Oblique impact of a tennis ball on the strings of a tennis racket

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Abstract

Measurements are presented of the friction force acting on a tennis ball incident obliquely on the strings of a tennis racket. This information, when combined with measurements of ball speed and spin, reveals details of the bounce process that have not previously been observed and also provides the first measurements of the coefficient of sliding friction between a tennis ball and the strings of a tennis racket. At angles of incidence less than about 40° to the string plane, the ball slides across the strings during the whole bounce period. More commonly, the ball is incident at larger angles in which case the ball slides across the string plane for a short distance before gripping the strings. While the bottom of the ball remains at rest on the strings, the remainder of the ball continues to rotate for a short period, after which the ball suddenly releases its grip and the bottom of the ball slides backwards on the string plane. The bounce angle depends mainly on the angle of incidence and the rotation speed of the incident ball. Differences in bounce angle and spin off head-clamped and hand-held rackets are also described.

Keywords: tennis strings, coefficient of friction, slide, grip, bounce

Introduction

The bounce of a tennis ball off a tennis court is usually a simple process in which the ball slides along the court for a short distance before bouncing. The ball slows down in a direction parallel to the surface by an amount that depends on the angle of incidence to the surface and on the coefficient of sliding friction between the ball and the surface (Brody, 1984). A complication arises if the ball is incident with sufficient topspin or if it acquires sufficient spin during the bounce, in which case the ball stops sliding and commences to grip the court (Cross, 2003). A ball will usually grip the court at some stage during the bounce if it is incident at an angle greater than about 20°. If the ball is incident without spin and if it grips the court then the horizontal speed of the ball after the bounce is typically about half the horizontal speed before the bounce, regardless of the angle of incidence.

The same level of understanding has not been reached regarding the bounce of a tennis ball off the strings of a tennis racket. The behaviour of a ball incident in a direction perpendicular to the string plane is well known and is well understood, but in match play the ball rarely strikes the strings at normal incidence. Elite tennis players apply topspin or backspin to almost every shot, in which case the ball is incident obliquely on the strings. The rebound angle and spin of the ball will then depend on the details of the interaction between the ball and the strings.

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Measurements have previously been made by several authors of the rebound spin and angle for an angle of incidence near 50° (Bower and Sinclair, 1999; Knudson, 1991; Goodwill and Haake, 2003) but the bounce process itself has received very little experimental or theoretical attention. To study the bounce process, at least several different angles of incidence are needed to determine if and when the ball slides, rolls or grips the strings when it bounces.

The bounce of a ball off the strings of a racket differs from the bounce off the court in many respects:

- The ball is usually incident at a large angle to the string plane, typically within about 40° of the normal.
- The strings have a low coefficient of sliding friction, so the string plane is 'faster' than most court surfaces.
- The string plane is more deformable than a court surface, having a stiffness about the same as that of the ball.
- The strings can move relative to each other within the string plane, in contrast to a typical hardcourt surface which has no tangential or perpendicular compliance.
- The mass of a hand-held racket is much smaller than the mass of a court, with the result that the bounce speed off the strings, if the racket is initially at rest and the ball impacts in the middle of the strings, is typically about half the bounce speed off a court. The bounce speed is considerably smaller for an impact near the tip of a racket.
- The racket rotates during the bounce. The bounce angle, therefore, depends on the angle through which the racket rotates.
- The racket usually approaches the ball at high speed and hence the bounce speed, spin and angle off the strings depends strongly on the initial speed and trajectory of the racket.

For all of these reasons, the bounce of a ball off the strings of a racket is a much more complicated process than the bounce off a court. An additional complication is that very little is known about the friction force acting between the ball and the strings, or whether the ball slides, rolls or grips the strings. The main purpose of the present paper is to present measurements of the friction force acting between the ball and the strings and to show how the friction force affects the bounce speed, spin and angle off the strings. The friction force was measured by allowing a racket head to move horizontally on rollers to measure its acceleration Since this is not the normal way a racket is used, additional results are presented to compare the bounce parameters with those of head-clamped and hand-held rackets.

Qualitative features of the bounce process

A simple model of the bounce process, based on a rigid ball approximation, is described by Brody (1984). Brody noted that the friction force acting backwards on the ball would slow it down in a direction parallel to the surface and the torque due to the friction force would act to increase its rotational speed. If v_x is the horizontal ball speed, ω is its angular velocity and R is the ball radius, then the rotational speed at any point on the ball circumference is $R\omega$ and the speed of any point in contact with the surface is given by $v_x - R\omega$. The ball will slide throughout the bounce period provided v_x remains larger than $R\omega$. However, if v_x decreases and ω increases during the bounce to a point where $v_x = R\omega$, then the bottom of the ball will come to rest on the surface. This situation arises when a ball rolls along a horizontal surface, in which case the friction force is essentially zero since there is no relative motion between the ball and the surface in the contact region. Brody assumed that the friction force on a bouncing ball would drop to zero during the bounce period if the ball entered a rolling mode, and that there would be no further change in v_x or ω during the remainder of the bounce period. As a consequence, Brody predicted that a ball incident without spin would slow down by at most 40% in a direction parallel to the surface and that the maximum spin of the ball would be that corresponding to the rolling condition, i.e. $\omega_2 = vx_2/R$ where subscript 2 denotes values at the end of the bounce period.

Brody's bounce model remained unchallenged by experimental data for 18 years, despite the fact that the model was qualitatively inconsistent with the known behaviour of highly elastic balls such as superballs, and it was inconsistent with the exceptionally low bounce speed of clay courts reported by players. Recent experiments Cross (2002a, 20002b, 2003) have shown that

(a) the Brody model remains valid for real balls if the ball slides throughout the bounce period, (b) a tennis ball incident without spin can slow down by as much as 60% in a direction parallel to a court surface and (c) real balls do *not* roll when they bounce since real balls (including steel balls) are flexible. Rather, all balls grip the surface when $v_x = R\omega$, as predicted and observed previously by Maw *et al.* (1976, 1981). A tennis ball without spin incident on a court surface will slide throughout the whole bounce period if it is incident at an angle less than about 18° to the surface, the relevant angle being larger than 18° if the ball slides on a slippery (fast) court and smaller than 18° if the ball slides on a rough (slow) court. At higher angles of incidence the ball slides for a short period, then it grips the surface for a short period, then it releases its grip, causing the contact area to slide backwards on the surface.

Ball grip on a stationary ball can be demonstrated if a heavy weight is placed on top of a tennis ball and if one attempts to pull the ball in a horizontal direction. For small values of the pulling force, the ball will stretch but static friction will prevent the ball from sliding if the coefficient of static friction is large enough and if the vertical force is large enough. Under these conditions, the bottom of the ball will remain at rest since the pull force is balanced by the static friction force acting backwards at the bottom of the ball.

In the case of a bouncing ball, the ball will grip if the vertical force arising from ball compression is large enough when the contact area of the ball comes to rest. The ball may grip for only 1–2 ms before releasing its grip, but that will be enough to reduce v_x to a value lower than that predicted by Brody, and to increase ω to a value higher than that predicted by Brody. During the time that the ball grips the surface, the static friction force acts backwards on the ball hence v_x continues to decrease and ω continues to increase. The static friction force acts backwards since the initial momentum of the ball carries the upper part of the ball forwards, so the upper part of the ball pulls forwards on the lower part. The ratio $R\omega/v_x$ is 1.0 when the ball first grips but it increases to a value greater than 1.0 while the ball retains its grip on the surface. At first sight this may seem to be inconsistent with the bottom of the ball remaining at rest while the ball grips. However, the remainder of the ball continues to rotate and to slow down in a direction parallel to the surface, due to the inertia and flexibility of the ball. As a result, the ball deforms elastically for a few milliseconds while the bottom of the ball remains at rest. When the ball commences its vertical rise off the surface, the normal reaction force drops and the ball releases its grip. After grip release the bottom of the ball slides backwards on the surface since $v_x - R\omega$ is negative during the latter part of the bounce. As a result, the friction force reverses direction during the bounce and the ball bounces off the surface with $R\omega_2 > v_{x2}$. Further details regarding this process can be found in Cross (2002b), in Brody *et al.* (2002) and in the original articles by Maw *et al.* (1976, 1981).

The same basic process can be expected for a bounce off the strings of a racket, apart from the added complications outlined in the Introduction. In particular, a ball that grips the strings will tend to drag the long main strings across the short cross strings and will drag the whole racket head in the direction of the incident ball. A parameter of interest in this respect is the tangential speed of the contact area of the ball relative to the string plane. The tangential speed can be positive, zero or negative. A positive or negative speed indicates that the contact area slides forwards or backwards respectively on the string plane. If the relative speed is zero then the ball grips the strings. A simple indication of whether a ball grips during the bounce is provided by the parameter $S_2 = R\omega_2/(v_{x2} - V_{x2})$ where subscript 2 denotes values after the ball bounces, $R\omega_2$ is the peripheral speed of the ball due to its rotation and V_{x2} is the speed of the racket head in a direction parallel to the string plane. In order to measure S_2, one needs to measure the speed and spin of the ball after it bounces, as well as the speed of the racket head. If the ball slides across the string plane during the whole bounce period then S_2 will be less than 1.0. If the ball rolls across the string plane during the latter stages of the bounce then the ball will bounce with $S_2 = 1.0$. If the ball grips the strings and then slides backwards until the ball bounces off the string plane then S_2 will be greater than 1.0 . An additional and even more informative indication of whether a ball grips the strings is obtained by measuring the friction force between the ball and the strings, as described below. One can calculate, from the measured friction force, the time at which the ball grips the strings, the time at which the ball releases its grip, and the time history of v_x and ω throughout the bounce.

