Bounce of an oval shaped football

Rod Cross*

Physics Department, University of Sydney, Sydney NSW 2006, Australia

Abstract

The bounce of an oval shaped football may appear to be erratic but it can be described by simple laws of physics. Projected at an oblique angle without spin or with backspin, a football usually bounces backward if the top end points backward on impact. Projected with topspin, a football usually bounces forward. A football incident with topspin will sometimes bounce to a much larger height than usual, and other times it will roll for a short distance before it bounces. These effects were examined by filming the bounce with a video camera, and explanations are provided in terms of the normal reaction and horizontal friction forces acting on the ball. A surprising feature is the fact that the friction force can reverse direction 3 or 4 times while the ball remains in contact with the surface.

I. INTRODUCTION

Unusual bounce effects are observed if an oval shaped football is projected obliquely onto a horizontal surface. The ball will sometimes bounce backward without significant rotation. Other times it will bounce forward or backward spinning rapidly in either a clockwise or a counter–clockwise direction. These effects are unusual in the sense that they differ from the familiar bounce of a spherical ball. A football is not alone in this respect. Unusual bounce effects are also observed when other elongated objects such as a pencil or a screwdriver are dropped or thrown to the ground.[1] The bounce behaviour of an elongated object differs from that of a spherical ball since the normal reaction force does not usually act along a line through the center of mass. Consequently the torque applied to an elongated object when it bounces depends on its orientation at impact and can be significantly larger than that on a spherical ball. An elongated object can also bounce without a significant change in spin if the torque due to the friction force is approximately equal and opposite the torque due to the normal reaction force.

Football players in Australia are required to throw the ball forward onto the ground if they wish to run more than 15 m with the ball, but can catch it on the run as it bounces back toward them, provided it is thrown at an appropriate projection angle and lands at an appropriate angle of inclination. This version of football is known as Australian Rules, it is played at a very fast pace, and it regularly attracts crowds of 80,000 people. A football kicked along the ground exhibits another curious effect. That is, it can bounce several times to a height of around 0.5 m and then suddenly bounce to a height of 2 m or more, as if its coefficient of restitution (COR) suddenly increased well above unity. Alternatively the ball might bounce to a height of less than 0.1 m after several previous 0.5 m bounces. This behaviour is quite unlike that of a spherical ball. A spherical ball projected downward at an oblique angle onto a horizontal surface normally bounces forward and upward with an angle of reflection approximately equal to the angle of incidence, and it bounces with a COR that is less than unity. The ball does not normally bounce backward, although a superball or a basketball can do so if the ball is projected near normal incidence with backspin.[2,3]

Measurements and calculations are presented below to describe the bounce of a football under several different initial conditions where the ball was allowed to spin about a transverse, horizontal axis. Video film was used to measure the incident and rebound speeds, spins and angles, and the data were used to examine the roles of the friction and normal reaction forces in determining the bounce behaviour. Direct time–resolved measurements of the friction and normal reaction forces were also made using a force plate designed specifically to measure the bounce of a ball.

II. BOUNCE GEOMETRY

A football has an approximately elliptical cross section defined by the relation $(x/a)^2 + (y/b)^2 = 1$ where a and b are the major and minor radii respectively. If the ball makes contact at coordinate $(x,\ y)$ with a horizontal surface then the long axis is inclined at an angle α to the horizontal given by $\tan\alpha = dy/dx = -(b^2x)/(a^2y)$. The radial distance from the contact point to the center of the ball is $R = (x^2 + y^2)^{1/2}$, the horizontal distance is given by $X = R\sin\beta$, and the vertical distance is given by $Y = R\cos\beta$, where the angle β is defined in Fig. 1. Note that x and X are different variables here, as are y and Y. As α is varied, X passes through a maximum value of $(a-b)$ at α typically about 38° for a football, depending on the ratio of a to b. Consequently, the torque on the ball due to the normal reaction force is a maximum when α is about 38° (or $180 - 38 = 142°$).

If the ball is rotating at angular velocity ω just before or after the impact then the contact point has a velocity component $v_\perp = R\omega\sin\beta = X\omega$, relative to the center of the ball, in a direction perpendicular to the surface. It also has a velocity component $v_\parallel = Y\omega$ in a direction parallel to the surface, relative to the center of the ball. The contact point of a spinning spherical ball also has a velocity component parallel to the surface but it has no perpendicular component relative to the center of the ball immediately before impact.

We assume that the center of mass (CM) of the ball is incident from left to right with velocity components v_{x1} and v_{y1}, as shown in Fig. 2, and that the CM rebounds with velocity components v_{x2} and v_{y2}. v_{x1} and v_{x2} are taken to be positive if the ball travels left to right, v_{y1} is taken to be positive if the ball is travelling downward toward the surface, and v_{y2} is taken to be positive if the ball is travelling upward away from the surface. The ball is incident at angular velocity ω_1 and rebounds at angular velocity ω_2, both assumed to be positive if the ball rotates in a clockwise direction as indicated in Fig. 2. The ball is incident at angle $\theta_1 = \tan^{-1}(v_{x1}/v_{y1})$ and rebounds at angle $\theta_2 = \tan^{-1}(v_{x2}/v_{y2})$.

Just prior to impact the vertical speed, v_{py1}, of the contact point on the ball is given by

$v_{py1} = v_{y1} + X\omega_1$ and the horizontal speed of the contact point is given by $v_{px1} = v_{x1} - Y\omega_1$. X can be positive or negative depending on the angle of inclination, α, but Y remains positive regardless of the angle of inclination. X is taken to be negative if the ball is inclined with $0 < \alpha < 90^\circ$ as depicted in Fig. 2. Immediately after the impact the vertical speed, v_{py2}, of the contact point on the ball is given by $v_{py2} = v_{y2} - X\omega_2$ and the horizontal speed is given by $v_{px2} = v_{x2} - Y\omega_2$. X and Y are not the same immediately before and after the impact since the ball rotates during the impact. In the experiments described below, the angular velocity of the ball was typically about 10 or 20 rad.s^{-1} and the impact duration was 15 ms. Consequently, the ball rotated by about 10° or 20° during the impact depending on the initial and final rotation speeds.

