Physics of overarm throwing

Rod Cross[a)]
University of Sydney, Sydney, NSW 2006, Australia

(Received 22 April 2003; accepted 24 October 2003)

Measurements are presented of the speed at which objects of different mass can be projected by an overarm throw. Light objects can be thrown faster than heavy objects, although the difference in speed is not as large as one might expect. For a factor of 60 increase in the thrown mass, there was a decrease of only 2.4 in the throw speed. The relatively small change in throw speed is due to the fact that the force that can be applied to a thrown object increases with object mass. Estimates of the muscle forces involved indicate that the increase in force with mass is primarily an inertial rather than a physiological effect. The total kinetic energy of the mass, hand, and the forearm was found to be almost independent of the object mass, and the throw speed is almost independent of the mass of the upper arm.
[DOI: 10.1119/1.1634964]

I. INTRODUCTION

It is intuitively obvious that one can throw a baseball faster than a brick because the baseball is lighter. If the force applied to each object is the same and if both objects are accelerated through the same distance, then both objects will have the same kinetic energy. In practice, one can apply a larger force to the brick, so the brick will have greater kinetic energy. The additional force on the brick is not quite large enough to propel it at the same speed as a baseball, but the percentage difference in speed is much smaller than the difference in mass. In this paper, measurements of throw speed versus object mass are described and simple models of throwing are presented to explain the results.

The results should be of particular interest to those involved in the teaching of elementary physics in life or sports sciences courses. One of the problems in teaching physics to such students is the difficulty of obtaining relevant and reliable data on the forces and energy involved in human movement. An example of this problem concerns the optimum angle of jumping, or of throwing a shot put to obtain the maximum range. The angle depends not only on the physics of the trajectory, but also on the fact that the applied biomechanical forces depend on the angle at which the force is applied.[1] In the context of throwing, the applied biomechanical forces also depend on the mass of the thrown object and its speed.

Throwing provides an interesting but relatively complicated departure from conditions normally encountered in the classroom. As physics teachers, we are used to simplifying real world problems so that, for example, the force applied to an object is independent of time and it is either independent of mass or is proportional to mass (in a gravitational field). Throwing at maximum speed involves conditions where the applied force increases with mass, but is not directly proportional to mass. The force also varies with time, the object moves in an arc of varying radius, and the force acts in a direction that is neither parallel nor perpendicular to the object path.

The primary physics questions of interest in throwing are how does the applied force vary with mass of the thrown object, and why does it vary with object mass? The purpose of this paper is to provide answers to these questions because they are not available in the teaching or the research literature. There is a suggestion in the physiology literature that heavy objects can be thrown only at low speed because muscles develop large forces only at low contraction speeds. However, it is shown that the main effect involves elementary physics rather than physiology.

For the throwing experiment, the objects chosen varied in mass by a factor of 60, from 57 g (a tennis ball) to 3.4 kg (a lead brick). Each object was thrown at least twice and up to four times by five male subjects at close to maximum possible speed. As expected, all subjects threw the tennis ball faster than the lead brick. A question of interest was whether one can identify a parameter that remains constant, such as the kinetic energy of the object, the total kinetic energy of the object plus the arm, or some other parameter such as the input power. The biomechanics of throwing is obviously important, but not the main consideration. There have been many studies of the biomechanics of throwing,[2–5] but in almost all cases the mass of the object thrown was not varied. An exception is described in Ref. 6 where the mass of the arm was varied by attaching weights.

II. EXPERIMENTAL PROCEDURE

The types of thrown objects are listed in Table I. Each object was thrown overarm by four male students and by the author. The small 200-g cylindrical brass mass was thrown by only two students because it became apparent that it was slightly dangerous to throw this mass at high speed toward a person holding a radar speed gun. None of the students excelled at a sport involving throwing, and none were particularly strong or athletic or frail. Each had their own style of throwing and maintained that style throughout the experiment. For example, Adam consistently leaned backward as his throwing arm swung forward, some stepped and leaned forward when throwing, and David stood with both feet firmly planted on the ground throughout each throw. No one lifted their front leg like a baseball pitcher.