Experimental arrangement

The arrangement used to measure the friction force between a tennis ball and the strings of a racket is shown in Figure 1. An old aluminium frame racket was cut in half to detach the handle, and the head was mounted in a horizontal plane on two cylindrical rollers. This particular head was used since the frame was of uniform thickness, unlike most modern rackets, and it could therefore move freely in a purely horizontal direction on the rollers. The rollers were made from solid aluminium rod covered with heat-shrink plastic tubing to reduce high frequency vibrations generated during an impact by metal-to-metal contact. The plastic tubing also helped to ensure that there was no slip between the head and the rollers.

A ball impacting obliquely on the string plane exerted a horizontal friction force on the strings,

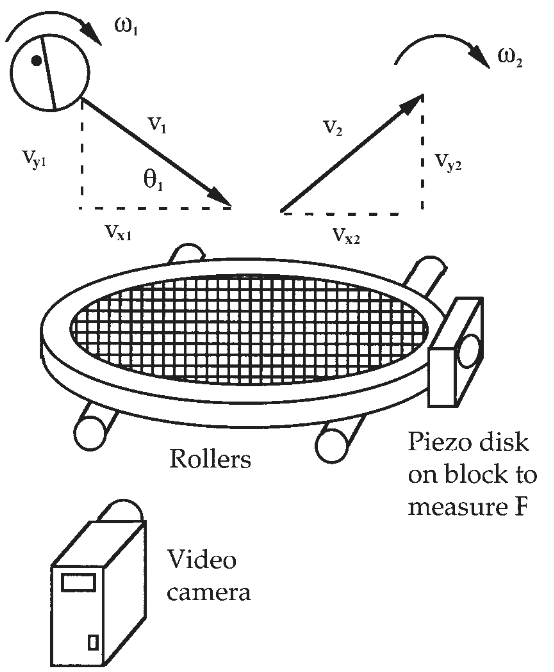


Figure 1 Experimental arrangement used to measure (a) the friction force, *F*, between a tennis ball and the strings of a racket (b) the velocity and spin of the ball before and after bouncing and (c) the horizontal speed of the racket head.

resulting in horizontal acceleration of the racket frame with almost no frictional resistance between the frame and the rollers. The acceleration of the frame was measured by means of a ceramic, piezoelectric disk attached to a flat block of insulating material glued to one end of the frame. The output voltage from the piezo disk is directly proportional to the acceleration of the frame and therefore provided a direct measure of the time-dependent friction force acting on the strings. The linearity of the piezo accelerometer was established by rolling a tennis ball at various speeds towards the racket frame to impact the frame head-on (at the end diametrically opposite the piezo disk).

The original nylon strings in the racket were left intact and the string tension was measured at 210 N using an ERT700 instrument (described by Brody *et al.*, 2002). The mass of the frame, strings and piezo assembly was 257 g, and the mass of each roller was 212 g. Since the frame moves at twice the linear speed of each roller, the effective mass of the frame and the two rollers was $M = 257 + 212 = 469$ g. If the frame translates at speed V, the linear momentum of the frame plus the two rollers is MV. The mass of the ball was 57 g. The horizontal momentum of the frame and rollers after each impact was measured to be equal and opposite the change in horizontal momentum of the ball, indicating that there was no slip between the frame and the rollers. For a ball incident without spin, the horizontal speed of the frame after each impact was typically about ten times smaller than the horizontal speed of the ball after the impact. The ball bounced at a lower horizontal speed when it was incident with backspin, transferring greater momentum to the frame, in which case the speed of the ball was sometimes comparable to the speed of the frame. The ball bounced at a higher horizontal speed when it was incident with topspin, transferring very little momentum to the frame. With a sufficient amount of topspin, the ball bounced with a larger horizontal speed than it had before the bounce, causing the frame to accelerate backwards.

The incident and rebound speeds, spins and angles were measured by filming each bounce with a JVC 9600 digital video camera. The speed of the racket frame after each impact was also monitored on the same recording. The maximum frame rate of this camera was 100 frames per second, and the minimum exposure time was 2 ms, which meant that only low speed impacts could be studied to determine the ball spin. The incident ball speed was typically 3 to 5 ms^{-1}, lower than the speeds normally encountered in the game of tennis. Quantitative differences between low speed and high speed impacts are described in the discussion section below, but there is no reason to expect that low speed bounces should be qualitatively different from high speed bounces. Each ball was thrown by hand onto the string plane to land near the middle of the string plane. Moderate topspin or backspin could be imparted by hand, and additional data was obtained by rolling a ball down an inclined plane to impact the strings with larger topspin. To assist with measurements of ball spin, a line was drawn around a circumference of the ball with a felt pen, and a single dot was marked near the line to help determine the orientation of the ball. Each ball was projected with the dot facing the camera. At least three images of the ball prior to and after each bounce were used to determine the ball speed and spin. A correction for gravitational acceleration was made to determine the vertical speed of the ball just before and after each bounce. Occasionally, a ball was projected incorrectly with its spin axis not accurately aligned in a direction parallel to the camera axis. Such impacts were not analysed. Since it was easy to throw and film 100 balls in about 50 minutes, it was also easy to select for analysis only those bounces that impacted in the middle of the string plane, with the spin axis correctly aligned and with the appropriate incident spin.

Output signals from the piezo were calibrated as a measure of the friction force, F, by equating the time integral of the piezo signal to the change in horizontal momentum of the ball. The time integral was terminated at 7.0 ms, which was the duration of the impact on the strings. The impact duration is typically about 5 ms for high speed impacts on a hand-held racket, but the impact duration was extended at low ball speeds and on a head-clamped racket due to the lower ball stiffness and the larger effective mass of the racket. The normal reaction force, N, was not measured, but the impact duration was measured by impacting a ball on a small piezo attached to the string plane. The impact duration varied from about 6.9 ms to about 7.2 ms for all incident ball speeds, since the normal incident ball speed remained relatively small and large changes in ball speed result in only relatively small changes in impact duration (Brody *et al*, 2002). A theoretical estimate of N is shown with the results below to allow comparison of magnitudes of F and N. At low ball speeds, N versus time is essentially a half-sine waveform (Cross, 1999; 2002a) the time integral of which can be equated to the measured change in vertical momentum of the ball.

Two additional experiments were performed to determine how the results obtained with the sawn-off head on rollers relate to the bounce off head-clamped and hand-held rackets. In these experiments, the friction force was not measured but the incident and rebound ball speeds, spins and angles were measured for low speed impacts on a Pro Kennex No. 24 graphite racket when (a) the head was rigidly clamped and (b) the handle was hand-held with the head free to recoil. For both additional experiments the racket was strung with a nylon string at a tension of 250 N. Even though the string tension in the aluminium head was lower than the string tension in the Pro Kennex racket, string plane stiffness was about the same given the larger head size of the Pro Kennex racket (775 cm^2 versus 515 cm^2). Small differences in string plane stiffness have very little effect on the rebound spin or speed of the ball (Bower & Sinclair, 1999; Brody *et al*., 2002). The string plane was horizontal, the ball was thrown by hand onto the string plane to impact near the middle of the strings and the bounce was filmed with the same digital video camera described above. A new Slazenger hardcourt ball was used for all three experiments. The Pro Kennex racket was 696 mm long, had a mass of 270 g and its centre of mass was located 378 mm from the butt end of the handle. The moment of inertia about an axis perpendicular to the handle, parallel to the string plane and passing through the centre of mass was I_{cm} = 0.0123 kg m^2. The racket rotates about this axis when a ball is incident perpendicular to the string plane and impacts the middle of the strings.

The head-clamped case was studied since all previous measurements of the oblique bounce of a ball off racket strings have been made either with a head-clamped or with a handle-clamped racket. However, no attempt has previously been made, either theoretically or experimentally, to show how such results relate to a hand-held racket. The bounce angle off a