III. VERTICAL AND HORIZONTAL BOUNCE SPEEDS

The vertical bounce speed of a football depends on the coefficient of restitution (COR), defined by the ratio v_{py2}/v_{py1}. For a spherical ball, the COR is usually defined by the ratio v_{y2}/v_{y1}. However, the COR can be defined more generally in terms of the speed of the contact point rather than the speed of the center of mass. For a spherical ball the two definitions are equivalent. In other situations, such as the bounce of a football or the impact of a bat and a ball, the COR is defined in terms of the normal velocity components at the point of contact.[4,5] As a consequence, the simple relation between COR and bounce height that exists for a spherical ball does not usually apply to an oval shaped football.

The COR of a football is typically about 0.8. If a spherical ball with a COR of 0.8 is dropped from a height of 1.0 m, it will bounce to a height of $0.8^2 = 0.64$ m, even if it spins about an axis when it is dropped. A football dropped from a height of 1.0 m may also bounce to a height of 0.64 m but only if it impacts the ground with its long axis parallel to the ground. More generally, the bounce height of a football will depend on whether $v_\perp$ is positive, negative or zero. For example, the bounce height will be zero if $v_{py1} = 0$ or if $X\omega_1 = -v_{y1}$, a situation that can arise if X or ω_1 is negative and if ω_1 is large enough.

For an oblique impact the contact point on the ball will generally strike the surface at finite horizontal speed and hence the ball will initially slide along the surface. The sliding motion will be opposed by a horizontal friction force F acting at the contact point. The direction of F shown in Fig. 2 is the usual direction when a spherical ball is incident from left to right, in which case the result is a decrease in horizontal ball speed and an increase in angular velocity in a clockwise direction. However, the initial horizontal speed of the contact point for a football is given by $v_{px1} = v_{x1} - Y\omega_1$ which can be positive, negative or zero depending on the magnitude and sign of ω_1. A similar situation can arise with a spherical ball, but a football is different in that the normal reaction force, N, also has a strong influence on the torque exerted on the ball. As a result, the magnitude, direction and duration of the friction force acting on the ball depends not only on the magnitude of N but also on the line of action of N.

Consider the situation shown in Fig. 2 where the ball is incident left to right and rotating clockwise before impact. If ω_1 is relatively small so that $v_{px1} > 0$, then F will act as shown to oppose sliding motion to the right. For a spherical ball, the effect of the friction force is to reduce the horizontal speed of the ball and to increase its rotation speed in a clockwise direction. If a spherical ball is incident without spin within about 60° of the normal, the bottom of the ball will come to rest during the bounce and grip the surface, since the increase in rotation speed and the decrease in horizontal sliding speed is sufficient to reduce the sliding speed to zero. For the situation shown in Fig. 2, F and N both exert clockwise torques on the ball with the result that the net torque acting on the ball is larger than it would be on a spherical ball under the same conditions. The angular acceleration is therefore larger so the contact point will come to rest sooner. Since the backward directed friction force acts for a shorter time, the ball will bounce with a larger horizontal speed. Conversely, if the ball is leaning backward when it impacts the surface, then N will act ahead of the CM and reduce the angular acceleration of the ball. The sliding phase will then persist for a longer period of time and the ball will bounce with a smaller horizontal speed than would a spherical ball.

Sliding friction acting on the bottom of a ball can reduce the horizontal speed of the ball to zero but it cannot reverse the direction of motion of the ball. The backward bounce of a football arises from a backward directed static friction force that persists after the bottom of the ball grips the surface. If the ball was perfectly rigid then the whole ball would come to rest when the contact region of the ball comes to rest, or it would commence to roll along the surface. However, a football is relatively flexible and will stretch horizontally when the contact region comes to rest due to the forward motion of the upper part of the ball. The upper part of the ball therefore exerts a forward force on the contact region while the ground exerts an equal and opposite backward force. Nevertheless, the net effect of static friction can be close to zero in some situations since the static friction force usually reverses direction before the ball bounces off the surface. If the time integrated static friction force is essentially zero then the bounce model described by Brody[6] can be extended to describe the bounce of a football, as outlined in the following Section.

IV. SIMPLE BOUNCE MODEL

The bounce shown in Fig. 2 is governed by the relations $F = -M dv_x/dt$, $N = M dv_y/dt$ and $FY + NX = I_{cm} d\omega/dt$ where M is the ball mass and I_{cm} is the moment of inertia of the ball about a transverse axis through the CM. For the conditions of the present experiment, the gravitational force was much smaller than N. Integrating over the whole time period of the bounce and assuming that X and Y remain essentially constant in time, we find that

$$\int F\,dt = M(v_{x1} - v_{x2}), \tag{1}$$

$$\int N\,dt = M(v_{y1} + v_{y2}), \tag{2}$$

and

$$I_{cm}(\omega_2 - \omega_1) = Y \int F\,dt + X \int N\,dt = MY(v_{x1} - v_{x2}) + MX(v_{y1} + v_{y2}) \tag{3}$$

A simple solution of Eqs. (1)–(3) can be found if the ball slides throughout the bounce period since then $F = \mu N$ where μ is the coefficient of sliding friction. Sliding behaviour persists throughout the bounce when a ball is incident at a grazing angle on a surface, but not when the ball is incident at angles near the normal or even at angles up to about 60° away from the normal. In the present experiment the ball always gripped the surface during the bounce, and a simple relation between F and N did not exist. An alternative solution of Eqs. (1)–(3) can be found in terms of the spin parameter $S_2 = Y\omega_2/v_{x2}$ which represents the ratio of the tangential speed of the ball at the contact surface to the horizontal speed of the CM immediately after the bounce. If the ball were to exit the surface in a rolling mode with $v_{x2} = Y\omega_2$ then S_2 would be 1.0. If the grip condition is maintained in such a way that the bottom of the ball is approximately at rest when it exits the surface then S_2 will be approximately 1.0. Substituting $\omega_2 = S_2 v_{x2}/Y$ in Eq. (3) yields the solution

$$I_o\, v_{x2} = I_{cm}\, Y\, \omega_1 + MY^2\, v_{x1} + MXY(v_{y1} + v_{y2}) \tag{4}$$

where $I_o = S_2 I_{cm} + MY^2$. Equation (4) shows that the horizontal ball speed and direction after the bounce has three separate and independent components depending on the magnitude and direction of (a) the incident ball spin, ω_1, (b) the initial horizontal speed, v_{x1} and (c) the inclination of the ball on impact. For the conditions of the experiment described below, all three components were of similar magnitude. Any two of the three components can be zero and the ball will still bounce forward or backward depending on the sign of the third component. For example, Eq. (4) with $S_2 = 1$ provides a good description of the bounce of a football dropped vertically without spin so that $v_{x1} = 0$ and $\omega_1 = 0$.

