Measurements of the horizontal and vertical speeds of tennis courts

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Abstract

Tennis courts are normally classified as fast or slow depending on whether the coefficient of sliding friction (COF) between the ball and the surface is respectively small or large. This classification is based on the fact that the change in horizontal ball speed is directly proportional to the COF if the ball is incident at a small angle to the horizontal. At angles of incidence greater than about 16° it is commonly assumed that the ball will roll during the bounce, in which case one can show that the ratio of the horizontal speed after the bounce to that before the bounce will be 0.645 regardless of the angle of incidence or the speed of the court. Measurements are presented showing that (a) at high angles of incidence, tennis balls grip or 'bite' the court but they do not roll during the bounce, (b) the bounce:speed ratio can be as low as 0.4 on some courts and (c) the normal reaction force acts through a point ahead of the centre of mass. An interesting consequence is that, if court A is faster than court B at low angles of incidence, then A is not necessarily faster than B at high angles of incidence. An exception is a clay court which remains slow at all angles of incidence. The measurements also show that the coefficient of restitution for a tennis ball can be as high as 0.9 for an oblique bounce on a slow court, meaning that the ball bounces like a superball in the vertical direction and that slow courts are fast in the vertical direction.

Keywords: court speed, coeffcient of friction, coeffcient of restitution, bounce

Introduction

The speed of a tennis court is of interest to players, to administrators of the game and to people involved in the construction and purchase of tennis courts, but it is a parameter that has proved difficult to measure on a routine basis. The International Tennis Federation (ITF) has sponsored the development of a device called the Sestee or Surface Pace Rating Apparatus (ITF, 1977) to measure court speed, which functions by projecting a tennis ball onto a surface and uses infrared beams to monitor the incident and rebound speeds and angles. The Sestee device costs around $50 000 and it is not sufficiently simple or portable to be used routinely by tennis clubs. It requires a trained person to operate it. For these reasons, the ITF has also sponsored the development of a simpler and cheaper device called the Haines Pendulum which can be used to measure the coefficient of friction between a tennis ball and a court surface. The ball is mounted on the end of a pendulum, brushes against the court at the bottom of its swing and rises to a height that depends on the coefficient of friction. The Australian and United States Tennis Associations are investigating the use of a commercial device called a Tortus to measure the coefficient of friction between a rubber

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pad and the court surface but it works reliably only on relatively smooth surfaces and is not suitable for measurements on grass or clay, or acrylic surfaces with small ridges. Measurements of court speed in this paper were obtained by filming the bounce of a ball with a video camera. This provides a measurement of court speed in both the vertical and horizontal directions (as does the Sestee) and it is the only technique currently available to measure the spin of the ball. An additional advantage of this technique is that it can be used by anyone with access to a digital video camera and a computer with a video card.

In tennis, the horizontal component of the ball velocity is usually much larger than the vertical component. The horizontal speed of a ball decreases suddenly when the ball bounces, by an amount that depends on several factors, including the angle of incidence, the type of court surface and the rate at which the ball is spinning when it hits the surface. Tennis courts can be classified as fast or slow, depending on whether the coefficient of sliding friction (COF) between the ball and the court is respectively small or large. The COF on a fast grass court is about 0.6 while the COF on a clay court is typically about 0.8. Grass is the fastest court used for major tennis tournaments and clay is the slowest.

An additional factor determining the playing characteristics of a court surface is the coefficient of restitution (COR), defined as the ratio of the vertical component of the rebound speed to the vertical component of the incident speed. The COR affects the bounce height off the court and it also affects the rebound angle and the change in horizontal speed during the bounce. The latter effect is due to an increase in the normal reaction force if the COR is larger, and hence the friction force is also larger. The rules of tennis specify that a ball dropped from a height of 100 inches (2540 mm) onto a hard surface such as concrete must bounce to a height between 53 and 58 inches (1350–1470 mm). This translates to a COR of $0.745 \pm 2.3\%$. It is well known that the COR decreases as the incident ball speed increases (Brody, 1979; Casolo *et al.*, 1994; Cross, 1999) or if the ball is dropped onto a soft inelastic surface such as carpet or grass. The decrease in the COR with ball speed is in fact not of much relevance for a fast serve, since the vertical component of the ball speed at impact is typically about the same as that for a vertical 100 inch drop. Less well known but of greater significance is the fact that the COR increases under some conditions for an oblique angle impact. At ball speeds and angles typical of those for a fast serve, the COR on grass varies from about 0.6 to about 0.9, depending on the condition of the surface. The corresponding COR for an oblique impact on clay varies from about 0.8 to about 0.9. Since the bounce height is proportional to the COR squared, a ball can bounce higher off a clay court than a grass court by a factor of $(0.85/0.6)^2 = 2.0$, and by an even larger factor if a ball is hit with topspin, since the ball is usually incident at a greater vertical speed when hit with topspin.

The modern game of professional tennis is played at a much faster pace than in the pre–1970s wood racquet era, due to technical advances in racquet design and to the increased strength, fitness and height of modern players. This is especially noticeable on fast courts. For example, the average first serve speed for men competing at Wimbledon is about 185 km h^{-1} (115 mile/h) while the average first serve speed for men competing at the French Open is only 160 km h^{-1} (Haake *et al.*, 2000). The court itself has no effect on the speed at which a player can serve the ball, but most players reduce their serve speed when playing on clay in order to impart more spin to the ball. An interesting question is whether players base this choice on the horizontal speed of the court or on the vertical speed (i.e. the COR), or on a combination of both factors. In this paper, the main emphasis is to identify those factors that have the greatest influence on the horizontal and vertical speed of the court. It is shown that the simplified bounce model described by Brody (1984) accounts for the behaviour of a ball incident at low angles, but the model is unsatisfactory at higher angles of incidence. Brody's model indicates that at large angles of incidence the change in horizontal speed of a ball should be the same for all courts. This is not consistent with the experimental data presented below.

Surface pace and the Brody bounce model

Consider a spherical ball of mass m, radius R and moment of inertia $I = \alpha mR^2$. For a thin spherical shell, $\alpha = 2/3$ but for a tennis ball of outer radius 33 mm and wall thickness 6 mm, $\alpha = 0.55$. We consider a situation where the ball is incident at speed v_1, at an angle θ_1 and at angular speed ω_1 on a horizontal surface, as shown in Figure 1. It can be assumed that the mass of the surface is infinite and that the impact force is much larger than the gravitational force. As shown by Brody (1984), the ball will slide throughout the bounce if θ_1 is sufficiently small, in which case

$$\frac{v_{x2}}{v_{x1}} = 1 - \mu(1 + e)\tan\theta_1 \tag{1}$$

where μ is the coefficient of sliding friction, v_x is the velocity component in a direction parallel to the surface, subscripts 1 and 2 denote values before and after the bounce respectively, and $e = v_{y2}/v_{y1}$ is the coefficient of restitution. The time taken for the ball to reach the player after it bounces therefore depends on a number of factors, but it is independent of the initial ball spin if the ball slides throughout the bounce. The effect of the court on the change in horizontal ball speed depends primarily on μ but it also depends to a small extent on e. In the remainder of this paper μ will be regarded loosely as a measure of the speed of the court, but the speed at any given value of θ_1 will be quantified in terms of the ratio v_{x2}/v_{x1}. It is shown below that this ratio is a more generally relevant measure of court speed since (a) it is directly related to the horizontal speed of the ball and (b) when the angle of incidence is greater than about 20°, the standard definition of court pace does not provide a valid indication of the change in horizontal ball speed.