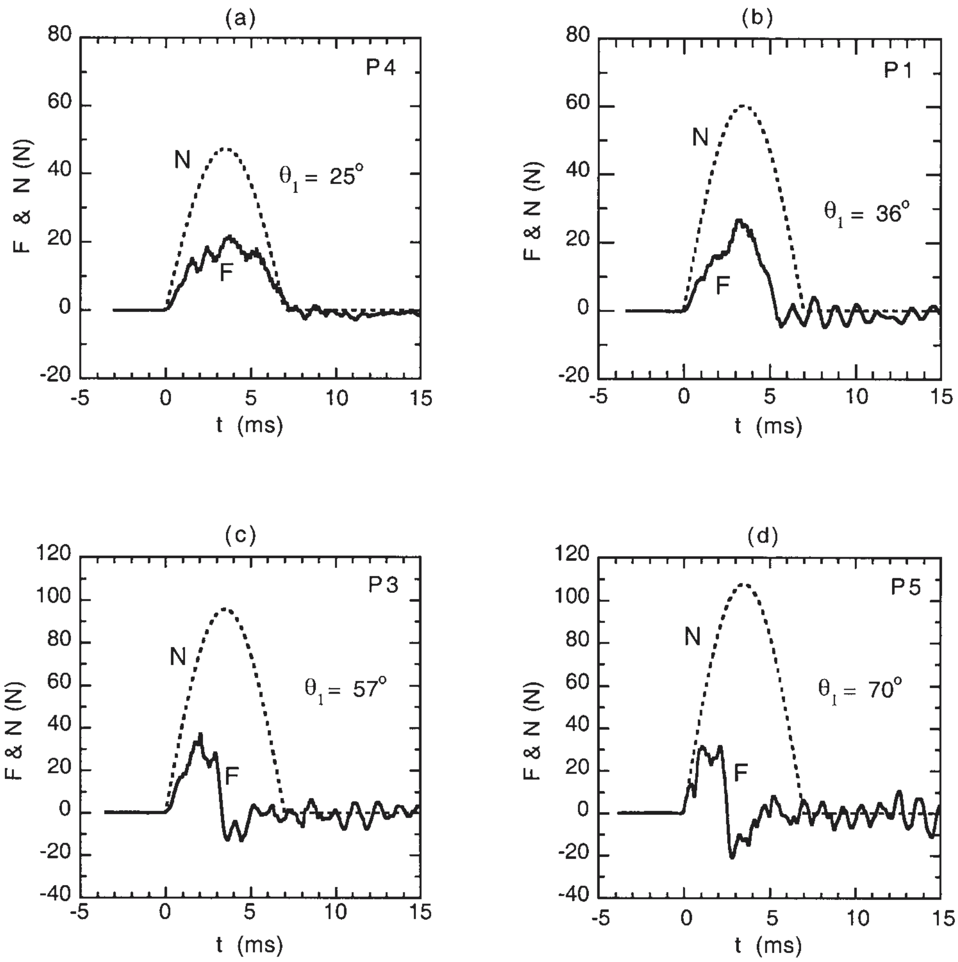


Figure 2 *F* and *N* versus time for a tennis ball incident with negligible spin at various angles on the strings of a tennis racket. The *N* waveform was assumed to be a half-sine wave of duration 7.0 ms. In (a), the rapid fluctuations in *F* occuring between $t = 1$ and $t = 5$ ms are due to movement of the strings within the string plane.

hand-held racket is typically about half of that from a head-clamped racket since the bounce speed in a direction perpendicular to the string plane is reduced by a factor of about two. The bounce angle also depends on the speed of the ball in a direction parallel to the string plane. The ball slides faster on a hand-held racket since the normal reaction force and the friction force are both reduced when the racket head is free to recoil.

Friction force results

Measurements of friction force

Measurements of the friction force for a ball incident without spin at several different angles of incidence, are shown in Figure 2. The force waveforms are contaminated by a 830 Hz component arising from high frequency vibrations of the racket frame. This component is due to the hoop mode, which corresponds to a vibration within the plane of the frame and which is excited by a tangential force rather than a force acting perpendicular to the string plane. The hoop mode is one where the shape of the frame alternately becomes more circular or more elliptical every half-cycle. The accelerometer tends to be more sensitive to high frequency vibrations than to low frequency vibrations, since for any given vibration amplitude the acceleration is proportion to the frequency squared. Nevertheless, the friction force component of the piezo signal is sufficiently different to be readily distinguished from the vibrational component. In general it was found that high frequency frame vibrations in the racket tended to be suppressed while the ball remained on the strings and were more obvious after the ball left the strings. One could attempt to filter out the 830 Hz component but it would compromise the representation of genuine, rapid changes in the actual friction force observed during each bounce.

Some of the small, rapid changes in F shown in Figure 2 represent sudden horizontal movements of the strings within the string plane. This effect was confirmed by bonding each of the cross strings to the main strings with a drop of superglue at each intersection point. This was done after all other measurements were completed, with the result that small, rapid changes in the F waveforms were eliminated. Given that players do not normally bond their strings at each intersection point, no further measurements with bonded strings were made.

In Figure 2a, the ball was incident at 25° to the string plane and F remained roughly proportional to N throughout the bounce. In can be inferred that the friction force was due to the ball sliding across the string plane and that sliding persisted throughout the 7 ms bounce period. The ratio of F to N yielded a time-average value of 0.43 ± 0.02 for the coefficient of sliding friction, μ.

Figure 2b shows the F and N waveforms for a ball incident at 36° to the string plane. F remained approximately proportional to N throughout most of the bounce period but it dropped to zero 2 ms before the end of the impact. At higher angles of incidence (Figures 2c and 2d) F dropped to zero at progressively earlier times during the bounce, and then reversed direction.

These results are qualitatively the same as those obtained for balls bouncing on a rigid surface (Cross, 2002b) and are consistent with measurements of ball spin and theoretical expectations. The measured values of S_2 for each of the bounces in Figure 2 were respectively (a) 1.15 (b) 1.27 (c) 1.03 and (d) 1.15, suggesting that the ball gripped the surface at all four angles of incidence. However, the F waveform in Figure 2a indicates sliding throughout the bounce. Measurements on a rigid surface indicate that, as the angle of incidence increases, the ball first starts to grip the surface not when $S_2 > 1.0$ but when S_2 is greater than about 1.15. A plausible explanation is that the normal reaction force towards the end of the bounce is too small to allow the ball to grip if $S_2 < 1.15$. At higher angles of incidence, the ball starts to grip the surface at an earlier stage during the bounce, at a time when the normal reaction force is relatively large.

Figure 3 shows the friction force acting on a ball incident with backspin and topspin. If a ball is incident with backspin then it slides across the string plane for a longer time before it grips the surface, since it takes additional time for the initial spin to reverse direction. The horizontal impulse on the ball is therefore larger, resulting in a larger speed reduction in a direction parallel to the string plane. Conversely, a ball incident with topspin commences to grip the strings earlier, resulting in a smaller reduction in the parallel component of the ball speed. Figure 3b illustrates an extreme example where the ball was incident with sufficient topspin to slide backwards at the beginning of the bounce, with the result that the parallel speed after the bounce was greater than the parallel speed before the bounce since the friction force on the ball acted to accelerate, rather than decelerate, the ball.

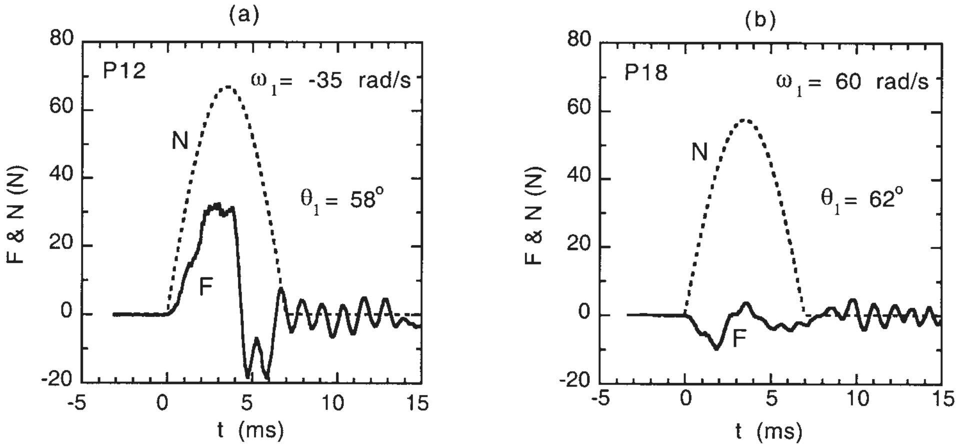


Figure 3 F and N versus time for a tennis ball incident on the strings with (a) backspin and (b) topspin.

Under normal playing conditions, the ball bounces off the court with topspin but it is usually incident on the strings with backspin relative to the string plane. At least, that is the situation when a player attempts to return the ball with topspin. If the player attempts to return the ball with backspin then a ball rising off the court may be incident on the strings with topspin relative to the string plane. The relative sense of the spin onto the strings depends on whether the ball is rising or falling as it approaches the strings, in a frame of reference where the string plane is vertical and at rest. A spinning ball that approaches the strings at right angles is incident neither with topspin nor backspin.

Analysis of friction force

The effect of backspin on the incident ball is of particular interest since it corresponds to the usual situation where a player hits an agressive topspin groundstroke. In that case, the ball will bounce with a relatively small velocity component in a direction parallel to the string plane. This should help the player to control the shot, given that the ball bounces off the strings in a direction almost perpendicular to the string plane (in a reference frame where the racket is initially at rest). We can examine the details of this process by considering a specific example, namely the bounce shown in Figure 3a. The ball was incident with v_1 = 3.27 m s^{-1}, θ_1 = 58.5°, ω_1 = –34.9 rad s^{-1} and bounced with v_2 = 2.52 m s^{-1}, θ_2 = 75.5°, ω_2 = +21.8 rad s^{-1}. The racket head translated horizontally at V_{x2} = 0.135 m s^{-1} after the bounce. These parameters indicate that v_{x1} = 1.71 m s^{-1}, v_{y1} = 2.79 m s^{-1}, v_{x2} = 0.63 m s^{-1} and v_{y2} = 2.44 m s^{-1}. Experimental errors in the speed, angle and spin measurements were typically about 2%.

The horizontal speed of the ball at any time t during the bounce is given by:

$$v_x = v_{x1} - \frac{1}{m}\int_0^t F\,dt$$

and the angular velocity of the ball is given by:

$$R\omega = R\omega_1 + \frac{R^2}{I_0}\int_0^t F\,dt$$

where I_0 is the moment of inertia of the ball about an axis through its centre of mass.