Occasionally, a football incident at an oblique angle will bounce vertically or almost vertically, with $v_{x2} \approx 0$. Under these conditions, the parameter S_2 approaches infinity. Equation (4) remains valid regardless of the value of S_2 but it is emphasised that not all bounces are characterised by a value of S_2 close to 1.0, even though the majority are. The exceptional case is one where the signs of ω_1, v_{x1} and X all conspire to produce a nearly vertical bounce.

The main difference between the bounce of a football and a spherical ball is described by the third term in Eq. (4) containing the XY product. This term is zero for a spherical ball, but it introduces a bias into the bounce of a football in the sense that there is an additional forward or backward component to the horizontal bounce speed that depends on the angle of inclination of the ball on impact. For an ellipse, XY has a maximum value of $(a^2 - b^2)/2$ at $\alpha = 45^\circ$, corresponding closely to the angle of inclination at which v_{x2} was found to be a maximum. Furthermore, if v_{x2} is approximately equal to $Y\omega_2$, then the rebound spin is also a maximum near $\alpha = 45^\circ$, as observed experimentally.

If the ball were to slide throughout the bounce period with $F = \mu N$ then μ would be given by the ratio of Eqs. (1) and (2),

$$\mu = \frac{(v_{x1} - v_{x2})}{(v_{y1} + v_{y1})}. \qquad (5)$$

However, if the contact region of the ball comes to rest during the bounce period then the friction force will drop to zero and reverse direction during the grip phase, in which case the average friction force during the whole bounce period will be less than μN. The expression on the right hand side of Eq. (5) then represents the average value of F divided by the average value of N which is shown in the graphs below as the quantity labelled COF. The value of COF is less than μ if the ball grips the surface during the bounce and it is negative if the ball bounces forward with $v_{x2} > v_{x1}$. The latter situation can arise if the incident ball is overspinning, with $Y\omega_1 > v_{x1}$, so that the bottom of the ball slides backward when it first contacts the ground, even though the ball as a whole is moving forward. Even in the latter situation, the magnitude of COF is typically less than μ since the contact region of the ball will usually slide to a stop and then grip the surface.

V. BOUNCE MEASUREMENTS

The football chosen for the present study had a major diameter of 286 mm, a minor diameter of 180 mm and a mass of 433 g, slightly fatter and slightly heavier than an official NCAA ball. It was filled with air to the recommended pressure of about 32 kPa so that it was firm and bounced well. All adult footballs have a length of about 290 mm and an allowed variation in length of about 10 mm, but the minor circumference can vary from about 530 mm for an American football or 550 mm for an Australian Rules football, up to about 600 mm for a Rugby Union football. A Rugby League football has a minor circumference of 565 ± 5 mm. The corresponding ball mass can vary from a minimum of about 400 g for an NCAA ball to a maximum of about 460 g for a Rugby Union ball.

The moment of inertia (MOI) of the chosen ball was measured after gluing a light metal tube to a pointy end so that it could be mounted as a pendulum with an axis coincident with the pointy end. The period of oscillation was 0.909 ± 0.002 s, giving a MOI about a transverse axis through the center of mass, $I_{cm} = 0.00385 \pm 0.5\%$ kg.m^2, in agreement with the theoretically expected value. For a solid ellipsoid of mass M and volume $4\pi ab^2/3$, $I_{cm} = M(a^2 + b^2)/5$. It is easy to calculate the corresponding MOI for a thin ellipsoidal shell of wall thickness t, by substracting the MOI of a solid ellipsoid with major radius $a - t$ and minor radius $b - t$. The result is that $I_{cm} = M(2a^3 + 3a^2b + 4ab^2 + b^3)/[5(2a + b)]$ when $t \ll b$, and it has a value of 0.00385 kg.m^2 for the chosen ball parameters.

The ball was projected by hand, from a height of about 1 m, at speeds between 4 and 6 ms^{-1} onto a concrete floor covered with low pile carpet. Each bounce was filmed at 100 frames/s using a JVC9600 digital video camera with an exposure time of 2 ms. A standard 30 frames/s camera would also have been suitable for this experiment but it would then be more difficult to determine from a given frame whether the ball was about to bounce or had just bounced. Video clips were transferred to a computer for analysis using Videopoint software to manually digitise the coordinates of the ball center of mass. The horizontal speed of the center of mass was determined to within 2% using a linear fit to the horizontal coordinates, and the vertical speed was determined to within 2% using a parabolic fit to the vertical coordinates, assuming a vertical acceleration of 9.8 ms^{-2}. The angular velocity of the ball was also determined to within 2% using a linear fit to angular displacement measurements obtained using a protractor against the flat computer screen. Bounces were analysed only if the long axis remained perpendicular to the optical axis of the camera before and after each bounce, as it did in most cases. The orientation of the ball on impact was determined to within about $\pm 5°$ and is plotted using the nearest 5° grid mark on the horizontal axis in Figs. 3–7.

The bounce speed, spin and angle of a football depends on at least four initial parameters. It depends on the incident speed, spin and angle, and it also depends on the orientation of the ball at impact. A football has two main axes that can define its orientation with respect to the incident plane, and it can also spin about three separate axes. However, for the purposes of the present experiment, the ball was projected so that its long axis remained in a vertical plane, and it was allowed to spin only about a transverse, horizontal axis. As a result, the ball remained in the same vertical plane before and after each bounce and the only relevant spin was either topspin or backspin. A large number of possible combinations of the four incident parameters was possible, but the present study was restricted to examining only a small subset of these combinations. The procedure adopted in each case was to vary the orientation of the ball while keeping the incident speed, spin and angle as constant as possible, subject to small variations due to the fact that the ball was projected by hand. Results were obtained for three different angles of incidence ($\theta_1 = 0°$, 20°, and 50°) and for three different values of initial spin ($\omega_1 = 0$, +17 rad.s^{-1} and −17 rad.s^{-1}). All bounce results are presented below as functions of the initial angle of inclination, α, immediately prior to impact.

