary that the first principal maxima, S_1 and S'_I of the diffraction pattern, be passed by the aperture since these correspond to the fundamental spatial frequency. This is the condition which replaces the Rayleigh condition for non-coherent light.

In general, of course, the object is not just a one dimensional grating. For a general two-dimensional object, however, all the above ideas still apply (one can imagine the object to be a series of gratings of different spacing set at various angles to one another). The diffraction by the object is now not as simple but notwithstanding this, a two-dimensional diffraction pattern is produced on the focal plane of the lens, a pattern which is the two-dimensional Fourier transform of the object and hence its spatial frequency spectrum.

3. APPARATUS

The apparatus consists essentially of an optical bench with various mounts used to support the different components of the optical system. Fig.2 shows the set-up which is used for the majority of the investigations.

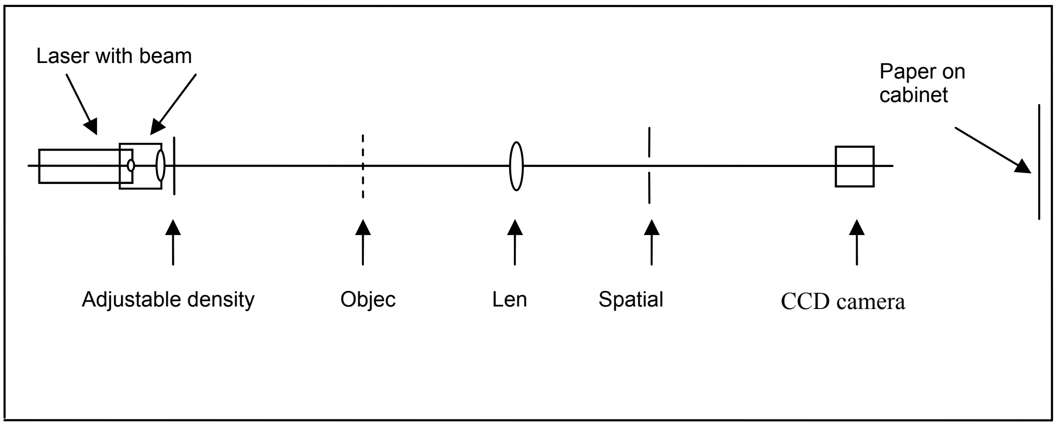

FIGURE 2
Experimental arrangement for examination of optical images.

In turn the items of the system are as follows:

1) **Laser:** this is a 1mW diode laser (λ=635nm) and is used as the coherent light source for the system. The height of the laser can be adjusted as can its tilt in the horizontal plane. It cannot be tilted in the vertical plane but it has been shimmed so that the beam is horizontal. Laser beam is expanded by the build in telescopic beam expander. It can be focused at desired plane or set as parallel by moving the second lens (by rotating it). Care should be taken in using the laser. DO NOT ALLOW THE LASER BEAM TO ENTER YOUR EYE.

2) **Laser current regulator:** the laser beam intensity can be adjusted by changing electric current supplying the laser diode. Additionally use 3) if beam intensity is still too high.

3) **Adjustable neutral density filter:** if the images observed are too bright the intensity of the beam should be decreased. This is achieved by rotating a neutral density filter in the mount next to the laser. IT IS VERY IMPORTANT TO KEEP INTENSITY OF THE LASER BEAM LOW TO AVOID SATURATION OF THE CCD CAMERA OR EVEN IT'S DEMAGE.

4) **Object:** for most investigations the object illuminated is a vertical line grating (one-dimensional object) or a mesh (two-dimensional object). Other objects are used, however.

5) **Lens:** the lens used has a focal length of approximately 310 mm.

6) **Spatial filter:** a variety of spatial filters are available for modifying the diffraction pattern in the focal plane of the lens. These consist of an adjustable slit, a rotating (iris) diaphragm, a black point, a periodic filter and a pinhole. The pattern at the focal plane is viewed using a magnifying glass.

7) **Screen:** the image is either formed on a white card mounted inside a cabinet in the line of the laser beam or on the chip of a CCD camera.

8) **CCD camera:** A CCD camera may be used to capture the image. The camera is mounted on an adjustable stand so that its position can be adjusted to obtain a sharp and clear image of an object. The camera can be rotated to face the beam or be removed away from the beam using a hinge mechanism.