It can be seen from Eqn. (1) that at low angles of incidence the reduction in the horizontal speed of the

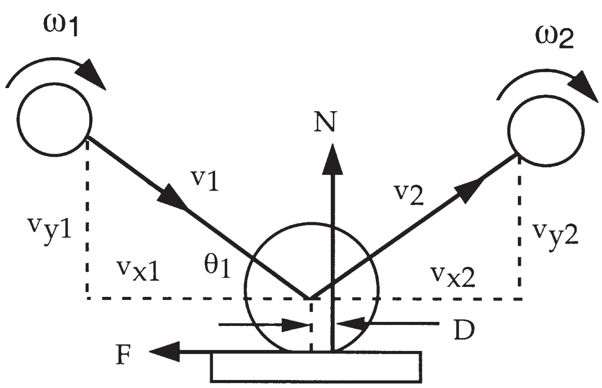

Figure 1 Geometry of the ball bounce.

ball is directly proportional to μ, hence courts with a low value of μ are fast and courts with a high value of μ are slow. It is on this basis that the ITF has adopted a definition of court speed or 'pace' that is based on the COF. The pace rating is defined by the relation (ITF, 2000)

$$\text{Pace} = 100\left[1 - \frac{(v_{x1} - v_{x2})}{(1 + e)v_{y1}}\right] \tag{2}$$

This definition reduces to Pace = $100(1 - \mu)$ if the ball slides throughout the bounce. A court with a Pace rating less than about 30 is classified as slow and a court with Pace greater than about 40 is classified as fast. The ITF (2002) defines slow as 0–35, medium as 30–45 and fast as > 40. In order to standardise and simplify Pace measurements, the ITF has adopted a procedure whereby the bounce of a tennis ball is measured for a ball projected from a ball launcher without spin at $v_1 = 30$ m s^{-1} and $\theta_1 = 16°$. Such a measurement yields a value for e and it also yields a value of μ if the ball slides throughout the bounce. The ball will slide throughout the bounce if μ is small enough but it is unlikely to do so if μ is larger than about 0.8.

The condition for the ball to slide throughout the bounce is given by $R\omega_2 < v_{x2}$, which yields the result (Cross, 2002*b*) that

$$\mu < \frac{1 - S}{(1 + 1/\alpha)(1 + e)\tan\theta_1} + \frac{D}{(1 + \alpha)R} \tag{3}$$

where $S = R\omega_1/v_{x1}$ is a dimensionless spin parameter and D is the distance between the line of action of the normal reaction force, N, and the centre of mass (CM) of the ball. It is assumed that $D > 0$ if N acts ahead of the CM as shown in Figure 1. If a rigid ball bounces on a rigid surface then $D = 0$ since there is only one point of contact between the ball and the surface, directly below the CM. However there are many points of contact when a flexible ball bounces and the distribution of N is not necessarily symmetric about the centre of the contact area. For example, any ball that rolls on a horizontal surface with $v_x = R\omega$ will eventually come to rest due to the fact that the coefficient of rolling friction is not exactly zero. N must then act ahead of the CM in order to generate a torque so that ω decreases when v_x decreases. This situation can arise if the ball and/or the surface on which it rolls is deformed asymmetrically (Hierrezuelo *et al.*, 1995).

Alternatively, the deformation may remain symmetric but the force distribution will be asymmetric if there are hysteresis losses in the ball (Tabor, 1994).

The bottom of a bouncing ball will come to rest on the surface during the bounce if at any time $v_x = R\omega$. Brody (1984) assumed that the ball would then start to roll and the friction force would drop instantaneously to zero. Since there is no further change in spin or horizontal speed if the friction force is zero, the final speed and spin of the ball are independent of the time at which the ball starts to roll and they are therefore independent of μ. In that case v_{x2} would be given by $v_{x2} = R\omega_2 = 0.645 v_{x1}$, regardless of the speed of the court.

The condition for a ball to enter a rolling mode is obtained by reversing the inequality in Eqn. (3). For example, if $e = 0.75$, $D = 0$ and $\omega_1 = 0$ then Eqn. (3) indicates that the ball will enter a rolling mode if $\mu > 0.203/\tan\theta_1$. For the standard $\theta_1 = 16°$ ITF pace test, rolling will commence if $\mu > 0.71$. If $e = 0.85$, then rolling will commence if $\mu > 0.67$. Under the standard test conditions and on courts with μ above 0.71, the Brody bounce model indicates that there should be no difference in the $v_{x2} = v_{x1}$ ratio and very little difference in Pace. If there are differences in Pace, then Eqn. (2) indicates that they can arise only from differences in the values of the COR. It is shown in the present paper that relatively large differences in Pace are observed on courts with $\mu > 0.7$, since tennis balls do not roll when they bounce, and they do not bounce with $D = 0$.

Effects due to finite *D*

In Brody's model there is no deformation of the ball or the surface on which it bounces. In reality, the situation is complicated by deformation of the ball and by possible deformation of the surface. As a result, the normal reaction force does not necessarily act through the centre of the ball. If we assume that D is finite as in Figure 1 then conservation of angular momentum about a point at the bottom of the ball is described by the relation

$$I\omega_1 + mRv_{x1} - mDv_{y1} = I\omega_2 + mRv_{x2} + mDv_{y2} \qquad (4)$$

Here and elsewhere in this paper the sign of v_{y1} is reversed so that v_{y1} is positive if the ball is incident in the negative y direction. Equation (4) allows D to be determined from measurements of the speed and spin of the ball before and after the bounce. Measurements described below indicate that D is typically about 4 mm for a low speed bounce but it can be as large as 11 mm when a ball impacts obliquely at high speed on a clay court.

A qualitative explanation for the finite positive value of D can be found by comparing the bounce of a ball with the behaviour of a vehicle when the brakes are applied. Friction on the wheels generates a torque about the centre of mass which results in rotation of the vehicle and a consequent shift of the weight forwards. The additional force on the front wheels provides a restoring torque and prevents the vehicle spinning like a ball. The weight of a ball is neglible compared with the normal reaction force but the front edge of the ball is forced down onto the surface as the ball rotates while the back edge rises upwards. While the ball is compressing, the front edge of the ball moves into the surface at a greater speed than the rear, the entry speed being a combination of the translational and rotational speeds. While the ball is rising up off the surface, the rear edge rises faster than the front edge. The amount of compression experienced by the ball depends on the local ball speed rather than its weight. Consequently, the normal reaction force will be larger at the front of the ball than at the rear, during the whole bounce period. Hysteresis losses in the rubber will also contribute to a shift in the normal reaction force. When an automobile tyre rolls, it is compressed at the front edge and expands at the rear edge. As a result of hysteresis, the force at the front edge is larger than the force at the rear, resulting in a shift in the line of action of the normal reaction force towards the front of the tyre (Tabor, 1994).

On a non-deformable surface such as concrete, a positive value of D has no direct effect on the horizontal speed of the ball (since N acts in the vertical direction) but it acts to decrease the total torque on the ball so the ball spin is reduced. As a result, the ball can slide throughout the bounce over an extended range of values of θ_1. For example, suppose that $e = 0.75$ and $\omega_1 = 0$. Then Eqn. (3) indicates that the ball will slide throughout the bounce if $\mu < 0.203/\tan\theta_1 + 0.645\,D/R$. For the standard ITF pace test and $D/R = 0.1$, the ball will slide throughout the bounce if $\mu < 0.77$.

A ball that bounces with finite D can bounce at a lower horizontal speed than one that bounces with $D = 0$, depending on the values of μ and θ_1. If μ and θ_1 are both relatively small then the ball will slide throughout the bounce regardless of the value of D, in which case v_{x2}/v_{x1} will depend on μ according to Eqn. (1), but it will not depend on D. Conversely, if μ and θ_1 are both relatively large then the ball will commence rolling or biting during the bounce, in which case v_{x2}/v_{x1} will depend on D but it will not depend on μ, unless D itself is a function of μ.

Effects due to biting

Real balls are not rigid and do not roll when they bounce. Instead, the bottom of the ball grips or bites the surface when $v_x = R\omega$. Instead of dropping instantaneously to zero, the friction force drops slowly to zero and then reverses direction when the ball bites, as shown by Cross (2002b). This effect is due to the fact that the bottom of the ball vibrates in a horizontal direction when it bites the surface, with a period that depends on the local tangential stiffness of the ball in the contact region. For a tennis ball, the horizontal vibration period is about 5 ms. The ball also vibrates in the vertical direction when it bounces but it bounces off the surface after one half period of oscillation (about 5 ms). A simple model describing these effects for a golf ball is given by Gobush (1994), and a detailed numerical solution is described by Maw *et al*. (1976; 1981).

Real balls bounce with $R\omega_2 > v_{x2}$ under conditions where the inequality in Eqn. (3) is reversed. For a tennis ball, $R\omega_2/v_{x2}$ is typically between 1.0 and 1.3 when the ball bites the surface. This is not radically different from the case where a ball rolls and hence experimental results for a tennis ball are qualitatively consistent with the simpler Brody model. The difference between a ball that rolls and a ball that bites is of greater significance if the ball stores a significant amount of elastic energy due to horizontal deformation and if that energy is recovered during the bounce. Such an effect is particularly evident in the case of a superball but it is of less significance for a tennis ball. Consequently one can treat biting and rolling as roughly equivalent for a tennis ball.