For practical reasons, only one ball was studied and it was launched by hand rather than by a mechanical device. About 200 individual bounces were analysed and about one third of those were discarded as being outside the desired range of launch parameters. With a little care and practice, it is possible to launch a ball by hand to within about 2% of any desired velocity and within 1° of any desired launch angle, which is better than most mechanical launchers can achieve. Evidence for the stated accuracy of a hand launch is the fact that most people can throw a tennis ball into a 12 inch diameter bucket nine times out of ten after sufficient practice, even if the bucket is 20 feet away. The same level of accuracy is required to hit a tennis ball from the baseline so that it lands within four feet of the other baseline.[5]

Other factors of significance in the bounce of a football, such as the effects of inflation pressure, the effects of variations in the shape or dimensions of the ball, and different rotation axes were not examined in this paper. Given that any sideways deflection of the ball induced by bouncing generates an even more unpredictable bounce, the effects of rotation about other axes would provide a useful extension to the present study.

VI. VERTICAL DROP RESULTS ($\theta_1 = 0$, $\omega_1 = 0$)

When a spherical ball is dropped vertically without spin onto a horizontal surface, it bounces vertically without spin. The only parameter of interest is the coefficient of restitution (COR), defined as the ratio of the vertical rebound speed to the incident vertical speed. When a football is dropped vertically onto a horizontal surface it can also bounce vertically without spin, but only if one of the two axes of symmetry is aligned perpendicular to the surface. Dropped from a height of 1.0 m, the ball landed at a speed of 4.43 ms^{-1}. The COR for a bounce on the side of the ball (the long axis being horizontal) was found to be 0.82 ± 0.01. The COR for a bounce on the end of the ball (the long axis being vertical) was 0.75 ± 0.01. When the long axis was inclined at an angle other than horizontal or vertical, the ball was observed to bounce sideways with topspin, toward the side to which it was leaning. Results of such measurements are shown in Fig. 3 as a function of the angle of inclination, α.

Fig. 3a shows the three ratios v_{y2}/v_{y1}, COR $= v_{py2}/v_{py1}$, and v_{x2}/v_{y1} as a function of the initial impact angle α. The COR varied smoothly from 0.82 to 0.75 as the impact angle α was varied from 0 to 90°, while the v_{y2}/v_{y1} ratio dropped to a minimum of 0.6 at about $\alpha = 50°$. The ratio v_{x2}/v_{y1} is a measure of the horizontal bounce speed normalised to the vertical drop speed. Since this ratio had a maximum value of 0.46 at $\alpha = 50°$ and since v_{y2}/v_{y1} had a minimum value of 0.6 at the same inclination, it is easy to calculate that the ball bounced at a maximum angle of 37° away from the vertical. The ratio v_{x2}/v_{y1} is consistent with Eq. (4) if allowance is made for the fact that α decreased by about 10°

during each bounce. At high values of α, excellent agreement is obtained using the value of α immediately after rather than immediately before the bounce.

The bounce angle $\theta_2 = \tan^{-1}(v_{x2}/v_{y2})$ and the scaled angular velocity ω_2 are shown in Fig. 3b as functions of α. Since ω_2 is directly proportional to v_{y1} at any given α, and since v_{y1} varied slightly from one bounce to the next, the values of ω_2 in Fig. 3b were scaled to a common bounce speed $v_{y1} = 4.5$ ms^{-1} by multiplying the experimental values of the angular velocity by the dimensionless ratio $4.5/v_{y1}$.

The angular velocity results in Fig. 3b are not consistent with Eqs. (1) – (3), being lower than expected at low values of α and higher than expected at large values of α. X and Y are easily calculated from the ball geometry, at least when the assumption is made that the ball contacts at a single point rather than over an extended region. For example, at $\alpha = 60°$ where $X = 4.1$ cm the expected value of ω_2 was 5 rad.s^{-1} whereas the observed value was 14 rad.s^{-1}. The discrepancy in this case can be resolved by increasing X by 1.1 cm. At other angles of inclination, the discrepancy could also be resolved by a similar or smaller change in X. A change in Y of about 0.5 cm, due to compression of the ball, has a much smaller effect on the theoretically expected rotation speed. Part of the explanation for the increase in the expected value of X can be attributed to rotation of the ball during the bounce. Rotation from $\alpha = 60°$ to $50°$ during the bounce would have the effect of increasing X by 0.85 cm. However, the length of the contact region observed on the video film was typically about 5 or 6 cm. The normal reaction force was therefore distributed over this length and was not applied at a single contact point.

Fig. 3c shows the spin parameter S_2 and a measure of the coefficient of friction, COF, between the ball and the carpeted floor, as given by Eq. (5). S_2 is approximately 1.0 for most angles of inclination, indicating that the ball grips or rolls during the bounce. The ratio $v_{x2}/(v_{y1} + v_{y2})$ is labelled as COF in Fig. 3c to indicate that it is an effective coefficient of friction averaged over the whole bounce period, rather than the actual coefficient of sliding friction, μ. When $S_2 = 1$ we find from Eqs. (1)–(3) that COF $= MXY/I_o$, which has a maximum value of 0.27 at $\alpha = 45°$, essentially as observed.

The fact that the maximum value of the observed COF was only 0.3 provides one indication that the ball did not slide throughout the impact period. A value of $\mu \approx 0.3$ indicates a relatively slippery surface, whereas the ball was dimpled to provide a good frictional grip for the player, and the carpet itself was not slippery. The observed COF was even lower than 0.3

at other angles of inclination of the ball. One possible explanation is that the friction force may have dropped to zero during the bounce if the ball commenced to roll on the carpet at some point in time. However, direct observation of the friction force showed that F did not drop to zero until the end of the bounce period, at least for a vertical drop without spin. Furthermore, the measured F/N ratio for an oblique bounce was approximately 1.0 during the sliding phase of the bounce, indicating that the actual value of μ was approximately 1.0.

VII. DIRECT MEASUREMENTS OF F AND N

Fig. 4a shows a measurement of F and N vs time for a vertical drop with $\alpha = 50°$. These waveforms were obtained with force plate apparatus described previously.[7]. Also shown are corresponding measurements of N and F for a ball incident at $\theta_1 = 22°$ without spin at three different angles of inclination, α. The apparatus used to obtain these waveforms was linear but uncalibrated, so the N and F values are given in arbitrary units. Nevertheless, we know from the video film data that the average value of F did not exceed $0.3\times$ the average value of N, so the relative amplitudes of the waveforms have been scaled accordingly. The waveforms were used primarily to provide a qualitative indication of the the time variations of F and N, and they also showed that the impact duration was 15.0 ± 0.5 ms for all conditions of interest in this experiment.