9) **Computer with associated software:** A computer is used for observing and analysing the image captured by the CCD camera. This computer is equipped with a frame grabber board which digitises the images from the camera. The programme used to view or capture images is called “Camera”. Captured images can be analysed using “Scion Image” programme.

4. GRATING DIFFRACTION PATTERN

The object to be used in the majority of the later investigations is the vertical line grating supplied. Here we examine the details of its diffraction pattern.

If the CCD camera at the end of the optical bench is facing the laser beam rotate it so that it is out of the path of the beam.

Check that the laser beam is horizontal (if it is not, notify your demonstrator).

To observe the diffraction pattern with greatest accuracy focus the laser beam onto a piece of paper fixed to the inside of the cupboard on the wall at the end of the optical bench. Place the vertical line grating in its mount, near to the beam expander and adjust its position until all lines are illuminated.

Observe the diffraction pattern on the paper. Measure the separation, x, of the maxima, the so-called orders of the diffraction pattern. Determine from this the separation, d, between the slits in the grating and its associated error. Use the formula which gives the angular positions, θ, of the different orders, n, viz.

$$d \sin\theta = n\lambda \qquad n = 0, 1, 2, 3 \ldots$$

where λ is the wavelength of the light (635 nm). Since θ is small this becomes

$$d = \frac{n\lambda D}{x} \qquad n = 0, 1, 2, 3 \ldots$$

where D is the distance from the slits to the diffraction pattern. Measure D using the tape measure supplied.

You will notice that a few of the maxima are either missing or weak. An order is missing if an interference maximum, determined by the above equation, occurs at the same place as a minimum of the diffraction pattern determined by the width of the slits. If the slit width is a, these diffraction minima occur at angles θ given by

$$a \sin\theta = m\,\lambda$$

There is conjunction between the diffraction minima and interference maxima, hence, when

$$\frac{d}{a} = \frac{n}{m}$$

For orders to be completely missing d/a thus has to be an integer, since n and m are integers. If d/a = 5, for example, this equation shows the 5th, 10th, 15th etc orders are missing. If d/a is not exactly an integer some orders will be weaker, rather than missing.

Estimate the first missing order (with its associated error) and hence obtain an estimate for the slit width a.

Measure the slit width, a, and separation of the slits, d, using the travelling microscope. Compare these values with those obtained from the diffraction pattern.

Now we will investigate the diffraction pattern of the vertical grating in the focal plane of the lens using the CCD camera and the “Camera” programme installed on the computer.

Remove the grating from the optical bench and adjust the beam expander so as to produce a parallel beam of light of approximately the same size all the way along the optical bench. Replace the grating on the optical bench so it is illuminated by the beam. Reduce light intensity to minimum using the laser current regulator and the adjustable neutral density filter. Place the lens on the optical bench in between the grating and the CCD camera around 310mm from the camera. Rotate the camera on the hinge to face a laser light. Run the “Camera” programme. Now you should see a diffraction pattern of the grating formed by the lens in its focal plane. Adjust position of the lens to observe a sharp image of the diffraction pattern. Change the position of the object moving it along the optical bench between the lens and the laser. What changes in the diffraction pattern do you observe?
Hint: Move the camera aside to observe higher frequency diffraction pattern not just a central part of the spectrum.

Repeat measurement that you did previously knowing that camera image width is ¼ of inch (6.35mm). X and Y position of the cursor and intensity of the image at this point can be read in the "Info" window ("View->Info").
Replace the vertical grating object with fine mesh. What is the main difference in the diffraction pattern between those two objects?
How a diffraction pattern in the focal plane changes when you replace the fine mesh with a coarse mesh?

5. IMAGE FORMATION WITH COHERENT LIGHT

In this section we look at the images of objects illuminated by coherent light and see how these images are modified by spatial filtering in the focal plane of the lens used to form the image. The set-up for these investigations is as shown in Fig.2.

5.1 Image of a one-dimensional grating

An image of the object is formed using the lens provided. The image should be observed using CCD camera.
Using the approximate focal length given (310mm) calculate the position of the lens and the object to achieve magnification of 0.5. Move the lens and the object to that position. Fine adjust position of the lens to have the image in focus. Note the image and its main features. How to change position of the lens to achieve image magnification of 2? Verify your guess.

From the object, lens and image positions determine the focal length of the lens exactly.

5.2 Spatial filtering

Place the spatial filter mount so that a white card held in it is at the focal plane of the lens. View the pattern on the screen and check, approximately, that it is the same as the diffraction pattern observed in Section 4.