Each thrower was instructed to throw each object in a horizontal direction as fast as he could toward another student located 70 feet away. The second student aimed a radar speed gun toward the thrower and read out the maximum speed for each throw. Each throw was also filmed at 100 frames/s using a JVC9600 digital video camera pointing at right angles toward the throwing arm. The velocity of the thrown object also was measured from the video film as a check on the speed recorded by the radar gun. In some cases

Table I. Mass, m, of each thrown object.

Object	m (kg)
Tennis ball	0.057
Cricket ball	0.16
Brass cylinder	0.20
Bocce ball	0.73
Lead block	1.42
Steel cylinder	2.10
Lead brick	3.40

the object was thrown upward at angles up to about 30° to the horizontal. The radar gun was pointed horizontally and therefore recorded only the horizontal component of the velocity. The throw speeds quoted below refer to the absolute speed, v, determined from the video film rather than the horizontal component, v_x. The heavier masses tended to be launched at higher angles to the horizontal, possibly because the throwers were accustomed to releasing a thrown object at a fixed time rather than at a fixed angle. Alternatively, the throwers may have had difficulty holding onto the heavy objects for a sufficient length of time to release them in a horizontal direction. The radar gun data proved to be redundant, but it provided useful feedback during the experiment because it encouraged the students to try harder to beat their previous throw or to beat the other students.

A simpler procedure would be to record the thrown distance rather than the throw speed, but the distance depends on the launch angle and also on the drag force through the air. For a tennis ball, the drag force reduces the horizontal speed of the ball by about 25% while it is in the air, regardless of the initial ball speed.[7]

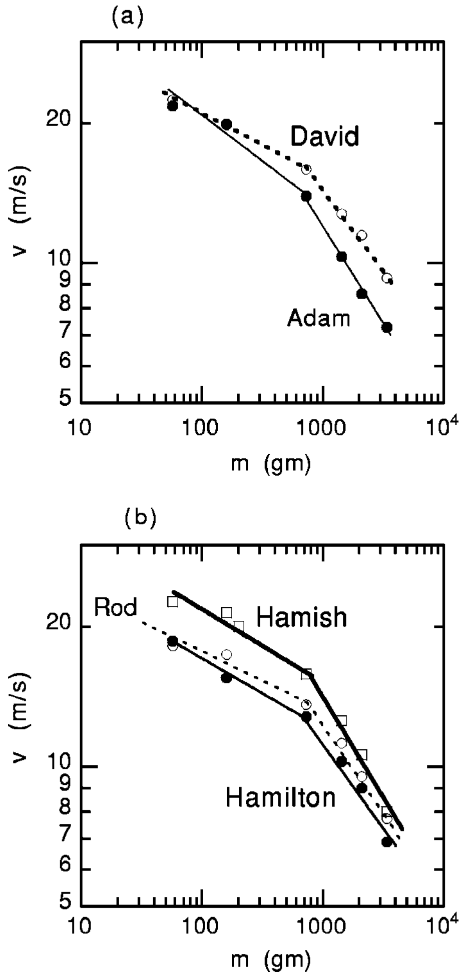

Fig. 1. Throw speed, v, vs thrown mass, m, for five male throwers. Straight line segments are separate fits to the $m<730$ g and $m>730$ g data for each thrower.

III. EXPERIMENTAL RESULTS

The speed of each thrown object as a function of its mass is shown for each subject on a log–log scale in Fig. 1. The speed of each object was taken as the average of the two fastest throws. If only two throws were attempted, the throw speed was taken as the fastest throw. By using a log–log scale, we can determine whether there is a simple power law dependence of the form $v=k/m^n$, where k is a constant, v is the speed, and m is the object mass. For very small values of m, less than say 10 g, we would expect that $n=0$ because the throwing speed will be limited only by the speed at which the thrower can swing the arm. At higher values of m, the data could be fit using two straight line segments, with $n \approx 0.15$ in the range 50 g$<m<$730 g, and with n about 0.4 when $m>730$ g. The fits are shown in Fig. 1 and the corresponding values of n are given in Table II.