Fig. 4a indicates that F is directly proportional to N throughout the bounce, although the F/N ratio is lower than one would expect for pure sliding. Given that the ball was incident vertically without any horizontal velocity component, a sliding phase is not expected in this situation. A similar situation arises if a long, slender object rests on a table and is allowed to fall from a near vertical position. The bottom end will slide backward if the coefficient of sliding friction is low, but the bottom end grips the table and the object pivots without sliding if the coefficient of friction is larger than about 0.4.[1] In the present context, the normal reaction force on the ball is due to vertical compression rather than its weight. There is no horizontal force arising from vertical compression but a horizontal static friction force is required so that the CM will accelerate forward as the ball rotates. The horizontal acceleration is proportional to $d\omega/dt$ which is proportional to N, hence F is proportional to N.

By contrast, Figs. 4b–d show an initial sliding phase, with $F/N \approx 1.0$, when the ball is projected with finite horizontal speed. After the ball grips the surface, F reverses direction in (b) and (c). In Fig. 4b, the reversal in F was sufficiently large that the ball bounced forward with a greater horizontal speed after the bounce than it had before the bounce, ie with $v_{x2} > v_{x1}$. In Fig. 4c, the time average value of F remained positive so the ball bounced forward with $v_{x2} < v_{x1}$. In Fig. 4d, F remained relatively large and positive throughout the bounce, with the result that ball bounced backward.

VIII. OBLIQUE BOUNCE WITHOUT INITIAL SPIN

Simplified bounce models exist for spherical balls where an oblique bounce can be completely specified in terms of the COR, the COF and the initial conditions. It is assumed in these models that the ball either slides throughout the bounce or it rolls. Such models cannot adequately describe the bounce of a football since the ball does not slide throughout the bounce, the ball tends to grip rather than roll, and since the grip duration depends on the ball orientation. As shown in Fig. 3, the bounce of a football needs to be described in terms of a relatively large number of parameters even for a simple vertical bounce. For an oblique bounce, the change in horizontal speed of the ball is an additional parameter of interest. The same parameters as those in Fig. 3 are shown in Fig. 5 for the case of an oblique bounce of the football, except that the parameter v_{x2}/v_{y1} is replaced by the more directly relevant parameter v_{x2}/v_{x1}.

Figure 5 shows results obtained when the ball was projected without spin at an angle $\theta_1 = 22° \pm 3°$. The ball bounced forward with topspin for inclination angles $-15° < \alpha < 110°$, and it bounced backward at inclination angles $110° < \alpha < 165°$. The actual direction of spin reversed when the ball bounced backward so the ball bounced backward with topspin. When the ball bounced forward, the horizontal speed after the bounce was generally higher than the incident horizontal speed and the vertical bounce height was relatively low. At $\alpha \approx 110°$ the ball bounced vertically upward with zero spin. Backward bounces occured at relatively low horizontal speed and low rotation speed but the ball bounced 2 or 3 times higher than it did when it bounced forward.

The results in Fig. 5 can be interpreted in simplified terms as being the same as those for a spherical ball of the same mass and COR, but with an additional strong bias in all bounce parameters represented by the results in Fig. 3. For example, a spherical ball incident without spin in a direction from left to right will bounce to the right with topspin. A football incident in this manner bounces at a greater horizontal speed and at a higher spin rate than a spherical ball if it is inclined forward, and with a smaller horizontal speed and smaller rate of spin if it is inclined backward. If one assumes that a football loses the same or a similar amount of kinetic energy when it bounces, regardless of its angle of inclination, then a consequence of the increased forward horizontal bounce speed and rate of spin will be a reduction in bounce height, as observed. Conversely, the ball bounces to a greater height when it bounces backward due to the reduced horizontal bounce speed and spin rate. In fact, the ratio of total kinetic after the bounce to that before the bounce varied from 0.57 (at $\alpha = 100°$) to 0.74 (at $\alpha = 50°$), the lowest energy losses occuring when the ball bounced forward.

IX. OBLIQUE BOUNCE WITH INITIAL BACKSPIN

Figure 6 shows results obtained when the ball was projected with backspin at an angle $\theta_1 = 20° \pm 3°$ and with $\omega_1 = -17.5 \pm 2.0$ rad.s^{-1}. In this case the ball bounced forward with topspin when it was leaning forward on impact (ie $0 < \alpha < 90°$) and it bounced backward with topspin when it was leaning backward (ie $90° < \alpha < 180°$). Compared with the results in Fig. 5, the ball bounced further back toward the thrower, at smaller θ_2 when $0 < \alpha < 90°$ and at larger θ_2 when $90° < \alpha < 180°$, and it bounced at smaller ω_2 when it bounced forward and at larger ω_2 when it bounced backward. As a result, the maximum bounce speed, spin and height was approximately the same for both forward and backward directed bounces.

Some backward bounces had the unexpected property that the COR was greater than 1.0. A slightly lower COR is calculated if one corrects for the small reduction in α during the bounce, but the COR still remains greater than 1.0. A possible explanation is that a backward bounce results in a significant horizontal stretch of the ball after it grips the surface, and that the ball receives an additional vertical impulse during the impact when the ball springs back to its normal shape. In this manner, kinetic energy associated with horizontal motion of the ball could be channeled into vertical motion.

X. OBLIQUE BOUNCE WITH INITIAL TOPSPIN

Figure 7 shows results when the ball was projected with topspin at an angle $\theta_1 = 50° \pm 3°$ and with $\omega_1 = 17.0 \pm 2.0$ rad.s^{-1}. In this case the ball bounced forward with topspin regardless of the angle of inclination at impact, α, but the bounce height varied strongly with α. When the ball landed with a forward inclination it tended to roll forward onto its top end before bouncing upward, as indicated in Fig. 7f. However, the ball sometimes bounced forward through the air at essentially zero vertical speed and then bounced upward as the top end spun around to impact the floor. The results shown in Fig. 7 are those pertaining to the initial low roll or bounce, not the subsequent high bounce that immediately followed the initial bounce.

When the ball landed with a forward tilt, the ball bounced or rolled forward with only a small loss in horizontal speed, a large increase in rate of spin, a low COR, and a low or even negative bounce height. The ratio of total kinetic after the bounce to that before the bounce was typically about 0.85 to 0.90. When the ball landed with a backward tilt, the ball bounced forward to a relative large height, with a large loss in horizontal speed, and a relatively small positive or negative change in rate of topspin. The ratio of total kinetic after the bounce to that before the bounce was typically about 0.7. All of these effects are consistent with expectations. When the ball lands with a forward tilt, the normal reaction force acts behind the ball CM and combines with the backward directed friction force to generate a large torque on the ball. The top end swings downward rapidly resulting in the ball entering a rolling mode. When the ball lands with a backward tilt the normal reaction force acts ahead of the CM and generates a torque that opposes the torque due to the backward directed friction force.