Adjust the lens position until the image is sharp and clear. From the "Selection" menu of the "Camera" programme choose "Select for FFT". This will select a central region of the image which horizontal and vertical size in pixels is a power of 2. This is necessary because "Scion Image" programme will use so called Fast Fourier Transform (FFT) algorithm designed for such images. When you are happy with the quality of the image save the selection to a file (in "BMP" format). This file will be used in later stages of the experiment.

Examine the effect on the image of spatial filtering in the focal plane. You should try removing different spatial frequencies using the masks of different width and passing spatial frequencies using the adjustable slit and the adjustable iris. In this context at least the following four cases should be tried:-

1) *Pass only the central frequency (mask the rest).*
2) *Pass the central and two side band frequencies closest to it.*
3) *Pass high frequencies only.*
4) *Pass one side only.*

Record your observations and give an explanation for all of the observed effects.

You should be looking for not only gross changes to the image, that is the introduction of false structure, but other effects such as a decrease in sharpness of the image and changes in the relative brightness of the bright and dark parts of the image.

5.3 Computerized image analysis

- *Now close the "Camera" programme and run the "Scion Image" software.*

- *Load the file that you have saved earlier. Perform a FFT on the image ("Process->FFT->FFT"). This will result in a diffraction pattern (in a new window) observed in the focal plane of the lens. The programme use to assign black colour to the highest values of the spectrum, so the image of FFT will look as inverted image you observe at the focal plane of the lens. To rectify this (if you wish to do so) choose "Edit->Invert" menu.*

- *Perform an inverse FFT on the FFT window. The result that will appear on a new window should be the same as the original image.*

- *Now close all windows except the main image window. Produce again a FFT of this image.*

At this stage we would like to remove some of the spatial frequencies from the diffraction pattern and examine its effects on the final image. To do so we will paint a mask on the FFT window. This can be done by painting with the brush or by selecting a shape like rectangle or ellipse and filling selection using "Edit->Fill" menu (Ctrl+F). Black areas will pass the corresponding frequencies and white areas will filter out the corresponding frequencies. It is not, however, possible to both pass and filter during the same inverse transform.

Repeat procedures 1-4 given in section 5.2 on the saved image using the FFT software and explain the results.

5.4 Image of a two-dimensional grating

Now close the "Scion Image" software and run "Camera" programme.
Use a slightly dusty fine mesh object. Observe its Fourier transform in the focal plane of the lens. Try to filter its image to have just a dust to be visible. What shape and size of the filter should be used?
What filter will be needed to achieve an image of the mesh without a dust visible?
Hint: See Hecht "Optics", Ch. 13.
Select a part of the image for FFT and save it to file.
Open this file in "Scion Image" and repeat all the spatial filtering you did above using digital processing.

5.5 Image of phase objects

The objects examined so far are what are known as **amplitude objects**. These are observable because of the variations they cause in the amplitude of the light waves. In contradistinction to these are **phase objects** those which are transparent, thereby providing no contrast with their surroundings, but which alter the phase of the waves. The optical thickness of such objects varies from point to point as either the refractive index or actual thickness, or both, vary.

Biological specimens are often of this nature. Various techniques have been developed to render such objects visible. All these techniques convert phase variations into amplitude variations which are visible. All of these, (apart from staining which has its drawbacks), rely on spatial filtering in the focal plane of a lens. The best of these is the phase contrast method, used in phase contrast microscopes, which makes use of a $\pi/2$ phase shifting filter.

Good results, however, can be obtained by just removing certain frequency in the diffraction pattern.

In the **Dark Field** technique, the central maximum is removed. This central maximum represents the incoming beam unperturbed by the object. If this is removed, we are simply left with the diffractive contribution of the object.

In the **Schlieren Technique** all maxima on one side of the pattern (and the central maximum) are removed.

For further information see Hecht section 13.2.

A close approximation to a true phase object would be the heated air above a hot wire. This should be investigated.

Place a wire heater in the object's mount. Using the computer display, observe the image of the wire. Adjust the wire that only its top is visible (the wire should be placed along the laser beam). Switch on a current to heat the wire. Investigate the effect on this image of using the Schlieren technique of masking in the focal plane of the lens. Comment on your results.

QUESTION:- Why won't we get the same result if we artificially remove the frequency components within the computer?