The bounce of a ball under any condition is completely determined from measurements of speed, angle and spin before and after the bounce. Alternatively, the bounce can be completely specified in terms of the measured values of the vertical and horizontal coefficients of restitution, together with a measurement of D, all three quantities being derived directly from measurements of speed, angle and spin. The vertical coefficient, e, is defined above and the horizontal coefficient, e_x, can be defined by the relation

$$e_x = -\frac{(v_{x2} - R\omega_2)}{(v_{x1} - R\omega_1)} \tag{5}$$

where $v_x - R\omega$ is the horizontal speed of a point at the bottom of the ball. This definition yields the result that $e_x = 1$ for a perfectly elastic ball with no energy losses. For such a ball, the speed of the ball at the point of contact with the surface is reversed by the bounce, in both the vertical and horizontal directions. By contrast, if a ball enters a rolling mode during the bounce then the point of contact comes to rest in the horizontal direction and then $e_x = 0$.

Unlike e, e_x can be positive or negative. The value of e_x characterises the bounce, as follows:

$e_x = -1$	frictionless surface	$v_{x2} = v_{x1}$, $\omega_2 = \omega_1$
$-1 < e_x < 0$	ball slides throughout bounce	$R\omega_2 < v_{x2}$
$e_x = 0$	ball rolls	$R\omega_2 = v_{x2}$
$0 < e_x < 1$	ball grips or 'bites' the surface	$R\omega_2 > v_{x2}$
$e_x = 1$	all elastic energy recovered	$R\omega_2 > v_{x2}$

If a ball grips the surface then $e_x = 0$, but if the elastic energy stored in the horizontal direction is not completely recovered then $e_x < 1$. The magnitude of e_x in this case provides a useful indication of how well the ball grips the surface and how much additional spin can be expected as a result of energy recovery. For example, a superball spins much faster than a golf ball of the same mass and diameter since e_x is about 0.6 for a superball but is only about 0.1 for a golf ball (Cross, 2002b).

Equations (4) and (5) can be combined to give

$$\frac{v_{x2}}{v_{x1}} = 1 - \frac{(1 + e_x)(1 - S)}{(1 + 1/\alpha)} - \frac{D(1 + e)\tan\theta_1}{(1 + \alpha)R} \tag{6}$$

and

$$R\omega_2 = R\omega_1 + \frac{(v_{x1} - v_{x2})}{\alpha} - \frac{D(1 + e)v_{y1}}{\alpha R} \tag{7}$$

Equations (6) and (7) remain valid regardless of whether the ball slides or rolls or bites but the essential physics is obscured by the fact that e_x, e and D are experimentally determined parameters. The expression for v_{x2}/v_{x1} given by Eqn. (6) is not of much value when a ball slides throughout the bounce, since e_x is then a complicated function of μ, D and S, as can be seen by comparing Eqns. (1) and (6). Equation (6) is better suited to situations where e_x is either zero or 1 or close to zero. For a tennis ball, e_x varies in a narrow range from zero to about 0.2 when the ball bites the surface.

The last term in Eqn. (6) resembles the last term in Eqn. (1) with μ replaced by $D/(1+\alpha)R$. The latter coefficient represents the coefficient of rolling friction, which is zero when $D = 0$ but which is finite when D is finite. When D is finite, a torque will act on the ball even if it rolls, causing both the linear and rotation speed to decrease until the ball bounces off the surface. The quantity $\mu_R = D/(1+\alpha)R$ can be formally identified as the coefficient of rolling friction by considering the motion of a ball that rolls with $v_x = R\omega$ and with finite D, as described by Hierrezuelo *et al.* (1995) and by Cross (2000). For a tennis ball, $\mu_R = 0.645D/R$.

Effects of finite D and e_x can be illustrated by a few numerical examples. Brody considered the case of a rolling ball with $\omega_1 = 0$, $e_x = 0$ and $D = 0$. In that case $v_{x2}/v_{x1} = 1/(1+\alpha) = 0.645$ regardless of the speed of the court. Now suppose that the ball is incident with zero spin and that it rolls with $e_x = 0$, $D/R = 0.1$ and $e = 0.75$. According to Eqn. (6), the ball will bounce at a horizontal speed given by

$$\frac{v_{x2}}{v_{x1}} = 0.645 - 0.113 \tan\theta_1 \tag{8}$$

A ball that rolls and that is incident at $\theta_1 = 20°$ will therefore bounce with $v_{x2}/v_{x1} = 0.604$. If it is incident at $\theta_1 = 30°$ then it will bounce with $v_{x2}/v_{x1} = 0.58$. If D/R increases to 0.3 and if $\theta_1 = 30°$ then $v_{x2}/v_{x1} = 0.45$. A further reduction in v_{x2}/v_{x1} arises if the ball bounces with $e_x > 0$. A typical value of e_x for a tennis ball that bites the surface is 0.1 in which case $v_{x2}/v_{x1} = 0.610 - 1.129(D/R)\tan\theta_1$ when $e = 0.75$. It can be seen that the effect of finite D is generally more significant than the effect of finite e_x for a tennis ball.

Previous measurements of court speed

There exists no sufficiently complete set of data for any tennis court that would allow the bounce of a ball to be determined under all or even most conditions of interest. Published data includes only a limited range of incident angles, spin and ball speeds for any given court. Nevertheless, sufficient data is available to make some useful and interesting comparisons with the theoretical estimates described above.

Extensive data sets obtained by filming the bounce of a ball on various surfaces are given by Thorpe & Canaway (1986), and by Pallis & Mehta (2000). In each of these studies, the ball was incident on a variety of tennis courts at speeds between 20 and 30 m s^{-1} and at angles of incidence from about 20° to 30°. A reliable measurement of the COF can be extracted for only one court, where the ball was incident at 17°. Nevertheless, the published data show clearly that

(a) The COR for an oblique bounce on most courts is typically between 0.8 and 0.9, despite the fact that the COR for a vertical bounce on a hard surface must be close to 0.75 for an approved ball. For an oblique bounce, a tennis ball can therefore bounce almost as high as a superball (at the same incident speed).

(b) Thorpe and Canaway measured the COR for a 100 inch (2540 mm) vertical drop on each court, obtaining a value of 0.65 for two different grass courts and a value of 0.77 on each of the other three courts tested. In all cases, the COR for an oblique bounce was found to be larger than the COR for a vertical bounce.

(c) The COR for an oblique bounce on grass courts varies over a wide range. Thorpe and Canaway obtained a value of about 0.75 for the COR on the grass centre court at Kooyong and a value of about 0.89 for one of the outside grass courts, while Pallis and Mehta found that the COR on their grass court was 0.6. This variability indicates that the condition of the grass and the underlying soil plays an important role in determining the COR on grass. The observed differences for oblique bounces on grass are almost certainly due to the fact that (i) the centre court at Kooyong was used only once a year (for the Australian Open), the grass was cut very short and the court was rolled frequently in order to produce a hard surface, (ii) the outside grass court was (and still is) used almost every day by club members, the grass was kept relatively long to minimise wear and the courts were rolled infrequently and (iii) the grass court tested by Pallis and Mehta was still moist after a shower of rain and the grass was not as immaculately groomed as the courts used at Kooyong or Wimbledon.

(d) For a ball incident without spin and at an angle of incidence of about 20°, v_{x2}/v_{x1} varies over a relatively narrow range from one court to the next, typically between 0.60 and 0.65, despite the fact that players tend to rate different courts as being of significantly different speed. The slowest court tested by Thorpe and Canaway was the outside grass court where the average value of v_{x2}/v_{x1} for 18 bounces (all at $\theta_1 = 20°$) was 0.51. Grass courts are normally regarded as fast, but the results obtained by Thorpe and Canaway and also by Pallis and Mehta show that grass courts can also be slow. Pallis and Mehta found that $v_{x2}/v_{x1} = 0.49$ on their grass court for balls incident with zero spin at $\theta_1 = 24°$.

(e) On those courts where the incident and rebound spin of the ball were measured, the data presented by Thorpe and Canaway indicate the D was typically between 4 and 8 mm. The courts concerned were Plexipave and En-tout-cas, the latter being a clay court constructed from crushed brick rather than actual clay.