XI. CONCLUSIONS

Anyone who has watched a game of football played with an oval shaped ball will have noticed that the bounce of the ball tends to be erratic. The bounce is governed by the same laws of mechanics that determine the bounce of a spherical ball, but the orientation of a spherical ball is normally irrelevant. The unpredictable bounce of a football is due to random variations in its orientation on impact. The angle of inclination introduces a strong bias in all bounce parameters for a football since the line of action of the normal reaction force, and hence the torque on the ball, depends on the ball inclination at impact. The additional bias can be determined experimentally by dropping a football vertically without spin. A football dropped in this manner bounces at a maximum angle of about 37° away from the vertical, toward the side to which it is leaning when it lands. Regardless of the angle of incidence, the effect of the additional bias is a maximum when the angle of inclination is 45°.

Rule–of–thumb bounce laws for a football depend to some extent on the magnitude of the spin of the incident ball and also depend on the angle of incidence. However, it can be concluded generally that

(a) A football projected at an oblique angle without spin bounces forward if it leans forward on impact and it bounces backward if it leans substantially backward (ie not close to horizontal or vertical).

(b) A football projected at an oblique angle with topspin will bounce forward with topspin, and it will bounce to a greater height if it leans backward on impact than if it leans forward.

(c) A football projected at an oblique angle with backspin will bounce with topspin. It has an approximately equal chance of bouncing forward or backward, and it will bounce to a similar height regardless of whether it bounces forward or backward.

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[*] Electronic address: `cross@physics.usyd.edu.au`

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Figure captions

Fig. 1. A football inclined at angle α to the horizontal. The contact point on the ground rotates at speed $R\omega$ relative to the CM.

Fig. 2. A football incident at angle θ_1 to the vertical bounces at angle θ_2. It is subject to a friction force F and a normal reaction force N. F is proportional to N during the initial sliding phase but not during the grip phase of the bounce.

Fig. 3. Results for a football dropped vertically from a height of 1.0 ± 0.05 m without spin. The ball bounced to the right for the orientation shown in inset (a). The friction force therefore acted to the right. The solid and dashed curves in Figs. 3 to 7 are polynomial fits to the data, not theoretical curves.

Fig. 4. Measured waveforms of N and F for (a) a vertical drop without initial spin and (b)–(d) a ball incident at $\theta_1 = 22°$ without spin at three different angles of inclination, α. The inset in each graph shows the positive directions of F and N acting on the ball.

Fig. 5. Bounce results for a football thrown obliquely without spin. The ball is more likely to bounce forward than backward unless it is deliberately projected so that $\alpha > 100°$, as is the usual case in Australian Rules football.

Fig. 6. Bounce results for a football thrown obliquely with backspin. The ball has an approximately equal chance of bouncing forward or backward. If it bounces backward, the COR can exceed 1.0.

Fig. 7. Bounce results for a football thrown obliquely with topspin. There were no backward bounces under these conditions. When $20° < \alpha < 60°$ the ball bounced vertically in a negative direction as indicated by the first low bounce in (f).

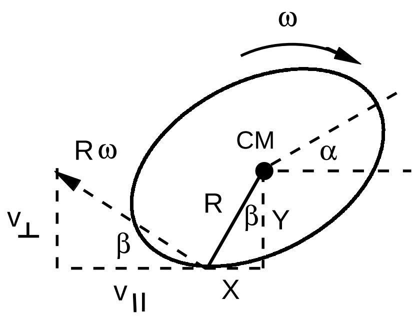
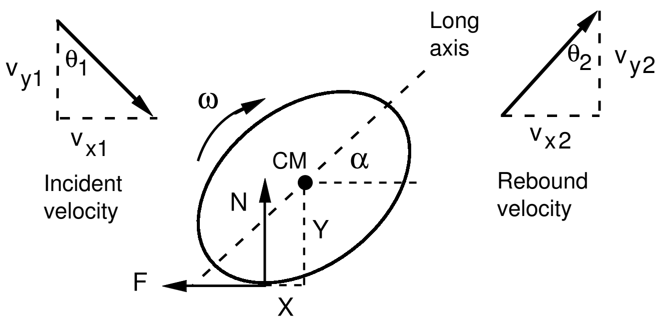


FIG. 1: A football inclined at angle α to the horizontal. The contact point on the ground rotates at speed $R\omega$ relative to the CM.



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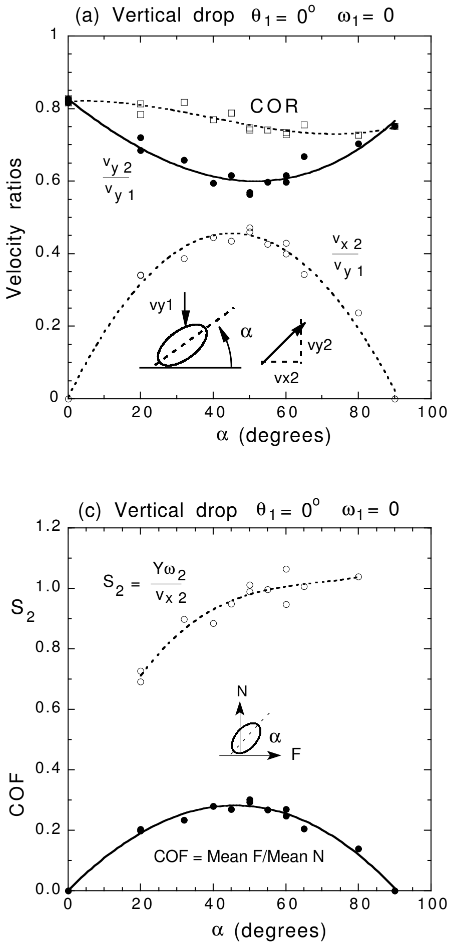