If the kinetic energy of each mass remained constant for each thrower, n would be 0.5. In practice $n<0.5$, a result that indicates that the mass of the arm is an important factor limiting the speed of a thrown object. If the maximum power input were constant,[7,8] then n would be 0.33. Even though Fig. 1 appears to show that there are two distinct regions for n, this distinction is a somewhat artificial result due to the small number of masses thrown. We could equally well fit a smooth curve to the data with n changing continuously, as described below.

IV. THEORETICAL MODEL

In principle, we could analyze the action of throwing by modeling each body segment as a series of connected rods, each with its own mass and moment of inertia. A measurement of the rotation speed of each segment would determine the torque acting on each segment. However, we will first consider a much simpler model in which all segments are replaced by a single "arm" of mass M, length L, and moment of inertia I. An object of mass m is located at one end of the arm and the other end is pivoted at the shoulder (or some other fixed point).

We assume for simplicity that the pivot point remains at rest. We also assume that the arm is rotated through a fixed angle θ by a torque τ before the mass m is released. The work, W, done by the applied torque is then

$$W=\int \tau \, d\theta=\frac{1}{2}mv^2+\frac{1}{2}I\omega^2, \tag{1}$$

Table II. Values of the exponent n in the relation $v=k/m^n$.

m range (g)	Adam	David	Hamilton	Hamish	Rod
50–730	0.20	0.14	0.14	0.16	0.12
>730	0.42	0.33	0.37	0.41	0.36

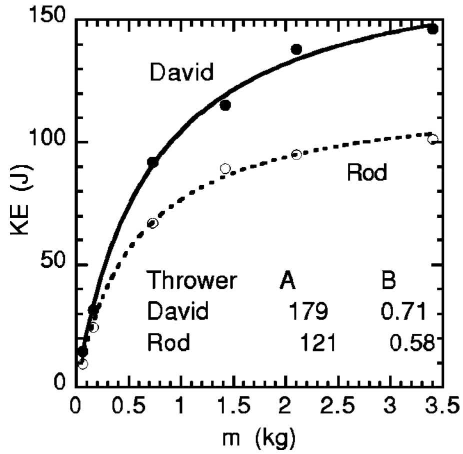

Fig. 2. Measured KE of each thrown mass vs m for David and Rod, with best fit curves defined by the relation $KE=Am/(m+B)$.

where $v=L\omega$ is the release speed of the mass m and ω is the angular velocity of the arm at the instant of release. If we further assume that the arm has a uniform mass distribution, then $I=ML^2/3$, and hence

$$W=\frac{1}{2}mv^2\left(1+\frac{M}{3m}\right), \tag{2}$$

so

$$KE=\frac{1}{2}mv^2=\frac{Wm}{(m+M/3)}. \tag{3}$$

The total available energy is shared between the arm and the mass m. The energy given to the mass m increases as m increases, provided that the torque applied to the arm (and hence the work done) does not decrease significantly as m increases.

The measured variation of the kinetic energy with m for two of the five throwers is shown in Fig. 2. Also shown are two-parameter fits to the data assuming that KE has the form $KE=Am/(m+B)$, where A and B are adjustable parameters. According to Eq. (3), we can identify A as a measure of the work done, and B is nominally one-third the mass of the single-segment arm. The fit was quite good for all throwers, indicating that the work done in throwing an object is essentially independent of the mass of the thrown object. Equation (3) also can be used to obtain good fits to the v vs m data in Fig. 1, and the result is that v and the exponent n both vary smoothly with m.

The small scatter in the data and the relative insensitivity of the fitted curve to B did not allow for a measurement of B to better than about $\pm 30\%$, but A could be determined to within about $\pm 10\%$. The actual arm mass for each thrower can be estimated as a simple proportion of total body mass. According to published data[2] the average mass of the upper arm, forearm, and hand for males is, respectively, 3.25%, 1.87%, and 0.65% of total body mass. Forearm plus hand mass is 2.52% of total body mass. Table III summarizes these data for the five throwers, together with the value of $M=3B$ obtained from the curve fits.