The time history of v_x and $R\omega$ throughout the bounce can therefore be determined by integration of the measured F waveform, and the result is shown in Figure 4. It can be seen that v_x = $R\omega$ = 0.70 m s^{-1} at t = 3.3 ms. The ball must therefore grip the surface at about this time, or slightly earlier (at t = 3.0 ms) when $v_x - V_x$ = Rω. Figure 4 shows that F suddenly reversed direction at t = 4.0 ms, indicating that the ball suddenly released its grip. During the grip period, from t = 3.0 to 4.0 ms, ω increased from 12 to 41 rad s^{-1} and v_x decreased from 0.85 to 0.33 ms^{-1}. The ball cannot rotate as a rigid body during this period since the bottom of the ball remains at rest on the strings while the remainder of the ball rotates (at an average angular velocity of 26 rad s^{-1}) through an angle of about 1.5°. The result is that internal stresses in the ball must build up to a point where the ball releases its grip, in which case the bottom of the ball will suddenly start to slide backwards on the string plane. The friction force at the bottom of the ball therefore changes from a positive to a negative force. The negative force acts to increase v_x and reduce ω during the time interval from t = 4 ms to t = 7 ms, so that the ball bounces with v_x2 = 0.63 m s^{-1} and ω_2 = 21.8 rad s^{-1}.

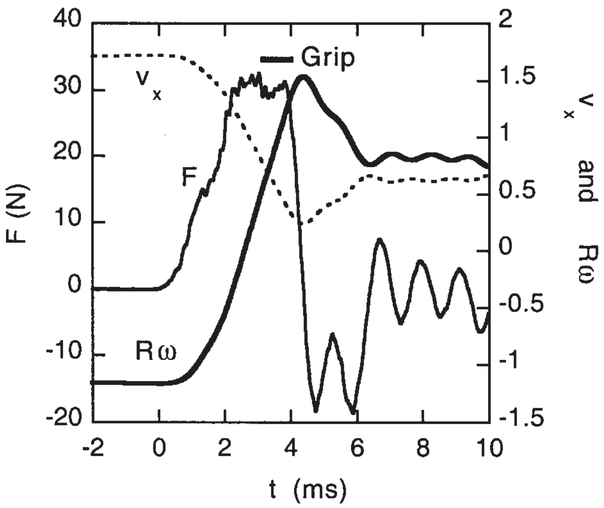


Figure 4 Analysis of the bounce shown in Figure 3a. The horizontal bar denotes the time interval during which the ball grips the strings.

The estimate of a 1.5° rotation prior to slip was consistent with a simple qualitative experiment. A 6 kg block was glued to the top of a tennis ball to simulate a 60 N load, and the bottom of the ball was placed on the strings of a tennis racket. The block was then pulled horizontally until the ball released its grip. Just prior to slipping, the top of the ball had moved about 1.5 mm horizontally while the bottom remained stuck, corresponding to a 1.3° rotation of the ball. In this experiment the ball slid forwards, in the direction of the pulling force. If a ball grips when it bounces, then one might expect that the ball would also slide forwards when it releases its grip, due to its linear momentum. The ball grips when $v_x = R\omega$. While the ball grips, $R\omega$ increases and v_x decreases. Consequently, the stress in the bottom of the ball arises primarily from ball rotation, so the ball slides backwards when the ball releases its grip.

Bounce theory

In the remainder of this paper, measurements and calculations are presented concerning the bounce speed, spin and angle off the strings of a racket under three separate conditions where (a) the racket head was mounted on rollers (b) the racket head was clamped and (c) the racket was hand-held. It is shown that (a) laboratory measurements of the bounce off a head-clamped racket do not provide a valid indication of the bounce speed, spin and angle off a hand-held racket and (b) there is no theoretical model currently available that would allow these parameters to be calculated accurately for a hand-held racket, even if the corresponding bounce parameters are known from measurements made on a head-clamped racket. The perpendicular component of the bounce speed off a hand-held racket can be calculated approximately from measurements made with a head-clamped racket, but the parallel component of the bounce speed is more difficult to calculate since it depends on the details of the ball grip process. To the author's knowledge, the bounce angle of a ball off a hand-held racket has not previously been measured and the results presented below are the first such measurements. Similarly, the spin and speed of a ball off a hand-held racket has not previously been measured under controlled conditions and when the ball is incident obliquely on the string plane. Other measurements have been made on a racket clamped by the the handle, but this method of clamping prevents the racket moving in a direction parallel to the string plane, i.e. in the direction of the friction force.

The results presented below also include the first measurements of the coefficient of sliding friction between a tennis ball and the strings of a tennis racquet. Such a measurement requires that the ball must be incident at a relatively low angle to the string plane so that the ball slides across the strings throughout the whole bounce period.

Geometry

Consider the bounce geometry shown in Figure 1 where a ball is incident obliquely on the strings of a stationary racket. The racket is not normally stationary when it impacts a ball, but the analysis is simplified if we consider the collision in the racket frame of reference. It is assumed that the strings are in the horizontal plane, and the ball trajectory is in a vertical plane defined by x and y co-ordinates. The y axis is taken as vertical and the x axis as horizontal. The racket may be hand-held or clamped around the head, or it may be mounted on rollers as in Figure 1. We can describe all three cases using the same approach, but the effective mass of the racket is different in each case. In this paper, the term ‘effective mass’ is taken to mean the ratio of the applied force at a given point to the acceleration of that point. The ball is incident with vertical speed v_{y1}, horizontal speed v_{x1} and angular velocity ω_1 and bounces with vertical speed v_{y2}, horizontal speed v_{x2} and angular velocity ω_2. The angular velocities are taken to be positive if the ball has topspin, and the linear velocities are taken to be positive when the ball is incident and bounces in the usual directions. We assume also that the impact point on the racket recoils at speed V_{x2} in the horizontal direction and at speed V_{y2} in the vertical direction. If the head is clamped then V_{x2} and V_{y2} are both zero. If the head is mounted on rollers then V_{y2} is zero.

If the ball slides across the string plane during the impact, it would normally cause the racket to rotate about its long axis. However, we will assume that the average position of the ball coincides with the centre of the strings, in which case we can ignore this rotation. For a hand-held racket, rotation of the racket about its centre of mass cannot be ignored, the result being that the effective mass of the racket at the impact point is less than its actual mass, as described below. If the head is clamped, then the effective mass of the racket is infinite. If the head is mounted on rollers then the effective mass of the head is infinite in the y direction and is equal to the actual mass of the head in the x direction.

Perpendicular COR

The ball speed in a direction perpendicular to the string plane can be specified in terms of the coefficient of restitution (COR), e_y, defined by the relation

$$e_y = \frac{(v_{y2} + V_{y2})}{v_{y1}}$$

or in terms of the apparent coefficient of restitution (ACOR), e_A, defined by

$$e_A = \frac{v_{y2}}{v_{y1}}$$

The COR is defined in terms of the relative speed of the ball and the racket at the impact point, before and after the collision. The ACOR is defined in terms of the ball speeds before and after the collision, ignoring the racket speed after the collision but assuming that the racket is initially at rest. The ACOR is equal to the COR if $V_{y2} = 0$, as it is when the head is clamped or mounted on rollers. For an impact at the vibration node near the middle of the strings, the COR for a hand-held racket is the same as that for a head-clamped racket (since there are no energy losses due to frame vibrations) but the ACOR values are different. For a hand-held racket, the COR and the ACOR are related by conservation of linear momentum in the y direction. If m is the mass of the ball and M_e is the effective mass of the racket in the y direction, then $mv_{y1} = M_e V_{y2} - mv_{y2}$ and hence

$$e_A = \frac{(e_y - m/M_e)}{(1 + m/M_e)}$$

The effective mass of a hand-held racket is given by (Brody *et al.*, 2002)

$$\frac{1}{M_e} = \frac{1}{M} + \frac{b^2}{I_{cm}}$$

where M is the actual racket mass, b is the distance between the impact point and the centre of mass (CM) and I_{CM} is the moment of inertia of the racket about its centre of mass. M_e is equal to M for an impact at the CM and is typically about $M/2$ for an impact in the middle of the strings.

Parallel COR

The ball speed in a direction parallel to the string plane can be specified in a manner that is analogous to that in a direction perpendicular to the string plane. That is, one can define a tangential COR and a tangential ACOR. The vertical COR is a measure of the energy losses in the ball and the racket arising from vertical compression of the ball, transverse vibrations of the racket frame and losses in the strings. The tangential COR is a measure of energy loss arising from ball deformation in the horizontal direction plus any horizontal or longitudinal vibration of the frame. It is defined in terms of the horizontal speed of the contact point (rather than the horizontal speed of the ball CM) and is given by (Cross, 2002a)

$$e_x = -\frac{(v_{x2} - R\omega_2 - V_{x2})}{(v_{x1} - R\omega_1)}$$

where R is the ball radius and $(v_x - R\omega)$ is the horizontal speed of a point at the bottom of the ball. For a perfectly elastic ball impacting on a massive surface, the horizontal and vertical speeds of the contact point are both reversed by the bounce and $e_y = e_x = 1$. If a ball starts rolling during the bounce then the contact point comes to rest with respect to the surface and then $e_x = 0$. If the ball slides throughout the bounce then the horizontal velocity of the contact point does not change sign and the ball bounces with $e_x < 0$.

Measurements of the tangential speed of the ball CM are quoted below in terms of the dimensionless ratio:

$$e_T = \frac{(v_{x2} - V_{x2})}{v_{x1}}$$

which is mathematically analogous to Eqn. (3) but it does not have the same physical significance. The difference arises from the fact that the vertical speeds of the ball CM and contact point are the same, but the horizontal speeds are not. The significance of e_T is that it can be used to compare experimental data obtained under different conditions where the head is either clamped or free to translate. For a sliding ball it would not be necessary to measure the horizontal ball speed relative to the surface on which it slides since the coefficient of sliding friction is independent of the speed of the ball or the surface and hence the horizontal ball speed is independent of whether the head is clamped or free. However, if the ball rolls or grips the surface, then the horizontal ball speed *does* depend on the speed of the surface. In the latter case, e_T provides a useful measure of the change in horizontal ball speed relative to the surface. For the clamped racket, $V_{x2} = 0$ and hence $e_T = v_{x2}/v_{x1}$. For the hand-held racket, V_{x2} was not measured accurately but it was clearly at least 10 times smaller than v_{x2}. The ball was projected not across the racket but along the racket in a direction from the tip towards the handle. In that case, the mass of the hand and arm contributed to the total effective mass of the racket in the x direction. Results for the hand-held racket are therefore quoted in terms of the ratio v_{x2}/v_{x1} since horizontal motion of the racket was neglible.