(f) The grass centre court at Kooyong had a COF of 0.64. On this court, the ball was incident at 17° and at an incident speed of 30 m s^{-1}. If one regards this as a relatively fast court then the COF of most other courts is likely to be around 0.7 or higher.

Author's measurements of court speed

Data on court speed obtained by the author are shown in Figures 2 to 6. The data were obtained by filming the bounce of new Slazenger Hardcourt balls on several different surfaces, using a JVC 9600 digital video camera operated at 100 frames/second. Bounces were filmed (a) outdoors on a Rebound Ace and a clay court and (b) in a laboratory on a smooth concrete block and on three different surfaces bonded to smooth, heavy blocks of concrete. Each of the concrete blocks had dimensions 30 cm × 30 cm × 4 cm and they were obtained from a building supply shop. The Rebound Ace surface used in the laboratory was a factory sample that was significantly smoother than the outdoor court. The other two blocks were covered with emery paper, one with P800 grade (nominally 800 silicon carbide particles per inch) and the other with P150. The P150 surface represents a moderately coarse grain emery paper, but one can rub one's hand firmly on the surface without cutting the skin or drawing blood. Nevertheless, it acted to rip 10 to 20 strands of cloth material from the ball each bounce, whereas the finer P800 surface did not. Instead, the ball left a yellow mark on the P800 surface, representing a polishing of the cloth rather than a ripping-out action. The fibres in the tennis balls were 0.03 mm in diameter; the average particle size in P800 is 0.022 mm; and the average particle size in P150 is 0.097 mm.

Two vertical ball cans spaced 1.0 m apart were used to calibrate the horizontal and vertical scales on the film taken outdoors. The ball was served at about 30 m s^{-1} or thrown at about 15 m s^{-1} to land close to the two cans. Individual bounces where the ball travelled accurately parallel to the centre line on the court were selected for analysis. Balls travelling at an angle greater than 4° across court were identified by markers placed on the baseline and were excluded from analysis.

The fastest shutter speed on the camera was 1/500 s, which resulted in a streaked image of the ball about two diameters long at the fastest ball speeds. The streaked image did not introduce a significant error in speed or angle measurements but it prevented measurements being obtained of ball spin. At low ball speeds it is possible to zoom up closer to the ball and obtain reliable measurements of both the ball speed and spin. At high ball speeds, it is necessary to zoom out to obtain enough data points to measure the ball speed, but then the ball image is too small and too streaked to measure its spin. The laboratory results were obtained by throwing a marked ball at a speed of about 7 m s^{-1} and with negligible spin so that the incident ball speed and the rebound speed and spin could be measured. The latter results were obtained by filming against a 14 mm grid (a 1.2 m × 0.6 m airconditioning vent) to calibrate the vertical and horizontal scales on the film. The spin and horizontal speed of the ball were each measured to within 1%. The vertical speeds just before and after impact were measured to within 2% after correcting for the acceleration due to gravity.

Rebound Ace is the court surface now used for the Australian Open and consists of a 1 mm thick acrylic upper surface on a sheet of foam rubber bonded to concrete or asphalt. The acrylic is mixed with sand to control the texture and speed of the surface and the rubber is used to provide a cushioning effect under foot. The speed of the surface also depends on how the acrylic/sand mixture is applied. If it is spread with a broom, then the speed depends on whether the broom is swept across the court, or along the court, or diagonally since the broom leaves tiny ridges in the surface. The clay court was constructed from crushed brick but the grain size was significantly smaller than for the En-tout-cas court tested by Thorpe and Callaway, resulting in a court that plays more like European clay. The advantage of the coarser and more porous En-tout-cas surface is that it dries rapidly after a shower of rain. European clay courts are generally constructed from a fine powder of crushed brick or tiles and can turn into mud when they get wet.

A surprising result of the measurements is that the bounce of a ball is quite variable even on surfaces that appear to be perfectly uniform. The ball itself is a reasonably uniform sphere and there is no obvious asymmetry in wall thickness or composition, apart from the seam on the ball. The scatter in the v_{x2}/v_{x1} and COR data is such that a single measurement at any given ball speed or angle cannot be regarded as typical. From an experimental point of view, it means that at least three bounces are required to characterise each surface at each angle of incidence. Alternatively, at least 15 bounces over a range of angles are needed to establish a reliable trend. From a player's point of view the variability in bounce provides an additional level of difficulty to contend with.

Variations in incident ball speed and angle can be minimised by using a mechanical ball launcher. No attempt was made to do so and all results were obtained either by hitting the ball with a tennis racquet or throwing the ball by hand, since the v_{x2}/v_{x1} and v_{y2}/v_{y1} ratios are not particularly sensitive to the incident ball speed and since the main objective was to measure these ratios as a function of the incident angle. From a practical point of view, it is easier and quicker to film 100 bounces at various angles when hitting or throwing the ball: it is less intrusive, there is no risk of damage to the court surface, and it allows anyone with a digital video camera to measure the speed of their own court. The disadvantage is that about half the bounces must be discarded as being unsuitable for analysis since it is essential that the ball trajectory lie in a plane closely perpendicular to the camera axis in order to obtain reliable measurements of the speed ratios and angles. In principle, one could use a second camera to correct for out of plane trajectories, but it is relatively time-consuming to analyse each bounce even without this correction.

Horizontal speed results

Results obtained by filming elite players serving down the centre line on the Rebound Ace and clay courts are shown in Figure 2, together with high speed bounces on P150 emery paper bonded to a slab of concrete. The players served at relatively high speed from the baseline or at a lower speed from points closer to the net. The players were instructed to hit the ball without significant topspin. Serves from the baseline landing near the service line at about 30 m s^{-1} were incident on the court at angles between 12° and 14°. Results at higher angles of incidence were generally obtained at lower speed, but there was sufficient overlap of speeds and angles to show that the v_{x2}/v_{x1} ratio was not sensitive to ball speed in the range 15 to 30 m s^{-1}. No results were obtained at high speed and low angles on the P150 surface since the players were unable to hit the small target reliably from a long distance. Low speed, low angle bounces on this surface are described below. Figure 2 shows the measured ratio v_{x2}/v_{x1} as a function of the angle of incidence, θ_1, together with the value of μ (the COF) determined from Eqn. (1). Each pair of data points corresponds to a single bounce and each graph shows the result of analysing up to 20 bounces at various angles of incidence from about 12° to about 34°. A reliable measurement of μ is possible only if the ball slides throughout the bounce, as it does at low angles of incidence. At higher angles of incidence the ball does not slide throughout the