The values for $3B$ listed in Table II are less than half the mass of the whole arm and are approximately equal to the mass of the forearm plus the hand for each thrower. An exception is the result obtained for Adam. He had an unusual throwing action where the hand was kept close to the shoulder throughout most of each throw, resulting in a relatively low moment of inertia about the shoulder axis.

Table III. Parameters deduced from the total body mass (BM) and the kinetic energy data.

Quantity	Adam	David	Hamilton	Hamish	Rod
BM (kg)	78	82	62	74	81
0.025 BM	1.95	2.05	1.55	1.85	2.02
$3B$ (kg)	0.90	2.13	1.53	1.50	1.74
A (J)	94	179	99	149	121

The data in Table III indicate that the mass of the upper arm is not a significant factor in determining the speed of the thrown objects, and that the speed is limited only by the mass of the hand and the forearm. This result was unexpected and suggests that the single segment model of the throwing arm may not be realistic. However, there is a simpler explanation. That is, the single segment model is realistic, and the upper arm does not play a significant role in limiting the speed of a thrown object. The angular velocity of the upper arm decreases substantially during the latter part of the throw, while the angular velocity of the forearm increases rapidly. The angular displacement of the two arm segments is shown as a function of time in Fig. 3 for two of David's throws. The apparent shortening of the arm segments in Fig. 3 indicates that the segments are inclined toward or away from the camera. The maximum acceleration of the thrown mass, and hence the maximum force on this mass, occurs during a time when the upper arm is almost at rest. Conversely, the acceleration of the thrown mass is relatively small while the upper arm is rotating at maximum angular

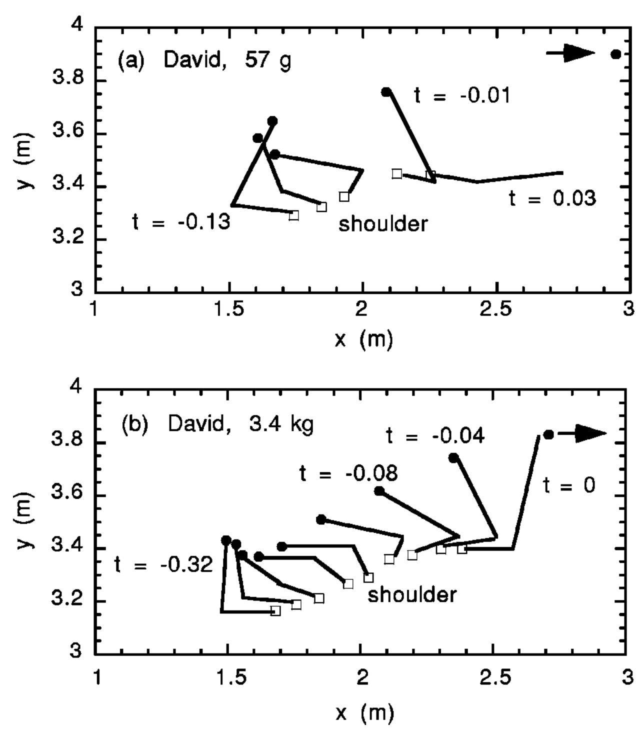

Fig. 3. Positions of the shoulder, upper arm, forearm, and thrown mass at 0.04-s time intervals. The (x,y) origin is a point at the bottom left corner of the video film. Time $t=0$ corresponds to the instant at which the mass was released.

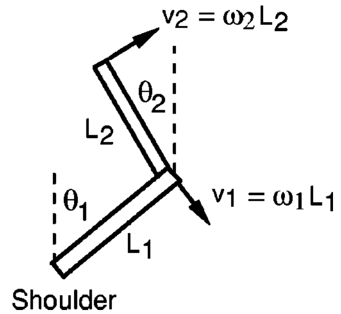

Fig. 4. Two segment model of throwing arm.

velocity. Consequently, the inertial properties of the upper arm do not play a major role in determining the ball speed. This interpretation is supported by an interesting result obtained by Southard,[5] who found that the speed of a thrown ball increased slightly when mass was added to the upper arm.