Ball spin

Measurements of ball spin are quoted below in terms of two dimensionless quantities, $S_1 = R\omega_2/v_1$ and $S_2 = R\omega_2/(v_{x2} - V_{x2})$, where v_1 is the incident speed of the ball. The parameter S_1 is the ratio of the peripheral speed of a point on the ball after the bounce to the incident speed of the ball. Values of ω_2 are not quoted below since they are directly proportional to the speed of the incident ball which varied slightly from one bounce to the next. The parameter S_1 removes this variability and allows different bounces to be compared to examine the effect on ball spin. S_1 can be regarded as a measure of the efficiency by which linear motion of the incident ball is converted to rotational motion of the rebounding ball. The parameter S_2 is the ratio of the peripheral speed of the ball to the horizontal speed of the ball after the bounce, relative to the string plane. A measurement of S_2 indicates whether the ball slides throughout the bounce ($S_2 < 1$) or whether it grips during the bounce ($S_2 > 1$), as described above.

Coefficient of friction

Regardless of whether the ball slides or grips when it bounces, the changes in ball speed in the horizontal and vertical directions are given by:

$$\int F\,dt = m(v_{x1} - v_{x2})$$

and

$$\int N\,dt = m(v_{y1} + v_{y2})$$

where m is the ball mass, F is the horizontal friction force acting at the bottom of the ball and N is the normal reaction force on the ball. The ratio of the time-average value of F divided by the time-average value of N can be regarded as a measure of the effective value of the coefficient of friction, COF, defined by the relation:

$$COF = \frac{(v_{x1} - v_{x2})}{(v_{y1} + v_{y2})}$$

If the ball slides throughout the bounce then COF is equal to the coefficient of sliding friction, μ, in which case $F = \mu N$ and hence:

$$\frac{v_{x2}}{v_{x1}} = 1 - \mu(1 + e_A)\tan\theta_1$$

Equation (12) shows that a ball will slide faster on a hand-held racket than on a head-clamped racket since e_A is smaller by a factor of about two for a hand-held racket.

If the ball starts to slide and then grips the surface then $F = \mu N$ at the beginning of the bounce but F drops to zero and may reverse direction during the bounce, in which case the time-average value of F is less than the time-average value of μN and hence $COF < \mu$. Measurements of COF, S_2 and e_x all provide useful indications of whether the ball slides throughout the bounce or whether it grips the surface during the bounce.

Shift in *N*

In theory, the torque acting on the ball is given by FR and the angular velocity of the ball will increase during the bounce according to the relation $FR = I_o d\omega/dt$ where $I_o = 3.41 \times 10^{-5}$ kg m^2 is the moment of inertia of a tennis ball of mass m = 57 g and radius R = 33 mm about an axis through its CM. The change in angular momentum during the bounce should therefore be given by $R \int F\,dt$, where $\int F\,dt$ is the horizontal impulse on the ball as given by Eqn. (9). Experimentally, it is observed that this impulse is typically too large to account for the observed change in angular momentum. The torque acting on the ball is therefore less than FR. Part of the reason is that R is reduced as a result of ball compression, particularly at high incident ball speeds. However, it is difficult to estimate the effect on the angular velocity, partly because R is difficult to determine and partly because the moment of inertia of a squashed ball is also difficult to determine. A simpler procedure is to assume that R and I_o remain constant, and to attribute the torque reduction entirely to a forward shift in the line of action of N, by a distance D. In that case the angular acceleration of the ball is given by $FR - ND = I_o d\omega/dt$. This overestimates the actual shift but it provides a simple method of calculating a single parameter, D, that can be used to quantify the combined effects of a real shift in N and a reduction in R.

Values of D as large as 11 mm have been calculated for a high speed bounce on court surfaces (Cross, 2003). On a court surface, a finite value of D can be attributed to the fact that the front edge of the ball is incident on the surface at higher speed than the back edge since the ball rotates into the surface at the front edge and out of the surface at the back edge. The effect is analogous to the forwards shift in weight of a vehicle when the brakes are applied, causing the vehicle to rotate about its CM. The same effect will occur on the string plane, but in addition the string plane is deformed by the incident ball in such a way that it will tend to resist forward motion of the ball. The normal reaction force can then be shifted forwards, in the same way that deformation of the surface resists forward motion of a rolling ball and shifts the normal reaction force forwards (Hierrezuelo *et al*. 1995).

The normal reaction force acts in a direction perpendicular to the string plane and has no direct effect on the tangential velocity. However, if N acts through a point ahead of the ball CM, it exerts a backwards torque on the ball, reducing its angular acceleration. As a result, the ball can slide for a longer time before it grips the strings. Since the friction force acts for a longer time, the tangential velocity of the ball is reduced.

Bounce speed and spin results

Ball incident without spin

Measured values of the ball speed and spin, for the racket head on rollers, are shown in Figure 5. Data were obtained over a large range of incident angles for a ball incident with zero or neglible spin (less than 5 rad s^{-1}). Depending on the incident ball speed and angle, the ball bounced with ω_2 between +30 and +100 rad s^{-1}. Each data point in Figures 5–7 was obtained with a measurement error of about 3%, but the main source of error in all measurements is the fact that no two bounces are the same, even if one projects a ball onto a smooth concrete slab. Even larger differences can arise when a ball impacts on the string plane of a racket due to the uneven nature of the surface. Sufficient data points were therefore obtained to provide a clear and statistically significant trend of each measured parameter as a function of incident angle or as a function of ball spin. The trend in each case is shown by the best fit polynomial curves in Figures 5–7, and the variation between successive bounces is indicated by the scatter in the experimental data points.

Figure 5a shows the dimensionless ratio $e_T = (v_{x2} - V_{x2})/v_{x1}$ and the parameter labelled COF which represents the effective coefficient of friction during the bounce. In Figure 5a, the sliding region corresponds to angles of incidence less than about 25° where μ is about 0.45. The coefficient of sliding friction between the ball and the strings is lower than that normally encountered between the ball and a tennis court where μ ranges from about 0.6 on a fast court to about 0.8 on a slow court (Brody *et al*., 2002). In that respect, the strings of a racket can be regarded as being very fast, at least for the low speed impacts studied in this paper. At higher angles of incidence e_T remains relatively constant at about 0.6 and the COF decreases as θ_1 increases, partly because the ball stops sliding at progressively earlier stages of the bounce (reducing the average value of F) and partly because N increases at high angles of incidence (since v_{y1} is larger).

Figure 5b shows the two dimensionless measures of the rebound spin, S_1 and S_2, as a function of the incident angle θ_1. The value of S_1 is small at both low

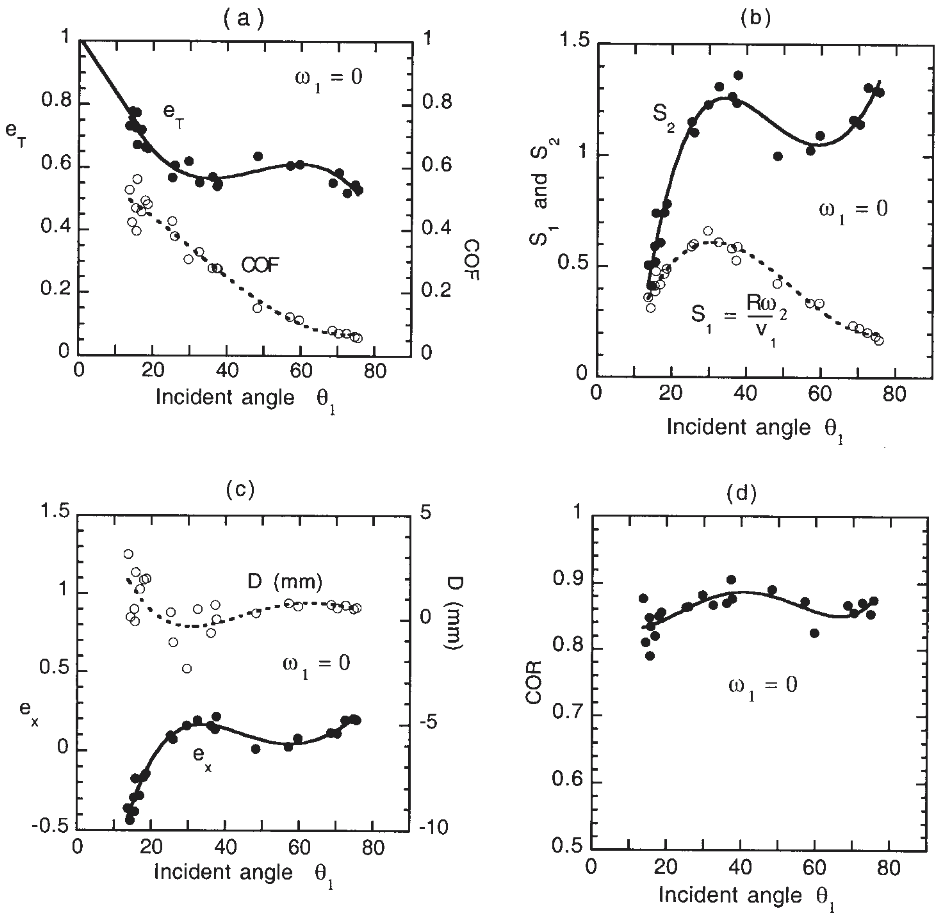


Figure 5 Data obtained from video camera measurements for a ball incident with neglible spin, as per Figure 1. Each data point represents a single bounce and each solid or dashed curve is a polynomial fit to the data.