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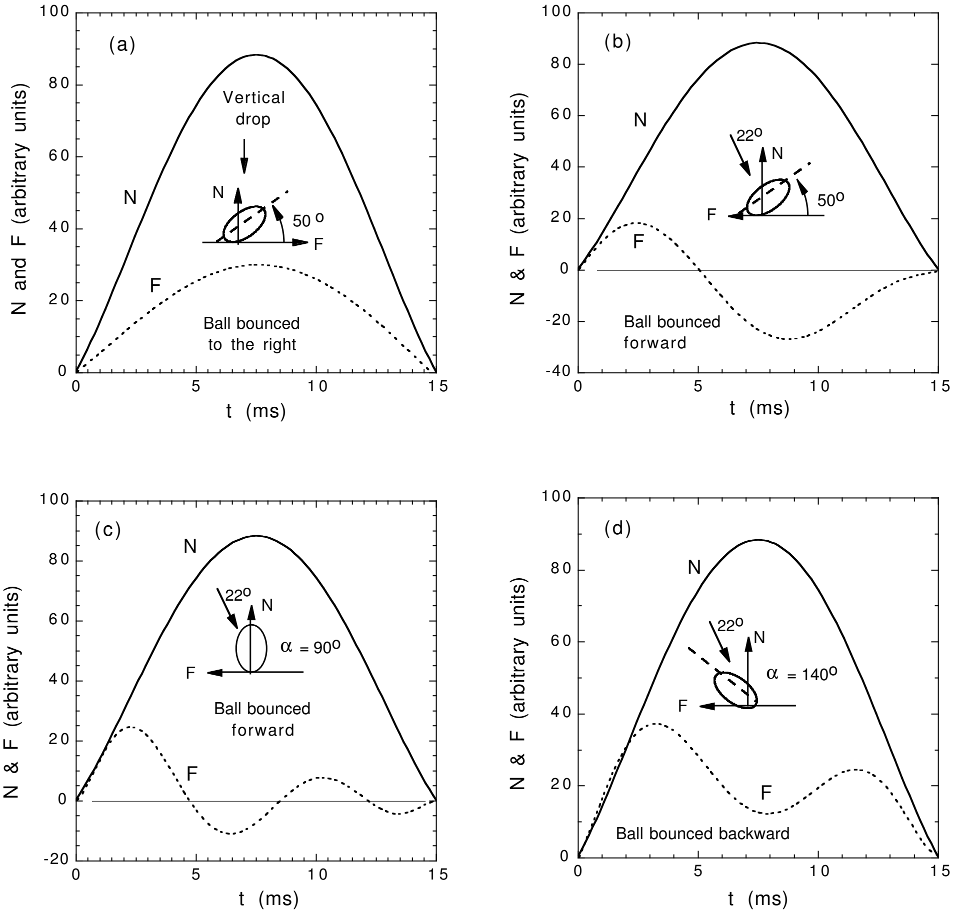


FIG. 4: Measured waveforms of N and F for (a) a vertical drop without initial spin and (b)–(d) a ball incident at $\theta_1 = 22^\circ$ without spin at three different angles of inclination, α. The inset in each graph shows the positive directions of F and N acting on the ball.

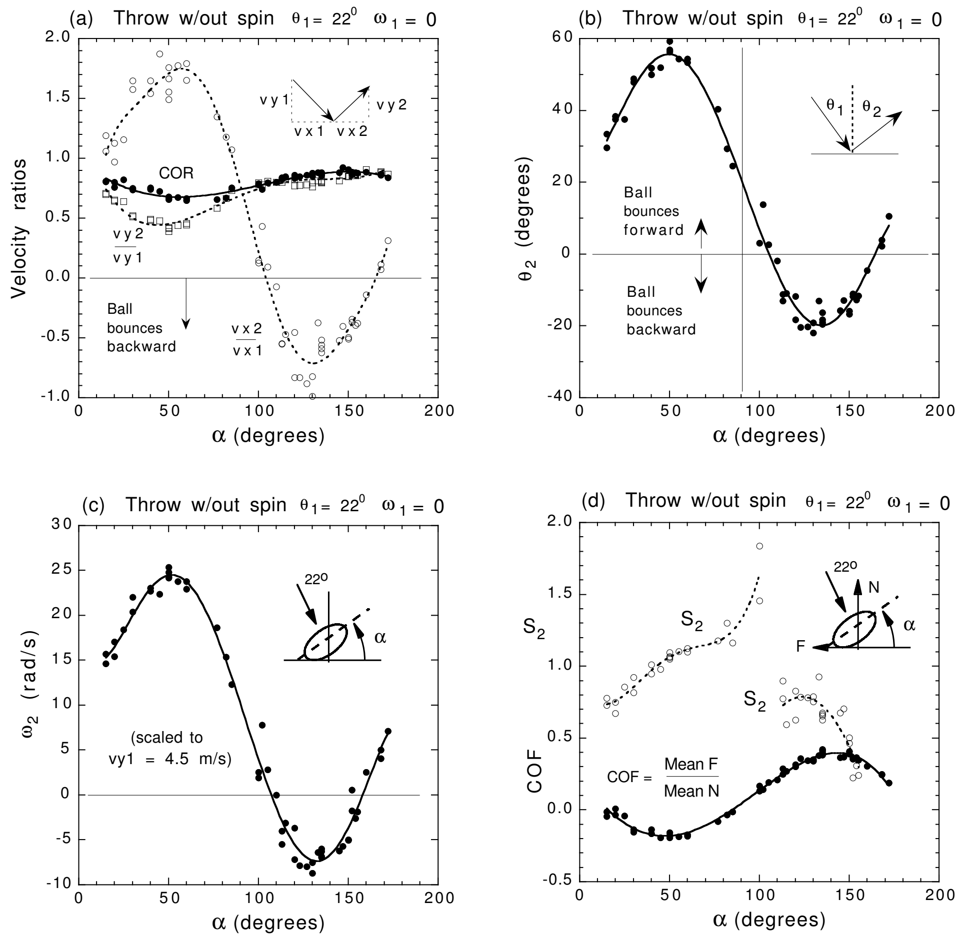
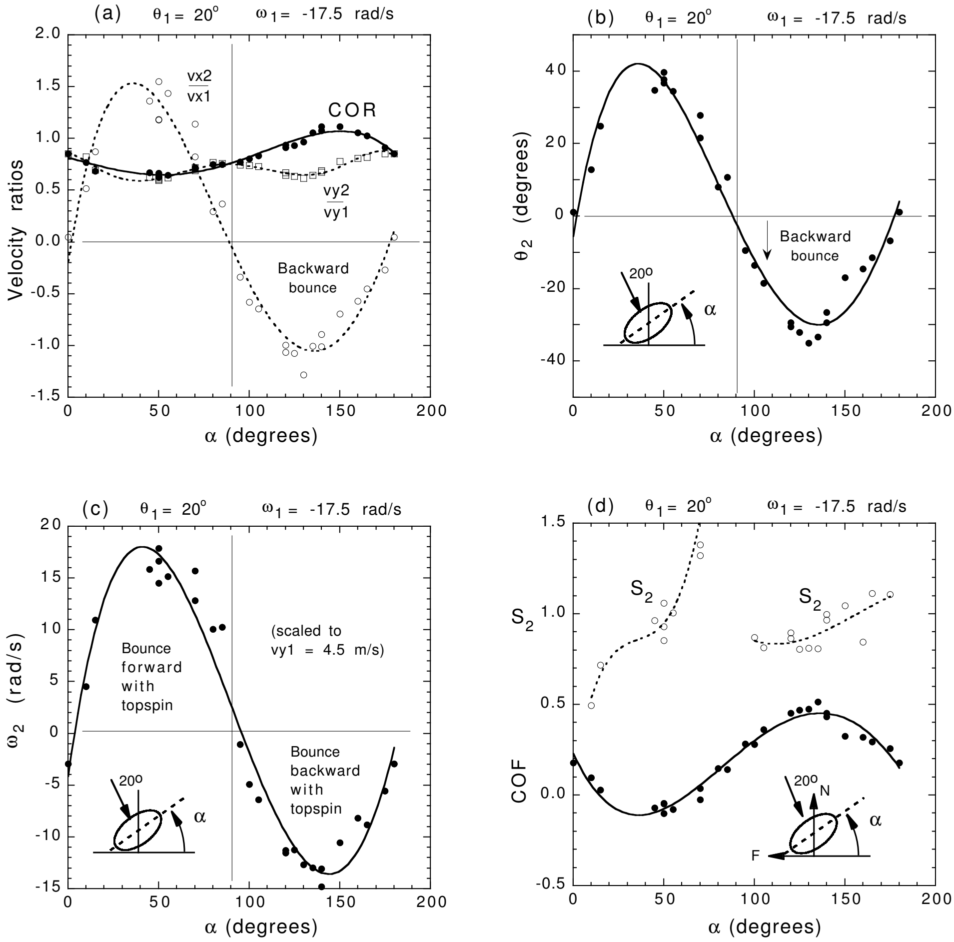