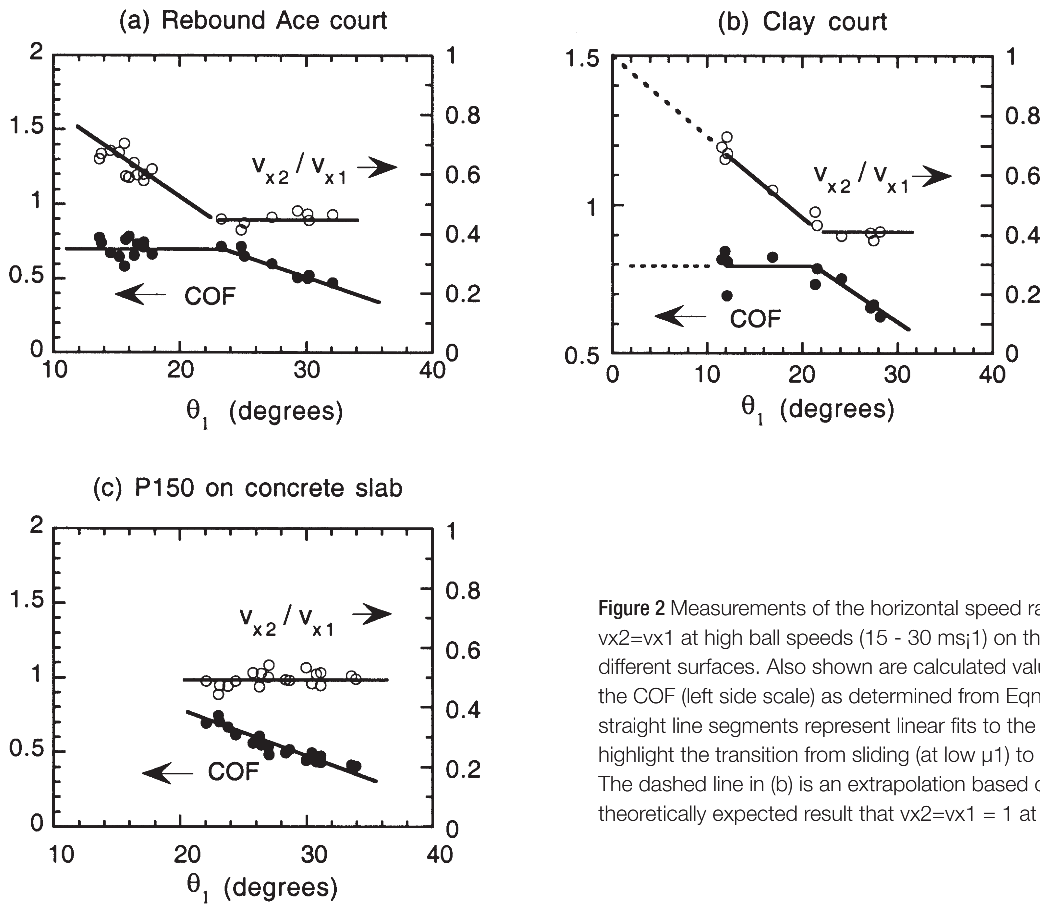

Figure 2 Measurements of the horizontal speed ratio vx2=vx1 at high ball speeds (15 - 30 ms¡1) on three different surfaces. Also shown are calculated values of the COF (left side scale) as determined from Eqn. (1). The straight line segments represent linear fits to the data to highlight the transition from sliding (at low µ1) to biting. The dashed line in (b) is an extrapolation based on the theoretically expected result that vx2=vx1 = 1 at µ1 = 0.

whole bounce period, in which case Eqn. (1) can be used to determine a time average or effective value of the COF. The values of the COF in Figure 2 represent the actual coefficient of sliding friction at low values of θ_1, but at high values of θ_1, the COF determined from Eqn. (1) underestimates μ. The effective value of μ is reduced when the ball grips the surface, since the friction force drops to zero during the bounce and then reverses direction. The solid lines in Figure 2 represent lines of best fit to the data, and are based on the assumption that the data can be fitted by two straight line segments. One could fit a single smooth curve through all the data points, but the two straight line segments were chosen because they provide a good fit and because of the expectation that there should be a significant change in slope when the ball stops sliding. The dashed lines in Figure (2b) are extrapolations based on the theoretical expectations that $v_{x2}/v_{x1} = 1$ at $\theta_1 = 0$ and that μ does not depend on θ_1 when the ball slides throughout the bounce.

The results in Figure 2 show that, for any given court, the observed v_{x2}/v_{x1} ratio depends on the angle of incidence in a manner that is qualitatively consistent with Brody's model. That is, the ratio decreases as θ_1 increases, up to a limit above which v_{x2}/v_{x1} is essentially independent of θ_1. At that limit, Brody assumed that the ball would enter a rolling mode but the ball spin data presented below show that the ball bites rather than rolls. Also at that limit there is a decrease in the effective coefficient of friction, since the ball ceases to slide throughout the bounce. Results obtained at low ball speeds on the four laboratory surfaces are shown in Figure 3. These results show the same general features as the high speed bounces in that v_{x2}/v_{x1} decreases as θ_1 increases, up to a threshold at which gross slip ceases and the ball begins to grip

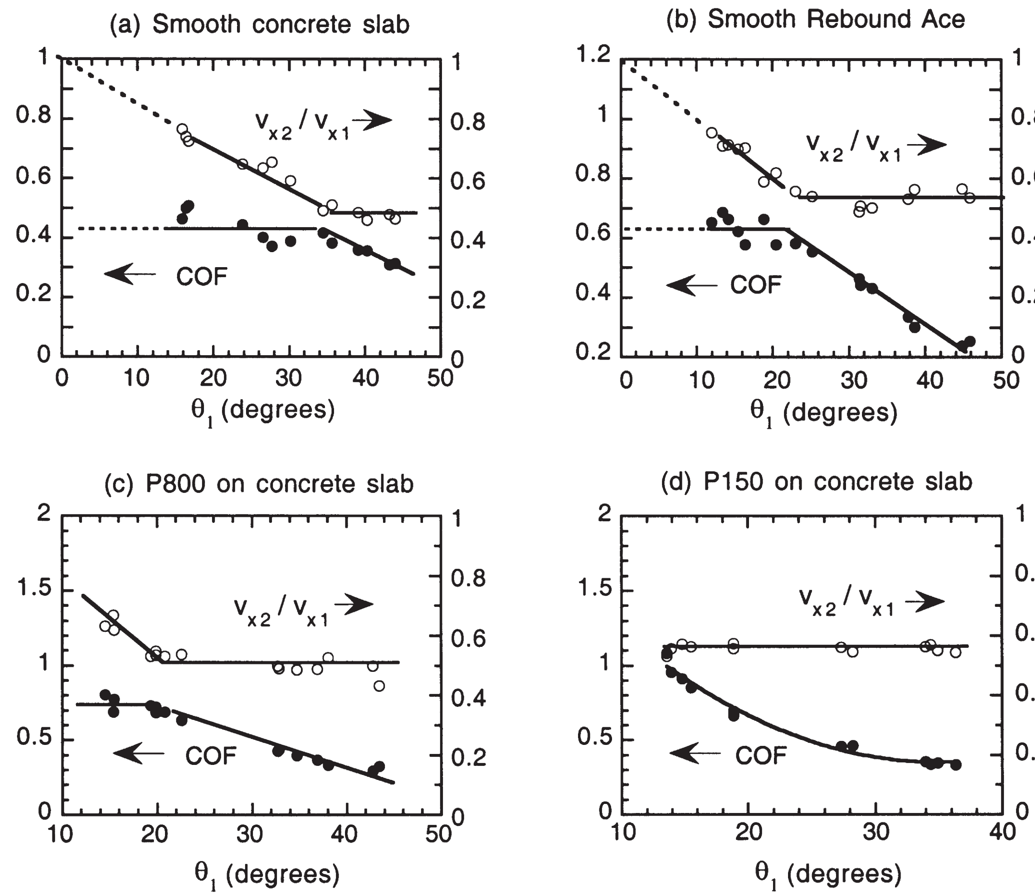

Figure 3 Measurements of the horizontal speed ratio $v_{x2} = v_{x1}$ at low ball speeds (6–10 m s^{-1}) on four different surfaces. The COF (left side scale) determined from Eqn. (1) represents the time average value of $F = N$ during each bounce. The straight line segments are as described in Figure 2.

the surface. A reliable measurement of μ was not possible on the P150 surface, but it is at least 1.0, as shown by the data in Figure 3d. For this surface, a better fit to the experimental data for the effective COF was obtained with a quadratic rather than a linear fit. The measurements shown in Figures 2 and 3 are summarised in Table 1. In Table 1, μ represents the average value of the COF for bounces in the low angle sliding mode, $S_1 = v_{x2}/v_{x1}$ at $\theta_1 = 16°$, S_2 is the average value of v_{x2}/v_{x1} in the high angle, biting mode and θ_T is the angle of incidence at which there is a transition from sliding to biting.

Table 1. Summary of results in Figures 2 and 3.

Surface	v_1 m s^{-1}	m	S_1	S_2	θ_T
Rebound Ace court	15–30	0.70	0.68	0.45	23°
Clay court	15–35	0.80	0.57	0.41	22°
P150	15–30	–	–	0.50	–
Smooth concrete	6–10	0.42	0.74	0.49	35°
Smooth Rebound Ace	7	0.62	0.70	0.54	23°
P800	6–9	0.73	0.65	0.51	20°
P150	7–10	> 1.0	0.56	0.56	< 13°

As expected, the fastest surface as indicated by the S_1 value in Table 1 is the smooth concrete slab, since it has the lowest COF, and the slowest surface is P150 since it has the highest COF. The other surfaces are ordered as expected. However, this is not the case for the S_2 values. The fastest surface at high angles of incidence is the P150 surface (at low ball speeds) and the slowest surface is the clay court. The court speed at high angles of incidence depends on the ball speed for the P150 surface. This may also be the case for the other surfaces, but the P150 surface was the only one tested at both high and low ball speeds. The four laboratory