The mass of the upper arm is more likely to be a significant factor in the game of cricket, where the ball is bowled rather than thrown. In that case the ball speed is limited by the fact that the arm must be kept straight, without bending the elbow. This constraint has the effect of increasing the moment of inertia of the whole arm (about the shoulder axis) and it also prevents any torque being applied to the forearm. To compensate for the reduced ball speed, the bowler is allowed to run at high speed toward the batter before releasing the ball.

V. TWO SEGMENT MODEL

Further insights into the physics of throwing can be obtained by considering a two segment model of the throwing arm, as shown in Fig. 4. Suppose that the forearm plus hand have mass M_2, length L_2, and the upper arm has mass M_1 and length L_1. We assume that both segments have a uniform mass distribution, the upper arm is pivoted at the shoulder (which remains fixed in space), the upper arm rotates at angular velocity ω_1, and the forearm rotates at angular velocity ω_2. The elbow rotates at speed $v_1=\omega_1 L_1$ with respect to the shoulder, and the hand rotates at speed $v_2=\omega_2 L_2$ with respect to the elbow. We assume for simplicity that both segments rotate in the same vertical plane, although each segment tends to rotate in a different plane in practice. Suppose that an object in the hand is released at a time when the upper arm is inclined at an angle θ_1 to the vertical, and the forearm is inclined at an angle θ_2 to the vertical. The release speed, v, is given by

$$v^2=v_1^2+v_2^2+2v_1v_2\cos(\theta_1+\theta_2), \tag{4}$$

and the total kinetic energy, KE, of the two arm segments is given by

$$\text{KE}=\frac{v_1^2}{2}\left(M_2+\frac{M_1}{3}\right)+\frac{1}{6}M_2v_2^2+\frac{1}{2}M_2v_1v_2\cos(\theta_1+\theta_2). \tag{5}$$

The most efficient throwing action results when KE is a minimum for a given value of v. If we express KE as a function of v and v_1, then it is easy to show that KE is a minimum when $v_1=0$, in which case KE$=M_2v_2^2/6$, regardless of the value of $\theta_1+\theta_2$. In hindsight, this result is obvious. If an object is thrown at maximum speed, then any motion of the upper arm would result in a loss of energy that could otherwise be used to propel the object. Such a result implies that kinetic energy and angular momentum is transferred from the upper arm to the forearm during the throw in such a way that the kinetic energy and the angular momentum of the upper arm is reduced to zero. A transfer of energy and momentum from one body segment to a more distal segment is known in the biomechanics literature as the kinetic link principle, whereby each body segment involved in any rapid movement transfers energy to the next in a sequential manner.[2–5] If the speed of the upper arm is zero when a mass is thrown, then the speed of the thrown mass is independent of the mass of the upper arm, as found in our experiment.

An essential and surprising feature of the sequential link between body segments, commonly overlooked in the biomechanics literature, is that more work is done by each segment than if it acted alone. For example, suppose that wrist action alone can propel a 140-g baseball at a speed of 3 m/s. The work done by the wrist is then 0.63 J. Such a result could be obtained if the wrist exerted a constant force of say 6.3 N acting over a distance of 0.1 m. A final flick of the wrist could therefore increase the throw speed from say 20 to 23 m/s, resulting in an increase in ball energy of 9 J. More work is done on the ball because the same 6.3-N force acts over a larger distance for the same time. The additional energy is extracted from the forearm which decelerates because an equal and opposite force of 6.3 N is applied to the forearm. It can be inferred from this example that maximum energy is transferred to a thrown object if the thrower activates muscles in the correct sequence (for example, legs, hip, shoulder, elbow, and wrist) with appropriate time delays between each group of muscles, rather than activating all muscles simultaneously. An extension of this concept is provided by a spear thrower[9] or a club or bat or racquet, each of which acts as an additional link in the chain and allows an object to be thrown or hit at a greater speed than by means of the hand alone.