and high angles of incidence and is a maximum near θ_1 = 30° in Figure 5b. If a player wants to maximize the spin of the ball then he or she should hit the ball as fast as possible to maximize v_1, in such a way that the ball is incident at 30° to the string plane. At least, this is the result inferred from the data in Figure 5b. For a hand-held racket, S_1 is a maximum near θ_1 = 40°, as described below. In Figure 5b the ball slides throughout the bounce with S_2 = < 1.15 at low angles of incidence. At higher angles of incidence, the ball bounces with a value of S_2 between 1.0 and 1.3. The same result was found previously for a tennis ball bouncing on various court surfaces (Cross, 2003). Other ball types (golf balls, superballs etc.) can bounce with higher or lower values of S_2 depending on the amount of energy stored and recovered as a result of the ball stretching in a horizontal direction. The value of S_2 also depends on the elasticity or compliance of the surface on which the ball bounces. If the strings are not interlaced (as in a so-called spaghetti strung racket) then the strings can stretch considerably in the horizontal direction with the result that the ball bounces with significantly more spin than in a conventionally strung racket (Goodwill and Haake, 2002).

Figure 5c shows the horizontal coefficient of restitution, e_x, and the distance D between the line of action of N and the centre of mass of the ball. It is assumed that D is positive if N acts through a point ahead of the centre of mass. The formula for D is given by Cross (2002b). A negative value of e_x indicates that the ball slides throughout the bounce and a positive value of e_x indicates that the ball grips the surface. For a superball, e_x is typically about 0.5 or 0.6, but for a tennis ball e_x is typically about 0.1 or 0.2 when the ball grips a rigid surface. The result in Figure 5c shows that e_x is neither enhanced nor reduced when a ball bounces off tennis strings. Figure 5c also shows that D is relatively small for a low speed bounce on tennis strings, but it may be considerably larger for a high speed bounce. If D is positive then N exerts a torque on the ball in a direction opposite the torque due to F, thereby reducing the ball spin.

Figure 5d shows that the coefficient of restitution (COR) was about 0.88 for large angles of incidence and about 0.84 at low angles of incidence. These results are consistent with previous measurements of the COR for perpendicular incidence on a head-clamped racket, although the decrease at low angles of incidence was surprising. Measurements of the oblique bounce of a tennis ball on various court surfaces indicate that the COR is usually enhanced at low angles of incidence (Cross, 2003). At low angles of incidence on tennis strings, and at low incident ball speeds, the ball brushes the strings only lightly and the normal reaction force is quite small. If the normal reaction force is small then a significant fraction of the ball compression will be due to compression of the cloth cover rather than the underlying rubber. The low COR observed at low incident angles may therefore indicate that energy stored in the cloth is recovered with low efficiency. Alternatively, energy losses may be enhanced at low angles of incidence due to vibration of the frame in the horizontal direction.

The overall conclusion from the results in Figure 5 is that a tennis ball bounces off tennis strings in a manner that is similar to the bounce off a tennis court and that is similar to the prediction of Brody (1984). The main differences are that the COR is greater off the strings than off a tennis court, the strings have a lower coefficient of sliding friction, and at high angles of incidence the ball spins about 20% faster than predicted by Brody. At high angles of incidence the ball bounces with e_T about = 0.6, as predicted by Brody. The agreement here is somewhat coincidental since Brody assumed that a tennis ball has an infinitely thin wall. If allowance is made for the fact that the wall of a tennis ball is about 6 mm thick, then Brody would predict that $e_T = 0.645$ at high angles of incidence. The reduction in e_T to about 0.6 and the additional spin is due to the fact that the ball grips rather than rolls.

Ball incident with topspin or backspin.

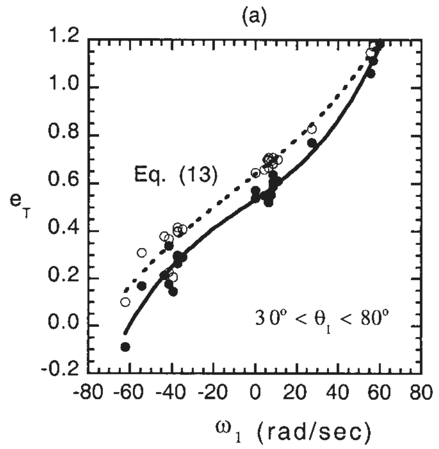
As shown in Figure 5a, e_T depends only weakly on θ_1 when θ_1 is greater than 30°. In practice, the ball is almost always incident on the strings of a racket with θ_1 = > 30° . All the data collected in this important range of angles are shown in Figure 6a, as a function of ω_1, the angular velocity of the incident ball. Figure 6a shows that e_T depends much more strongly on ω_1 than on θ_1. A ball incident without spin slows down by about 40% in a direction parallel to the string plane. If the ball is incident with sufficient topspin then the ball can bounce with a larger parallel speed than it had before the bounce. If the ball is incident with sufficient backspin, then it can bounce at right angles to the string plane, in which case e_T is zero, or it can bounce backwards, in which case e_T is negative. The significance of this result is that the bounce angle off the strings depends strongly on the magnitude and direction of the spin of the incident ball. Most tennis players are aware of this and will adjust their swing if they know that the ball is spinning as it approaches.

There is no detailed theoretical model of ball grip for a tennis ball that one can use to analyse the results in Figure 6a. However, the results are qualitatively consistent with the model proposed by Brody (1984). Using the same assumptions as those made by Brody, but allowing for finite wall thickness and for finite spin of the incident ball, it can be shown (Cross, 2002b) that:

$$v_{x2}/v_{x1} = 0.645 + 0.355R\omega_1/v_{x1}$$

This relation is compared with the experimental data in Figure 6a. The fit is reasonably good, indicating that the effect of the ball gripping the strings is qualitatively similar to that of a rolling ball. Equation (13) does not take into account the fact that N can act through a point ahead of the ball CM. This is not a serious omission at low ball speeds but the observed values of v_{x2}/v_{x1} at high ball speeds are considerably lower than predicted by Eqn. (13), as described in the discussion section.

Figure 5b shows that the spin efficiency parameter S_1 depends on θ_1, so it is not appropriate to plot S_1 versus ω_1 to include all data with θ_1 = > 30°. However, Figure 6b shows the corresponding graph for a subset of the data within the narrow range of incident angles $55° < \theta_1 < 65°$. Regardless of the direction of the spin of the incident ball, the ball bounced with topspin and hence with a positive value of S_1. As one might expect, Figure 6b shows that a ball incident with topspin



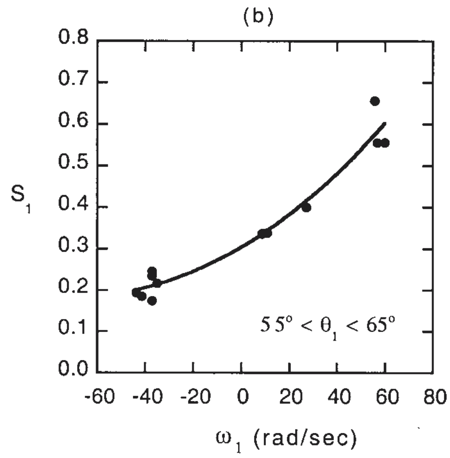


Figure 6 Measurements of (a) e_T and (b) S_1, as a function of incident ball spin. Experimental data points in (a) are indicated by solid dots. Theoretical data points, using Eqn. (13) plus the experimental values of v_{x1}, are indicated by open circles. All curves are polynomial fits to the data.

bounces with greater spin than a ball incident with backspin. What Figure 6b does not show is that at large values of ω_1, the spin after the bounce is less than the spin before the bounce, i.e. $\omega_2 < \omega_1$.

Bounce off Pro Kennex racket

Results obtained by filming low speed, oblique bounces of a ball on the Pro Kennex racket are shown in Figure 7. In all cases the string plane was horizontal, the ball was thrown by hand at a speed of 3–5 m s^{-1} to impact near the middle of the strings, and the ball was incident with neglible spin (less than 5 rad s^{-1}). Two sets of results were obtained, one where the head was rigidly clamped and one where the racket was held at the butt end of the handle by an assistant. Results obtained with the head-clamped racket are essentially the same as those obtained in the first experiment where the head was free to move horizontally on rollers. The value of e_T was almost identical and so was the S_1 ratio. For the head-clamped racket, e_T is equal to v_{x2}/v_{x1} since there was no horizontal motion of the racket. The measured COR was slightly higher on the head-clamped racket than when the head was supported on rollers, probably because vibrational losses in the frame were reduced.