FIG. 5: Bounce results for a football thrown obliquely without spin. The ball is more likely to bounce forward than backward unless it is deliberately projected so that $\alpha > 100°$, as is the usual case in Australian Rules football.



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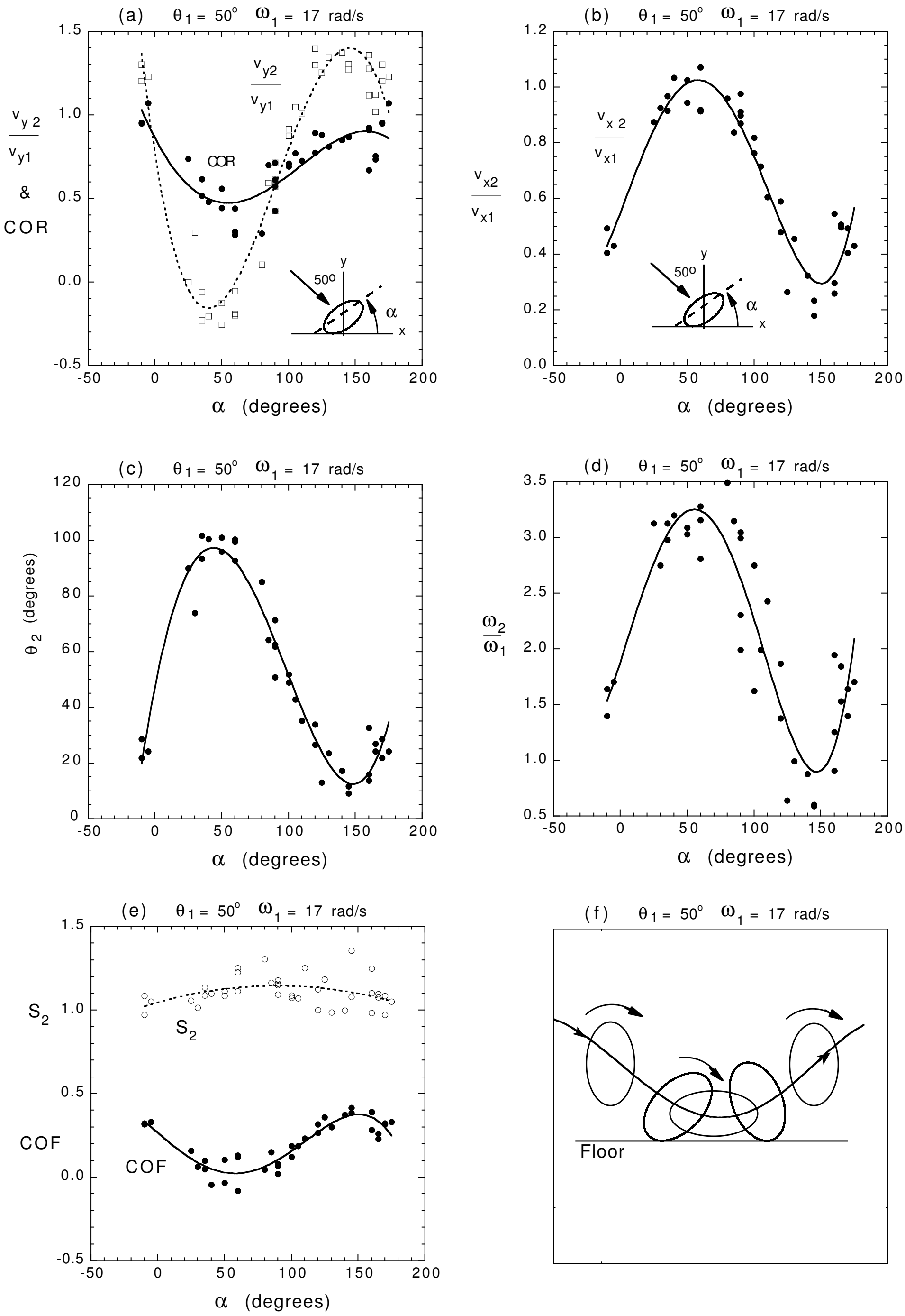

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