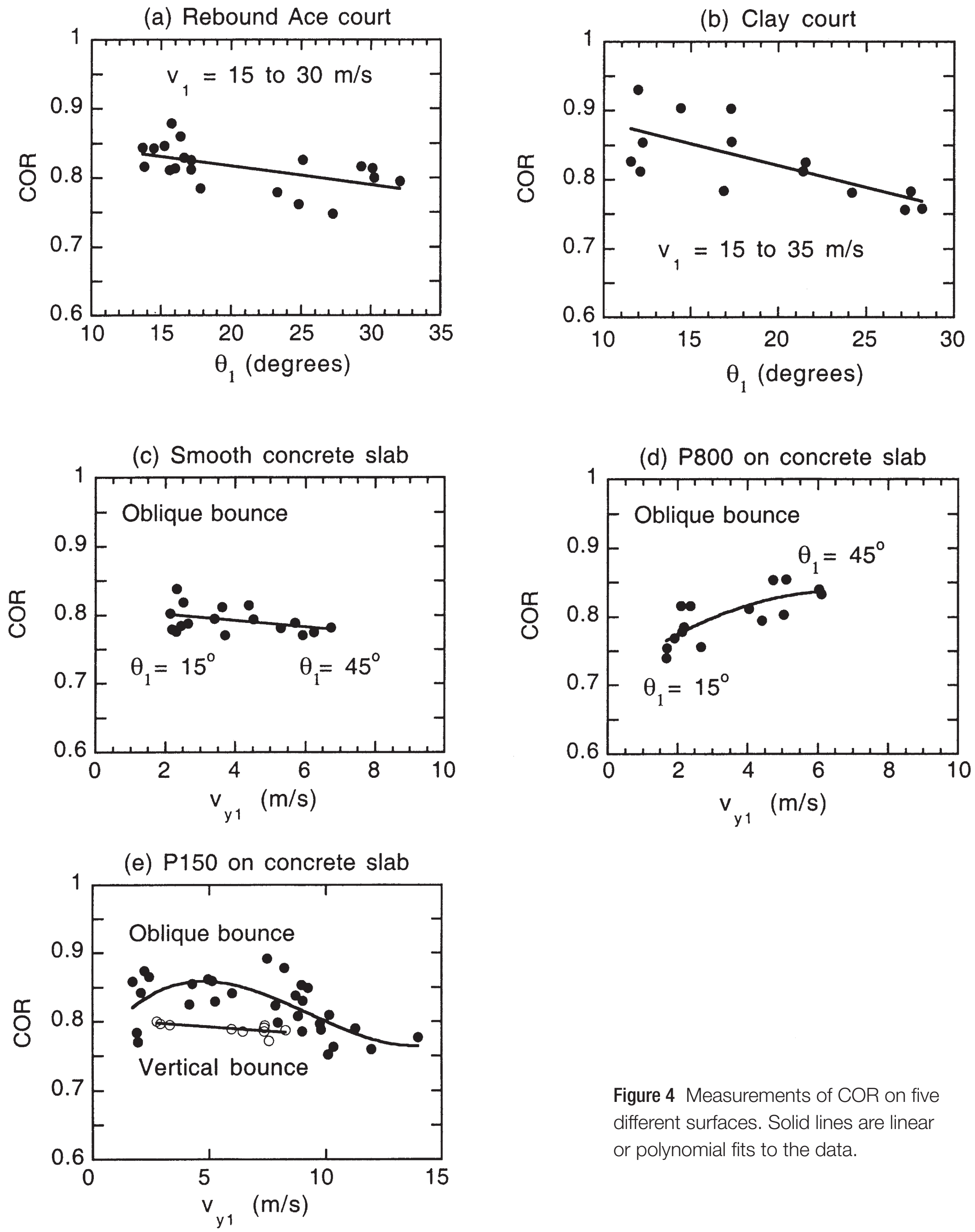

Figure 4 Measurements of COR on five different surfaces. Solid lines are linear or polynomial fits to the data.

surfaces were all tested at essentially the same range of ball speeds. Of these surfaces, the slowest at high angles of incidence was the smooth concrete slab. There is almost a complete reversal in the order of court speeds from low to high angles of incidence. A noteable exception is the clay court which remains slow at all angles of incidence.

Vertical speed results

Measurements of the COR, for the same bounces as those in Figures 2 and 3, are shown in Figure 4. The COR for the outdoor court surfaces is plotted in Figures (4a) and (4b) as a function of the incident angle. On the outdoor courts, the data at low angles was obtained at relatively high ball speeds (25 to 35 m s^{-1}) and the data at higher angles was obtained at a lower speed (about 15 m s^{-1}). The COR measured on three different laboratory surfaces is shown in Figures (4c), (4d) and (4e). The smoother Rebound Ace COR values are not shown since they are similar to those on the smooth concrete slab. Both the low and high speed data for P150 are combined into one graph (Figure 4e) to show the variation of COR on this surface with v_{y1}. Also shown in Figure 4e is the COR for a vertical bounce on the P150 surface. There is very little scatter in the data for a vertical bounce, but there is much more scatter for an oblique bounce, on all surfaces.

Measurements of ball spin, D and e_x

Measurements of ball spin for each of the laboratory surfaces are shown in Figure 5. The spin is plotted in terms of the dimensionless ratio $R\omega_2/v_{x2}$. This ratio remains less than 1.0 if the ball slides throughout the bounce and it would be equal to 1.0 if the ball commenced to roll during the bounce and continued to roll for the remainder of the bounce period. In fact, $R\omega_2/v_{x2}$ was observed to be greater than 1.0 at high angles of incidence, indicating that the ball bites the surface. When a ball bites the surface, the centre of mass of the ball continues to move forwards but the bottom of the ball slides backwards as it bounces off the surface, since $R\omega_2 > v_{x2}$.

The results in Figure 5a indicate that the ball bites the Rebound Ace surface when the angle of incidence exceeds 20° and it bites the smooth concrete slab when the angle of incidence exceeds 32°. Figure 5b shows that the ball bites the P800 surface when the angle of incidence exceeds 16°. The boundary between sliding and biting was not established for the P150 surface but it is at an angle of incidence less than 14°. These results are consistent with the transitions from sliding to biting shown in Figure 4.

For each of the bounces in Figure 5, values for D and e_x were obtained using Eqns. (4) and (5). The results are shown in Figure 6. The average value of D was 3.7 mm for the smooth concrete slab, 3.6 mm for the Rebound Ace surface, 4.2 mm for the P800 surface and 4.1 mm for the P150 surface, with typical variations in D of about 1 mm between bounces and over the range of angles investigated. There was no consistent variation of D with angle of incidence and the average value of D for each surface does not depend significantly on μ.

If the measured values of D, e and μ are substituted in Eqn. (3), then one finds that the ball should bite at angles of 29.7°, 19.7°, 16.7° and 11.8°, respectively, on the smooth concrete, Rebound Ace, P800 and P150 surfaces. It is assumed here that $\mu = 1.0$ for the P150

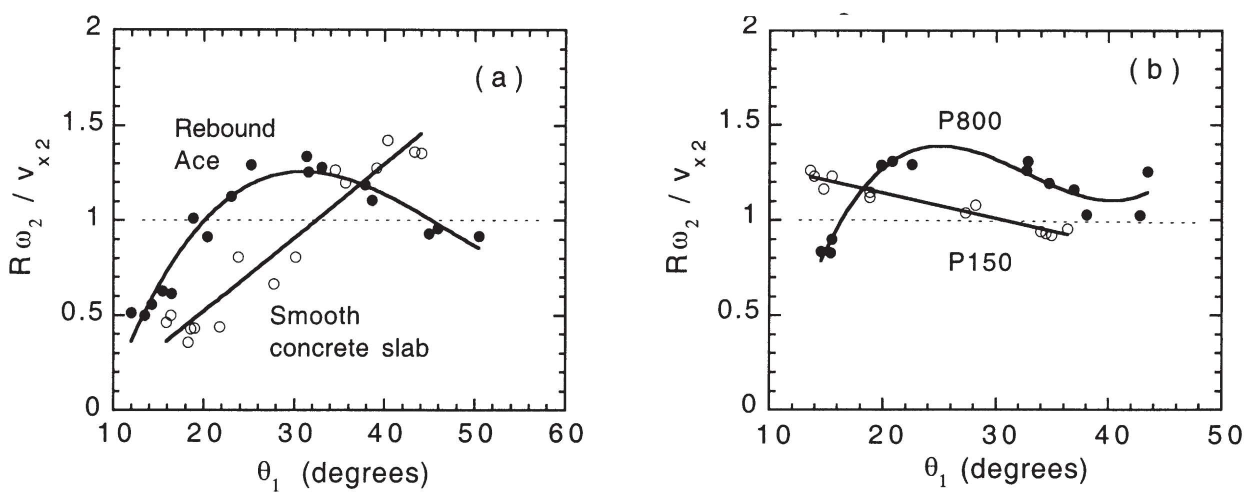

Figure 5. Measurements of ball spin on four different surfaces. A ball that rolls during a bounce would bounce with R!2=vx2 = 1. If the ball bites then R!2=vx2 > 1. Solid lines are linear or polynomial fits to the data.