Significant differences were found for the hand-held racket, most noticeably in the measured values of v_{y2}/v_{y1}, but also in the values of v_x2/v_x1. For a head-clamped racket, the v_{y2}/v_{y1} ratio is equal to the COR. The experimental value of the COR was 0.91 ± 0.01, regardless of the angle of incidence, as shown in Figure 7c. For a hand-held racket, the v_{y2}/v_{y1} ratio is equal to the apparent coefficient of restitution (ACOR). The relation between the COR and the ACOR is given by Eqns. (5) and (6). A theoretical estimate of the ACOR, based on the measured COR, is shown in Figure 7d. Not all incident balls landed in the middle of the string plane (16 cm from the tip), some landing short or long or slightly wide. Bounces landing more than 2 cm away from the long axis of the racket were not analysed and are not represented in any of the data in Figure 7. All bounces landing within 2 cm of the long axis are included in the data shown in Figure 7, even if they landed 5 cm short or 5 cm long in a direction parallel to the long axis. Bounces landing about 11 cm from the tip of the hand-held racket had an ACOR of about 0.30, and bounces landing about 21 cm from the tip had an ACOR of about 0.48. In all cases, the observed ACOR was equal to or slightly less than the theoretical estimate. The discrepancy can be attributed to the fact that impacts not coinciding with the vibration node near the centre of the strings generate transverse vibrations in the racket frame, thereby lowering the COR. In addition, impacts away from the long axis cause the racket to rotate about its long axis, thereby lowering the effective mass of the racket and hence reducing the ACOR.

A consequence of the reduced value of v_{2y}/v_{1y} for a hand-held racket is that the ball bounces at a lower rebound angle, as shown in Figure 7e. The rebound

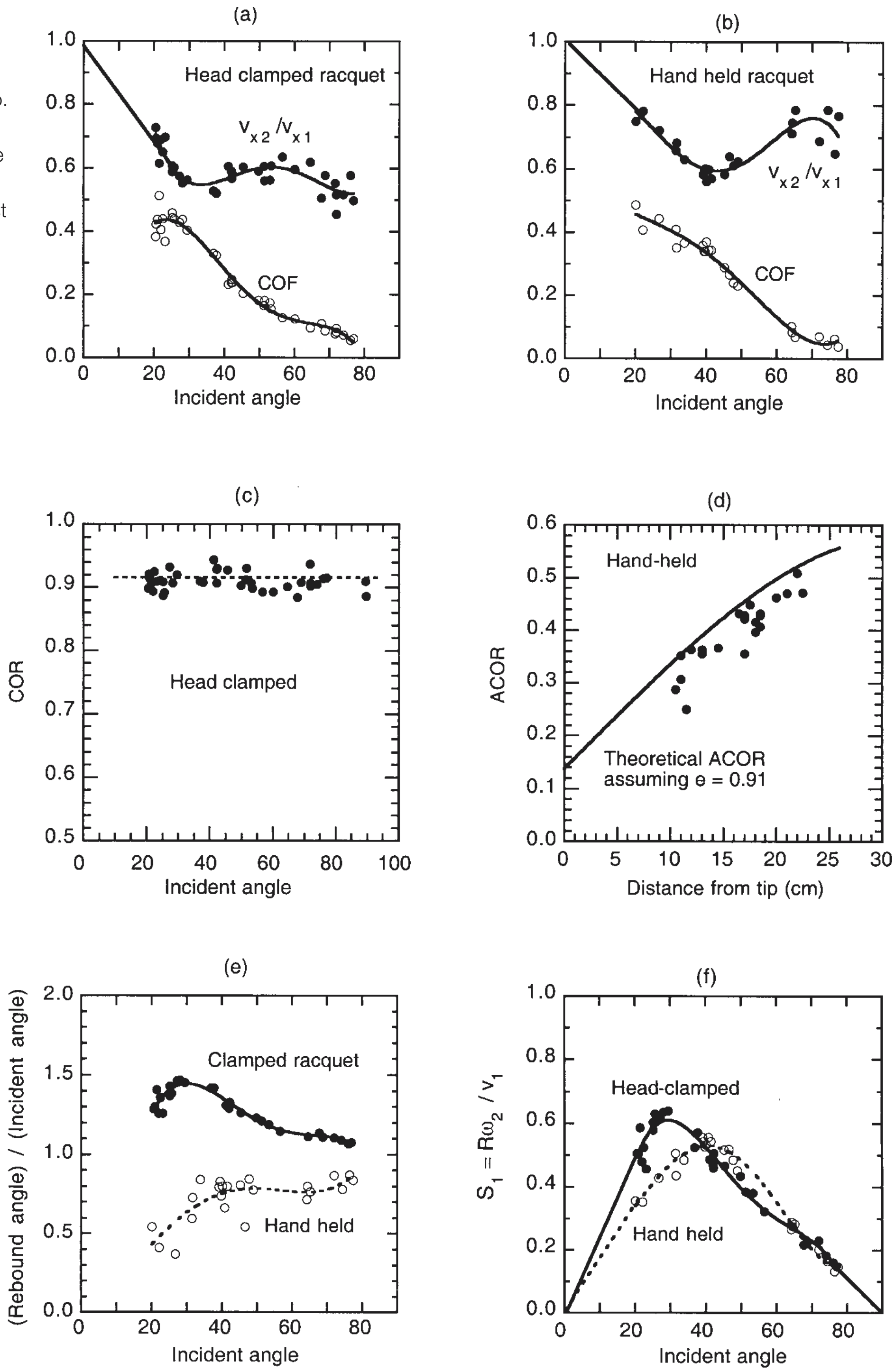


Figure 7 Measured bounce parameters for a ball bouncing obliquely on the strings of a Pro Kennex No. 24 racket with the head firmly clamped or when the handle was hand-held and the head was initially at rest and free to recoil. Curves are polynomial fits, with linear extrapolations to $\theta_1 = 0$ in (a), (b) and (f) to show the theoretically expected trend.

angle also depends on v_{x2}, which is larger for the hand-held racket, thereby enhancing the difference in the rebound angle.

The v_{x2}/v_{x1} ratio for a hand-held racket (Figure 7b) differs from that for a clamped racket (Figure 7a) in two respects. First, the sliding region extends to about $\theta_1 = 40°$ for the hand-held racket whereas it terminates at about $\theta_1 = 30°$ on the clamped racket. In both cases, the coefficient of sliding friction is about 0.45. The extended sliding region is easily accounted for, since the normal reaction force on the hand-held racket is reduced (at any given v_1 and θ_1). The time integral of N determines the change in v_y which is smaller for a hand-held racket since v_{y2} is smaller. As a result, the friction force is reduced and hence the ball spin is reduced at low angles of incidence, as shown in Figure 7f. In addition, v_{x2} is increased as indicated by Eqn. (12) and as shown in Figure 7b. The ball must therefore be incident at a higher angle of incidence on a hand-held racket before it starts to grip the strings.

Another difference between the hand-held and clamped rackets is that v_{x2}/v_{x1} is significantly higher on the hand-held racket at high angles of incidence. This is not consistent with Eqn. (13) and is not easy to explain. A possible explanation is that the ball releases its grip at an earlier stage during the bounce, due to the smaller value of N. In that case, the average friction force is smaller and hence the ball bounces at a higher speed in a direction parallel to the string plane. One might expect that the ball should therefore bounce with reduced spin on the hand-held racket. In fact, it bounces with the same spin at large θ_1 as shown in Figure 7f. The most likely explanation is that at large angles of incidence the calculated value of D is about 0.5 mm on the clamped racket and about −1.0 mm on the hand-held racket. Positive values of D act to reduce the spin while negative values act to increase the spin.

Measurements of S_2 for the Pro Kennex racket are shown in Figure 8. These results confirm the fact, deduced from the data in Figures 7a and 7b, that the ball grips the strings when $\theta_1 = > 30°$ on the clamped racket and when $\theta_1 = > 40°$ on the hand-held racket.

Discussion

Measurements of the friction force and other bounce parameters described above show that a tennis ball bounces off tennis strings in essentially the same manner as it bounces off a tennis court. That is, the ball slides throughout the bounce at low angles of incidence and it grips the surface at high angles of incidence. The ball subsequently releases it grip, more rapidly on strings than on a rigid surface. Maw *et al.* (1976, 1981) describe the grip release phase as being gradual and progressive because the normal reaction force at the edge of the contact area is less than that at its centre. Consequently, the outer contact region can commence to slide backwards on the surface while the central region remains stuck. At least, this is the situation that one might expect for a ball bouncing on a rigid surface. When a tennis ball is gripped by the strings of a racket, it is likely to be gripped firmly and more uniformly at all points in contact with the strings since the strings conform to the shape of the ball (or vice-versa). Consequently, when the strings release their grip on the ball, the transition to the backward sliding phase is relatively rapid. The process can be likened to a block of wood on an inclined plane, where the block will suddenly release its grip if the incline angle is gradually increased.

Arguments are sometimes presented that natural gut strings provide a better grip on the ball since since natural gut is more elastic and ‘cups’ the ball more firmly. That may well be the case, but it does not guarantee better performance or greater ball spin since the final spin at exit depends on the magnitude and duration of the friction force acting backwards on

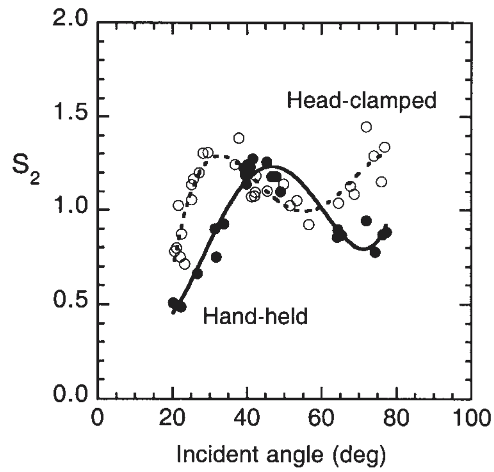


Figure 8 Measurements of $S_2 = R\omega_2/v_{x2}$ for the Pro Kennex racket when hand-held (black dots) or head-clamped (open circles). Curves are polynomial fits to the data.

the ball after the strings release their grip. Indeed, Goodwill & Haake (2003) found no significant difference in ball spin off natural and synthetic gut strings. They also found no significant difference in ball spin off thin and thick strings, contrary to common belief.

