Quantum Error Correction

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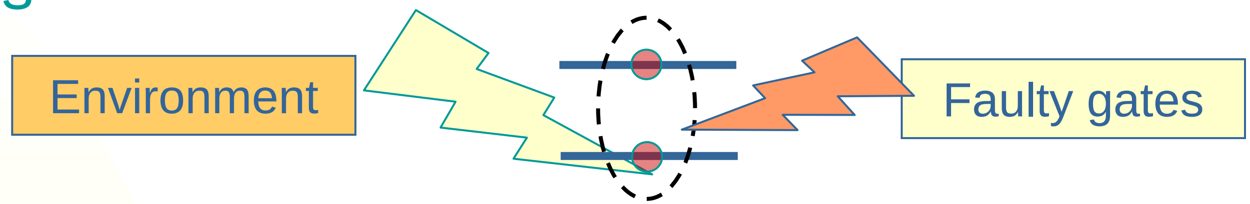
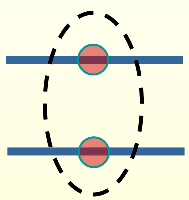
Decoherence – Quantum Errors

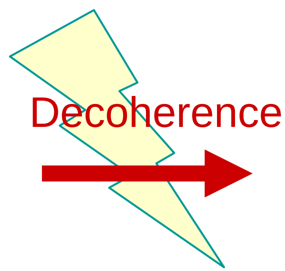
- Environment, or imperfect gates/operations, can cause errors



- Quantum superpositions are extremely fragile



$$\psi = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$



$$P(g) = |\alpha|^2 \qquad P(e) = |\beta|^2$$

Quantum superposition of ground and excited states → Probabilistic mixture of ground and excited states

Quantum Error Correction

- Pre-1995: most people believed that quantum computing would be impossible, due to errors
- Usual tricks of error correction won't work...
  - quantum states can have a "continuum" of errors (same problems as analog computing?)
  - measuring a qubit to detect errors will destroy the information
- Shor (1995): quantum error correction is possible
- Two "tricks":
  - Continuous errors can be decomposed into a finite basis

    i.e., use the fact that qubits are more like bits than continuous (analogue) variables
  - Special measurements can be done to detect errors *without* measuring the quantum information

Errors on a qubit

- General expression:

Prob of error i ⟶ $p_i\,,\quad U_i$ ⟵ Error i as unitary

- Each U_i is a 2x2 unitary matrix

$$U = \begin{pmatrix} u_{00} & u_{01} \\ u_{10} & u_{11} \end{pmatrix} \qquad U^{\dagger} = U^{-1}$$

- Decompose any 2x2 unitary matrix:

$$U_i = a_i I + b_i X + c_i Z + \mathrm{i} d_i XZ$$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Errors on a qubit

- Decompose any 2x2 unitary matrix:

Both bit flip and phase flip

$$U_i = a_i I + b_i X + c_i Z + \mathrm{i} d_i XZ$$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$I|\psi\rangle = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

Does nothing

$$X|\psi\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \beta \\ \alpha \end{pmatrix}$$

"Bit flip"

$$Z|\psi\rangle = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \alpha \\ -\beta \end{pmatrix}$$

"Phase flip"

Errors on a qubit

- Decompose any 2x2 unitary matrix:

Arbitrary error: $U_i = a_i I + b_i X + c_i Z + \mathrm{i} d_i XZ$

No error | Bit flip | Phase flip | Both bit flip and phase flip

- Any error on a qubit is:
  - a convex (probabilistic) sum of...
  - quantum superpositions of...
  - no error, a bit flip, a phase flip, and both
- Basic idea of quantum error correction:
  - measure properties of the *error*, not the encoded quantum information

Example: ECC for bit flips

- "Classical" ECC for bit flips:
  - encode a logical bit in three bits

$$0_L = 000$$

$$1_L = 111$$

  - if a bit flip occurs on one (and only one bit)...