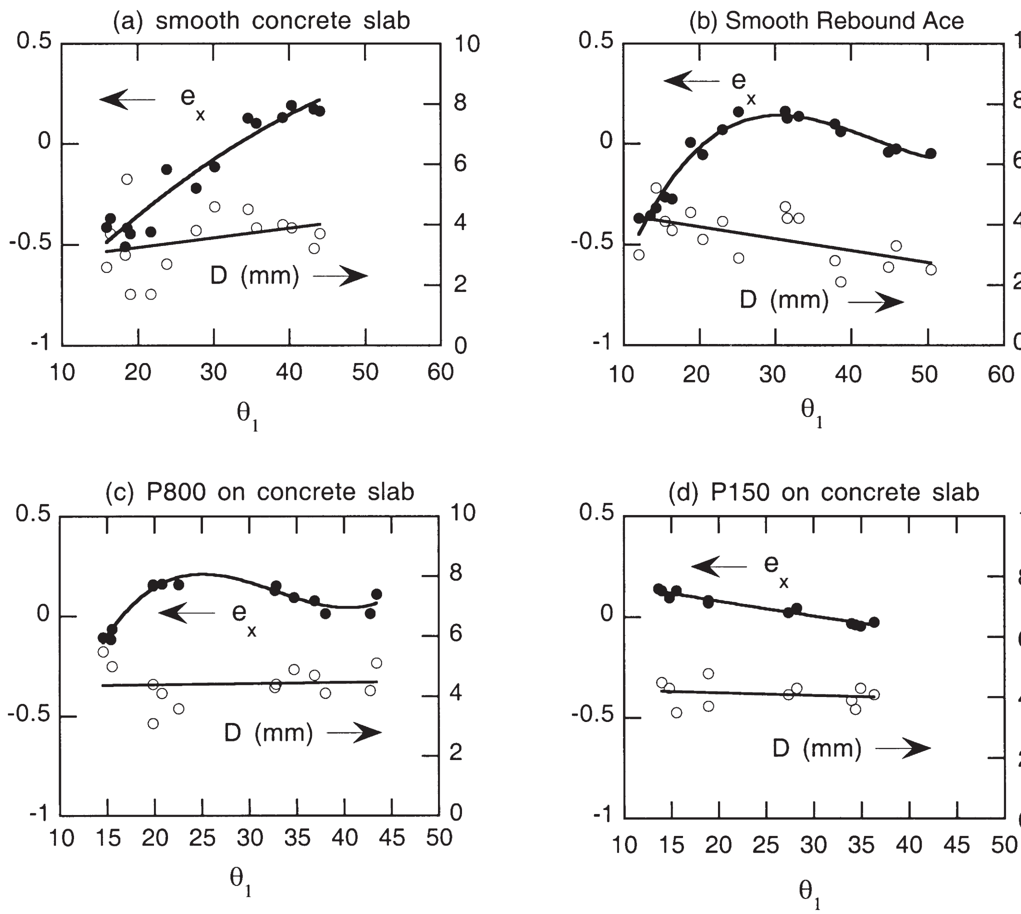

Figure 6. Values of D and e_x calculated from the spin and speed data in Figs. 3–5.

surface and the values for e were taken as 0.79, 0.80, 0.82 and 0.85, respectively. These estimates are consistent with the the transitions from sliding to biting shown in Figures 3 and 5.

The value of v_{x2}/v_{x1} at each transition point can be estimated from Eqn. (1). The corresponding values for the four laboratory surfaces are, respectively, 0.57, 0.60, 0.60 and 0.61. These values are all higher than the corresponding S_2 values listed in Table 1 (i.e. 0.49, 0.54, 0.51 and 0.56, respectively). However, the S_2 values in Table 1 were obtained by averaging the data at high angles of incidence where the ball bites and where e_x is typically about 0.1. An exact description of the S_2 values in Table 1 is given by Eqn. (6), using the measured values of D and e_x. This does not provide an improved theoretical estimate of v_{x2}/v_{x1}. Rather, the observed values of v_{x2}/v_{x1} agree exactly with Eqn. (6) since the experimentally determined values of D and e_x are based on the same equations as those used to derive Eqn. (6).

Based on the above measurements of e_x, one can estimate values of D for the high speed bounces on the P150 surface and on the Rebound Ace and clay courts shown in Figure 2. For these surfaces, $v_{x2}/v_{x1} = 0.50$, 0.45 and 0.41, respectively, at θ_1 in the range from 25° to 30°. If one assumes that on each of these three surfaces the ball was incident with negligible spin and bounced with $e = 0.8$ and $e_x = 0.15$ at $\theta_1 = 25°$ then from Eqn. (6) we find that $D = 5.6$ mm on the P150 surface, $D = 8.6$ mm on Rebound Ace and $D = 11.1$ mm on clay. These estimates do not depend strongly on the assumed value of e_x or on the assumed value of the initial spin factor S, provided that they remain small, since these quantities appear in Eqn. (6) as $1 + e_x$ and $1 - S$. Similarly, the estimates of D do not depend strongly on e, since e appears in the term $1 + e$ in Eqn. (6). Consequently one can be reasonably certain that the low values of v_{x2}/v_{x1} observed for high angle bounces on each surface are due primarily to relatively large values of D rather than unusually large values of e_x. Even for a superball, e_x does not exceed 0.5 (Cross, 2002b). The enhanced value of D observed on clay can probably be explained by the fact that loose particles on the surface are swept ahead of the ball to form a mound. This will act to increase the compression of the ball near the front edge and it will also provide an additional horizontal force component acting backwards on the ball (Hierrezuelo *et al.*, 1995).

Discussion

The results presented above were obtained over a wider range of ball speeds, incident angles and surface speeds than obtained previously and they highlight some significant discrepancies with the rigid ball bounce model described by Brody. In particular, it has been shown above that

(a) the COR for an oblique bounce is generally larger than that for a vertical bounce
(b) the normal reaction force does not act through the centre of the ball
(c) at high angles of incidence the ball grips the surface instead of rolling.

A surprising result found for all surfaces is that at high angles of incidence, the v_{x2}/v_{x1} ratio is independent of the angle of incidence. This is particularly evident for the P150 surface. Equation (6) indicates that v_{x2}/v_{x1} should vary with θ_1 due to the tan θ_1 term, but e, e_x and D all vary with θ_1 in such a way that v_{x2}/v_{x1} remains essentially independent of θ_1. This is predicted for a rolling or rigid ball but it is not obvious to the author why it is also the case when the ball bites the surface.The enhancement of the COR for an oblique bounce is also surprising, but there are several possible explanations. Ideally, one might expect that the COR for an oblique bounce would be the same as that for a vertical bounce, at least if the vertical component of the incident ball speed is the same. In fact, the COR for an oblique bounce can be as large as 0.9. Two suggestions have previously been made to explain the high COR, one involving deformation of the surface and one involving the effect of ball spin. If the ball forms a small depression in the court surface then it will be deflected upwards by the front edge of the depression. If the ball is spinning fast enough, ball deformation might be reduced (Cross, 2002a). Both of these suggestions are inconsistent with some of the data in Figure 4. The two emery surfaces bonded to concrete can be regarded as perfectly rigid, and the COR is enhanced even at low ball speeds where the ball spin is relatively small.

An approved ball has a COR between 0.73 and 0.76 when incident normally on a concrete slab at a vertical speed of 7.06 m s^{-1}. All the balls used in this experiment were taken fresh from a new can and had a COR between 0.77 and 0.79 under these conditions, which partially explains the high COR values shown in Figure 4. However, a large fraction of the bounces shown in Figure 4 have a COR of 0.8 or larger. There is a slight enhancement in the COR even on the smooth concrete slab, and a larger enhancement on the P800 and especially on the P150 surfaces. These results indicate that the COR is enhanced on surfaces with a large coefficient of friction, even when the surface is perfectly rigid.