The angle of incidence at which a ball will grip the surface is typically about 20° on a court surface, about 30° on a head-clamped racket and about 40° on a hand-held racket. These values depend on the coefficient of sliding friction, on the COR or the ACOR and on the magnitude and direction of the incident spin, but can be regarded as typical. In practice, the ball is usually incident on the strings at an angle greater than 45° so the ball will usually grip the strings. In that case, one cannot use conventional models of ball bounce (e.g. Brody, 1984) to predict the rebound angle or spin. In the absence of any quantitative model of how a tennis ball grips the strings, one must be guided by experimental data. The data in this paper was obtained at low ball speeds. Data obtained at higher ball speeds by Bower & Sinclair (1999), Knudson (1991) and Goodwill & Haake (2003) are compared with the results in this paper in Table 1. In each case, the authors obtained data at only one angle of incidence. In Table 1, v_1 is given in m s^{-1}, ω_1 is given in rad s^{-1}, $e_A = v_{y2}/v_{y1}$ and $e_T = (v_{x2} - V_{x2})/v_{x1}$. The tabulated data are representative values averaged over a range of experimental conditions (e.g. different string tensions or string types) and are meant to indicate typical values obtained by the different authors under different clamping conditions and at different ball speeds. To make a direct comparison between each set of measurements, a value of θ_2 was calculated assuming that a ball is incident at $\theta_1 = 50°$ and bounces with the values of e_A and e_T given in the Table.

As shown in Table 1, the ball bounces at a larger angle (closer to the normal) when it is incident with backspin and it bounces at a much smaller angle when the racket is hand-held or when the handle is clamped. In general, these two effects have a much bigger influence on the rebound angle than those due to variations in string tension, string type or ball speed. However, for any given topspin forehand, where the ball is incident with backspin and the racket is hand-held, the last two effects may be more significant. In practice, the significance of these effects is somewhat diminished because the rebound angle off the strings depends strongly on the trajectory of the racket. For example, suppose that a difference in string tension results in a 3° difference in rebound angle when measured on a racket initially at rest in the laboratory. If the same racket is swung towards the ball, then the rebound angle off the racket may differ by only 1°, depending on the initial speed and trajectory of the ball and the racket. This effect was overlooked by Bower & Sinclair (1999) and is described in more detail by Cross (2000) and by Brody *et al.* (2002).

Table 1 Bounce parameters observed by different authors

Author	How held	v_1	θ_1	ω_1	e_A	e_T	θ_2
Cross	On rollers	5	30 – 80	0	0.88	0.57	61.5°
Cross	On rollers	4	30 – 80	–40	0.88	0.20	79.2°
Cross	Head clamped	5	30 – 80	0	0.91	0.57	62.3°
Cross	Hand-held	5	40 – 80	0	0.40	0.70	34.3°
B & S	Handle clamped	20	45	0	0.50	0.51	49.4°
Knud	Handle clamped	19	64	0	0.44	0.70	36.8°
G & H	Head clamped	24	50	0	0.91	0.46	67.0°
G & H	Head clamped	41	50	0	0.86	0.39	69.2°
G & H	Head clamped	24	50	–400	0.85	0.20	78.8°
G & H	Head clamped	41	50	–400	0.82	0.28	74.0°

Several studies have shown that the COR and the ACOR for perpendicular incidence both decrease as the ball speed increases, a result that can be attributed mainly to higher fractional ball losses at high impact speeds. The energy loss in the strings is much smaller than the loss in the ball. For example, Casolo (1994) found that the COR for a ball incident normally on a clamped racket decreases from about 0.90 at low ball speeds to about 0.80 at an incident speed of 40 m s^{-1}, while Goodwill & Haake (2003) found that the COR decreased from 0.91 at 24 m s^{-1} to 0.86 at 41 m s^{-1} for a ball incident at 40° to the normal.

Bower & Sinclair (B & S in Table 1) projected a ball at q1 = 45° onto the middle of the strings of a handle-clamped racket, allowing the head to recoil and vibrate. The ball was incident without spin at speeds from 16 to 24 m s–1. The string tension was 180, 225 or 270 N, and the clamping position on the handle was varied to simulate variations in frame stiffness. The measured value of eA varied from 0.45 for a flexible racket strung at high tension, to 0.54 for a stiff racket strung at low tension. The corresponding values of vx2/vx1 varied over a narrow range from 0.50

to 0.52. These values were not quoted directly but can be calculated from the quoted rebound angles. In Figure 7b, the corresponding value of vx2/vx1 at q1 = 45° is 0.60 . The difference is probably due the fact that e_T decreases as the ball speed increases, a result that is apparent from the data given by Goodwill & Haake (2003).

Knudson (Knud in Table 1) describes an experiment where a ball was projected at 19.1 m s^{-1} without spin at θ_1 = 64.4° onto the middle of the strings of a handle-clamped racket. e_T ranged from 0.69 to 0.72, similar to the results for the hand-held racket shown in Figure 7b. In a subsequent experiment (Knudson, 1993) the ball was projected without spin at the same angle but at a higher speed (28.9 m s^{-1}) and the ball bounced at an angle closer to the normal, indicating that e_T was lower than in his first experiment. The rebound speed was not quoted so no quantitative estimate can be made of e_T in the second experiment.

Goodwill & Haake (G & H in Table 1) projected a high speed ball at θ_1 = 50° onto the string plane on a head-clamped racket. The ball was incident at speeds up to 41 m s^{-1} and with ω_1 varying from +100 to –600 rad s^{-1}. The value of e_T, for a ball incident with zero spin on a racket strung at a tension of 70 lb (154 N) was 0.46 at v_1 = 24 m s^{-1} and decreased to 0.39 at v_1 = 41 m s^{-1}. These values are considerably smaller than the value 0.60 measured on the head-clamped racket in this paper, and indicate that e_T decreases as the ball speed increases. This effect can be attributed to a larger deformation of the ball and the string plane at high ball speeds, with the result that D is also larger. For the head-clamped results in this paper, D = 0.5 mm for a ball incident without spin at θ_1 near 50°. The results presented by Goodwill and Haake show that D increased from 3.8 mm at v_1 = 24 ms^{-1} to 5.6 mm at v_1 = 41 ms^{-1} while e_x remained constant at about 0.05. Consequently, one can attribute the reduction in v_{x2}/v_{x1} directly to an increase in D, as described by Cross (2003).

Goodwill & Haake also found that e_T decreases as the amount of backspin increases. At ω_1 = –400 rad s^{-1}, e_T was 0.20 at v_1 = 24 m s^{-1} and 0.28 at v_1 = 41 m s^{-1}. While both of these values are lower than those obtained for a ball incident with zero spin, e_T *increased* with ball speed, whereas it decreased with ball speed when ω_1 = 0. This effect can be explained qualitatively using the rolling ball model (Brody, 1984). Even though this model is quantitatively incorrect at high ball speeds, Eqn. (13) shows correctly that v_{x2}/v_{x1} decreases as ω_1 becomes more negative, but for a given amount of backspin v_{x2}/v_{x1} increases as v_{x1} increases. The latter effect presumably outweighed the effect due to finite D since D decreased significantly when the ball was incident with backspin.

Conclusions

When a tennis ball is incident on the strings of a tennis racket at an angle within 50° to the normal, the ball commences to slide and then grips the strings. While the contact area remains at rest on the strings, the ball continues to decelerate in a direction parallel to the string plane and the angular velocity continues to increase due to the static friction force acting backwards on the ball. The ball then releases its grip and the contact region starts sliding backwards on the string plane. The sudden reversal in direction of the friction force acts to accelerate the ball in a direction parallel to the string plane and to decrease the angular velocity. The ball bounces with topspin off the strings but the spin is lower after the bounce than it was during the bounce.

The coefficient of restitution in a direction perpendicular to the string plane is typically about 0.90 for a low speed bounce near the middle of the strings but it decreases to about 0.85 for a high speed bounce. This information, when combined with measurements of racket mass and moment of inertia, is sufficient to calculate the ball speed in a direction perpendicular to the string plane, for a hand-held racket at any given swing speed, provided the impact is near the middle of the strings. Alternatively, a measurement of the ACOR at various impact points on the string plane provides sufficient information to calculate the perpendicular component of the ball speed for an impact at any given point on the string plane. In the latter case, it is not necessary to know the racket mass or its moment of inertia, although both parameters affect the measured ACOR values.

The bounce speed of a ball in a direction parallel to the string plane is more difficult to calculate since it cannot be characterised by a single number analogous to the COR. At low ball speeds and for an impact on a head-clamped racket, the ball speed can be estimated from Eqn. (13). In that case, v_{x2}/v_{x1} is essentially independent of the angle of incidence but it depends strongly on the magnitude and direction of the spin of the incident ball. If the ball is incident at high speed then the v_{x2}/v_{x1} ratio is reduced substantially due to deformation of the ball and the string plane. If the racket is hand-held, the v_{x2}/v_{x1} ratio increases since the normal reaction force is reduced and hence the friction force is reduced. The v_{x2}/v_{x1} ratio has not yet been measured for a hand-held racket when the ball is incident at high speed with backspin or with topspin. Furthermore, there is no theoretical model incorporating ball grip that would allow the ratio to be predicted or to be deduced from laboratory measurements on a head-clamped racket. There is therefore a need for such measurements to be made. The simplest procedure would be to film high speed bounces off a handle-clamped racket, but the best and most realistic procedure may well be to film players on the court under actual or simulated playing conditions.

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