    e.g., flip 2nd bit: $000 \rightarrow 010 \qquad 111 \rightarrow 101$
  - can measure the "error syndrome" (flip on 2nd bit)...
  - and correct
- Key idea for extending to the quantum case:

Ask:	**Don't ask:**
Has a bit been flipped? If so, which bit is different?	What is the value of each bit, then determine error

Example: QECC for bit flips

- Encode a logical bit in three bits

$$|0_L\rangle = |0\rangle_1|0\rangle_2|0\rangle_3 = |000\rangle$$

$$|1_L\rangle = |1\rangle_1|1\rangle_2|1\rangle_3 = |111\rangle$$

- Quantum error $U = aI + bX$ occurs on 2nd qubit

$$U|0_L\rangle = a|000\rangle + b|010\rangle$$
$$U|1_L\rangle = a|111\rangle + b|101\rangle$$

$$U(\alpha|0_L\rangle+\beta|1_L\rangle) = a(\alpha|000\rangle+\beta|111\rangle)+b(\alpha|010\rangle+\beta|101\rangle)$$

- Measure error syndrome: what *subspace* is it in?

$\{|000\rangle, |111\rangle\}$ No error

$\{|010\rangle, |101\rangle\}$ Error on 2nd qubit

$\{|100\rangle, |011\rangle\}$ Error on 1st qubit

$\{|001\rangle, |110\rangle\}$ Error on 3rd qubit

Example: QECC for phase flips

- Define two orthogonal states

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \qquad |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

- Observe: $Z|+\rangle = |-\rangle$, $Z|-\rangle = |+\rangle$ Phase flip acts like a bit flip on these states

- QECC for phase flips

$$|0_L\rangle = |+\rangle_1|+\rangle_2|+\rangle_3 = |+++\rangle$$

$$|1_L\rangle = |-\rangle_1|-\rangle_2|-\rangle_3 = |---\rangle$$

- Measure error syndrome: what *subspace* is it in?

$\{|+++\rangle, |---\rangle\}$ No error

$\{|+-+\rangle, |-+-\rangle\}$ Error on 2nd qubit

$\{|-++\rangle, |+--\rangle\}$ Error on 1st qubit

$\{|++-\rangle, |--+\rangle\}$ Error on 3rd qubit

QECC for any error on a qubit

Arbitrary error: $U_i = a_i I + b_i X + c_i Z + \mathrm{i} d_i XZ$

No error | Bit flip | Phase flip | Both bit flip and phase flip

- Key: concatenate codes

$$|0_{L1}\rangle = |0\rangle_1 |0\rangle_2 |0\rangle_3 = |000\rangle$$

$$|1_{L1}\rangle = |1\rangle_1 |1\rangle_2 |1\rangle_3 = |111\rangle$$

- Define:

$$|+_{L1}\rangle = \frac{1}{\sqrt{2}}(|0_{L1}\rangle + |1_{L1}\rangle)$$

$$|-_{L1}\rangle = \frac{1}{\sqrt{2}}(|0_{L1}\rangle - |1_{L1}\rangle)$$

- Level 2 code: one logical qubit in 9 physical qubits

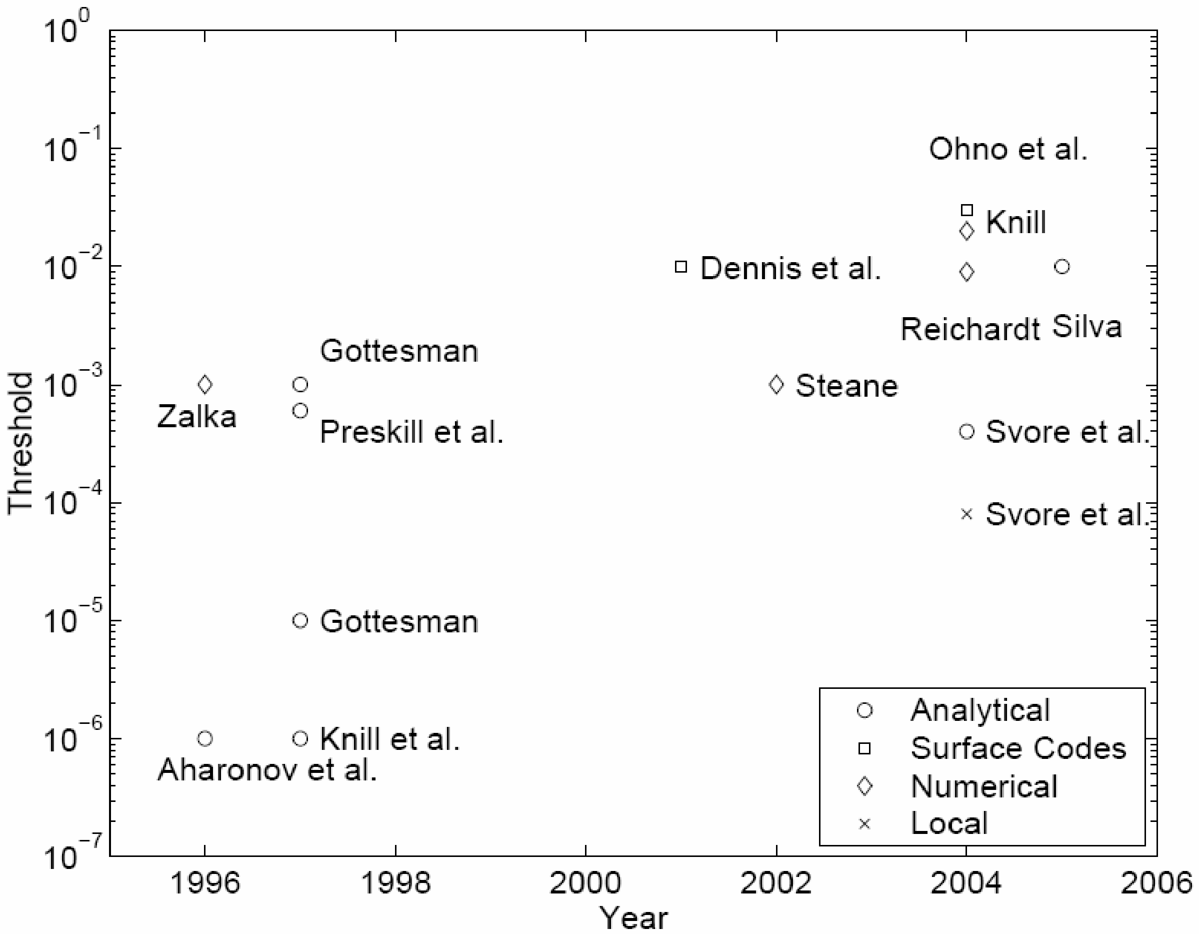
$$|0_{L2}\rangle = |+_{L1}\rangle_1 |+_{L1}\rangle_2 |+_{L1}\rangle_3$$

$$|1_{L2}\rangle = |-_{L1}\rangle_1 |-_{L1}\rangle_2 |-_{L1}\rangle_3$$

Recent progress

- Steane (1996): a 5-qubit QECC protecting against independent single-qubit errors
- Many “classical” ECCs (e.g., turbo codes) have been generalised to quantum codes
- Fault tolerant quantum computation:
  - Can a quantum computer built out of faulty parts function?
  - Multiple levels of concatenation
  - Thresholds for quantum computation: 10^{-6} to 10^{-2}
  - Devices are improving, optimism is increasing...

Theoretical progress



Better codes,

Better architectures,

Better simulations

What to believe?

What can devices achieve today?

K. M. Svore, A. W. Cross, I. L. Chuang and A. V. Aho,

http://www.arxiv.org/abs/quant-ph/0508176 (August 2005)