A possible explanation is that the ball deforms in a horizontal direction during the bounce, as well as in the vertical direction. If one attaches a string to each side of a ball and pulls horizontally then the ball will stretch horizontally. If the ball is then placed on a surface and both strings are cut simultaneously, the ball will bounce vertically since the ball will exert a vertical force on the surface as it springs back to its original spherical shape. Consequently, any elastic energy stored as a result of horizontal deformation will act to enhance the vertical rebound speed of the ball. This effect is likely to be more pronounced if a ball is incident on a surface with a high coefficient of friction and if the horizontal speed of the ball is significantly larger than the vertical speed. This is consistent with the data in Figure 4. The largest enhancement in the COR is observed at low angles of incidence and on surfaces with a large coefficient of friction. An exception is the result on the P800 surface where there is a slight reduction in the COR at low angles of incidence. Most people involved in the game of tennis are agreed that clay courts are much slower than grass courts, at least when referring to courts prepared for major tournaments. An interesting question is how this perception of court speed arises, given that different court surfaces make very little difference, with a high speed serve, to the transit time of the ball from end of the court to the other. The results presented above provide a possible answer to this question. In terms of the Brody bounce model, the maximum speed reduction of a ball bouncing obliquely on a court surface, with zero incident spin, is given by $v_{x2}/v_{x1} = 0.645$. This is the value expected for a ball that enters a rolling mode, and it is the same for all courts regardless of the speed of the court or the angle of incidence. According to this model, differences in court speed can be expected only if the ball slides throughout the bounce, in which case v_{x2}/v_{x1} is given by Eqn. (1). It can be seen from Eqn. (1) that the fractional change in v_{x2}/v_{x1} increases with μ but it also increases with θ_1. Consequently, the biggest change in the horizontal speed of the ball, and hence the most noticeable difference in court speed, will arise at relatively large values of θ_1, provided that the ball slides throughout the bounce. Balls that are incident at high values of θ_1 are generally hit higher over the net and at lower speed, so the time differences on fast and slow courts will also be more noticeable in both an absolute and a relative sense.

Suppose that a ball is served at around 200 km h^{-1} (55.5 m s^{-1}) with little or no spin. The ball slows down through the air and will land with v_{x1} about 40 m s^{-1} and at an angle θ_1 of about 12°. On most courts such a ball will slide throughout the bounce, in which case $v_{x2}/v_{x1} = 1 - 0.213\mu(1 + e)$. If $e = 0.75$ then the ball will bounce with $v_{x2} = 31.1$ m s^{-1} on a court with $\mu = 0.6$ or at $v_{x2} = 28.1$ m s^{-1} on a court with $\mu = 0.8$. Since the distance from the service line to the baseline is 18 feet = 5.49 m, a ball landing on the slower court will take about 19 ms longer to cross the baseline than the ball on the faster court. This difference may seem rather small but it makes an important difference to the player since the ball travels 59 cm in 19 ms at a speed of 30 m s^{-1}. If the player does not make any allowance for the change in ball speed, he or she will hit the ball much too early or much too late. A separate question is whether the player could pick the difference in ball speed as being due to a difference in horizontal court speed or whether it is due to the ball being served at a different speed or at a different angle of incidence or a difference in COR. For example it would be possible for two courts to have different values of μ and for the ball to bounce with exactly the same speed if $\mu(1 + e)$ is the same on both courts.

A more significant difference between fast and slow courts would result if the fast court has a low value of e and the slow court has a large value of e. If $v_{x1} = 40$ m s^{-1} and $\theta_1 = 12°$, then $v_{x2} = 31.8$ m s^{-1} on a court with $\mu = 0.6$, $e = 0.6$, while $v_{x2} = 27.1$ m s^{-1} on a court with $\mu = 0.8$, $e = 0.9$. In this case, the ball on the slower court takes 29 ms longer to cross the baseline than on the faster court. The player must then make a bigger adjustment for the change in ball speed and an even larger adjustment for the change in bounce height, given that the ball would bounce about twice as high on the slower court.

As described above, differences in horizontal court speed should be more significant and more obvious at low ball speeds since the time between the bounce and the ball reaching the player is longer. However, this is not necessarily the case according to the Brody bounce model. A ball hit at low speed over the net is incident on the court at an angle θ_1 typically greater than 20°, in which case the ball is likely to roll when it bounces, especially if the ball is hit with topspin. For example, if $D = 0$, $\theta_1 = 20°$, $\omega_1 = 0$ and $e = 0.75$ then the ball will roll if $\mu > 0.56$, which includes almost all court surfaces. In this case, a ball that rolls would bounce with $v_{x2}/v_{x1} = 0.645$ on all court surfaces, regardless of the speed of the court.

A ball incident with zero spin at a low speed and at a high angle of incidence will bounce at a different speed on a different surface only if D or e_x is non-zero. For example, if $D = 0.01$ m (the other parameters listed immediately above being held fixed) then the ball will bite only on courts with $\mu > 0.75$. In that case, a ball incident on a court with $\mu < 0.75$ will slide throughout the bounce and will bounce with a greater horizontal speed than on a court with $\mu > 0.75$. Furthermore, the horizontal bounce speed is significantly decreased for a ball that rolls or bites with $D > 0$. For example, if $D = 0.01$ m, $\omega_1 = 0$, $\theta_1 = 20°$, $e = 0.75$, $e_x = 0.1$ and $\mu = 0.8$, then $v_{x2}/v_{x1} = 0.485$. The same ball incident on a court with $e = 0.75$ and $\mu = 0.6$ will slide throughout the bounce and will bounce with $v_{x2}/v_{x1} = 0.62$. A ball incident on the service line with

v_{x1} = 20 m s^{-1} would then take about 0.443 s to cross the baseline on the faster court or 0.566 s on the slower court, a difference of 123 ms. Such a difference would be much more noticeable than the 19 or 29 ms differences quoted above. The speed of a court as perceived by a player is therefore likely to be based on low speed, high angle bounces rather than high speed, low angle bounces, in which case the court speed depends on both D and μ rather than μ alone.

Conclusions

Thorpe & Canaway (1986) remarked in their study of court speed that the ability of players to pick differences in court speed was puzzling since the measured differences in court speed were quite small on the courts tested. For a fast serve, the time differences are also very small. The time taken for the ball to cross the baseline when served at high speed on a fast court is typically only 20 ms less than on a slow court. It has been shown in this paper that relatively large differences in court speed will be noticed by players if their perception is based on low speed, high angle bounces rather than on high speed, low angle bounces. At incident angles greater than about 20° the ball bites the surface and the speed reduction is significantly larger than previously expected. In Brody's rigid ball model, the v_{x2}/v_{x1} ratio is equal to 0.645 on all court surfaces when the ball is incident with zero spin at high angles of incidence. Real tennis balls are flexible, with the result that the v_{x2}/v_{x1} ratio can be as low as 0.4 on some courts. This is partly due to the fact that the horizontal coefficient of restitution is greater than zero when the ball bites the surface but the main effect is that the normal reaction force on the ball acts through a point shifted by a distance D towards the front of the ball. For a clay court, D is about 11 mm, but on other surfaces D is smaller. The torque ND acts to reduce the ball spin and to extend the range of incident angles over which the ball slides before it starts to bite. The result is that the v_{x2}/v_{x1} ratio drops below 0.645 by an amount that increases as D increases. An additional result found in this study is that courts that are slow at low angles of incidence are not necessarily slow at high angles of incidence.

The vertical speed of a court can vary widely between different courts, as observed by others. It was found that the COR is enhanced even on rigid surfaces, particularly on slow surfaces and at low angles of incidence. This suggests that the increase in the COR is due to horizontal deformation of the ball, in which case some of the kinetic energy of the ball due to its horizontal motion can be chanelled into the vertical direction. Other explanations are not excluded, but horizontal deformation of the ball is probably a significant factor.

The current standard measure of court speed is the surface pace rating defined by the ITF. This provides a measure of horizontal court speed at angles of incidence less than 20° but it is not unique since it is affected by the vertical speed of the court. Furthermore, it does not provide a valid indication of court speed at high angles of incidence. An improved measure of court speed is suggested by the results in this paper, whereby the vertical and horizontal speeds could be specified by measured values of the COR and the v_{x2}/v_{x1} ratio at both low and high angles of incidence. Suitable angles would be 16° and 30°.

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