VI. FORCE MEASUREMENTS

Measurements of the instantaneous forces applied to throw an object were made by analyzing the video film of each throw. Plots of the x and y coordinates of a thrown object were used to obtain the v_x and v_y velocity components, and the latter components were used to obtain the a_x and a_y acceleration components. Small errors in the (x,y) coordinates can lead to large errors in the (a_x,a_y) components, unless particular care is taken with the data analysis procedure. A positional error of only 2 or 3 mm in y can easily produce a result where the vertical acceleration due to gravity is a factor of 2 larger or smaller than the accepted value. In the present case, this type of error was minimized by obtaining the velocity and acceleration components in two separate stages, prior to release ($t<0$) and after release ($t>0$), and by matching all components at $t=0$. Best fits were made to the data for the following polynomial functions, after the x, y, and t origins were shifted so that $x=y=0$ at $t=0$:[10]

$$x=\begin{cases} v_{x0}t+b_2t^2+b_3t^3+b_4t^4+b_5t^5+b_6t^6 & (t<0) \\ v_{x0}t & (t>0) \end{cases}, \quad (6)$$

$$y=\begin{cases} v_{y0}t+c_2t^2+c_3t^3+c_4t^4+c_5t^5+c_6t^6 & (t<0) \\ v_{y0}t-4.9t^2 & (g>0) \end{cases}. \quad (7)$$

The b and c terms are coefficients determined by the curve fitting procedure, and v_{x0} and v_{y0} are the velocity components of the thrown mass at $t=0$. The second derivatives of x and y were matched by setting $b_2=0$ and $c_2=-4.9$ so that $a_x=0$ and $a_y=-9.8\ \mathrm{m/s^2}$ at $t=0$. Using this procedure, it was possible to obtain good fits to the data, with regression coefficients greater than 0.999 in all cases.

Throwing involves accelerated motion under conditions where the applied force is neither parallel nor perpendicular to the direction of motion. Relatively large forces can be generated in a direction perpendicular to the path of a thrown object, but they do not act to increase the speed of the object or to increase its kinetic energy. The force acting in a direction parallel to the path of the object is given by $m dv/dt$. For the largest masses, the parallel force includes a significant component due to gravity. The F_x and F_y components of the force applied by the thrower were therefore determined from the relations $F_x=ma_x$ and $F_y-mg=ma_y$. The parallel or tangential component $F_T=F\cos\phi$ was then determined from the angle ϕ between the $\mathbf{F}$ and $\mathbf{v}$ vectors.

Typical results of the F_T calculations are given in Fig. 5 for two of David's throws. F_T increases to a maximum well before the instant of release and decreases to zero at release. F_T is a maximum at a time when the forearm is horizontal and commencing its rapid forward rotation. The object is released at a time when the forearm is approximately vertical and rotating at maximum speed. The peak value of F_T acting on the 57-g ball was 25 N (45 times its weight) and the peak value of F_T on the 3.4-kg mass was 195 N (5.8 times its weight). There is a factor of 60 difference in the thrown mass, a factor of 2.3 difference in the throw speed (19.5 vs 8.5 m/s in Fig. 5), a factor of 7.8 difference in the peak force, and a factor of 11.3 difference in the kinetic energy of the thrown mass.

A simple estimate of the tangential force applied to each object also was obtained from measurements of the speed at release, v, and the time over which each object was accelerated. The throw duration, T, was taken as the time interval between release (forearm approximately vertical) and the time at which the forearm was horizontal and commencing its rapid forward rotation. The average tangential force applied during the time T was calculated from the relation $\bar{F}=mv/T$. The values of $\bar{F}$ obtained by this method were essentially the same as the peak F_T values obtained by the first method. The latter result was coincidental, but can be attributed partly to the fact that the assumed throw duration was less than the actual duration, and partly because the change in speed during the time T was less than v. At the nominal start of the time interval T, the thrown object was moving forward at about one-third of its final speed and the tangential force was at its maximum value. Prior to this time there was no clearly identifiable start of the throw period because the thrown object was almost stationary for a significant time interval while the elbow moved rapidly forward.

Figure 6 shows the force, $\bar{F}$, and throw duration, T, as a function of the thrown mass, averaged over three throws for each mass thrown by David. Similar results were obtained for all throwers, with force and throw duration both increasing with mass. Because each thrown mass was accelerated

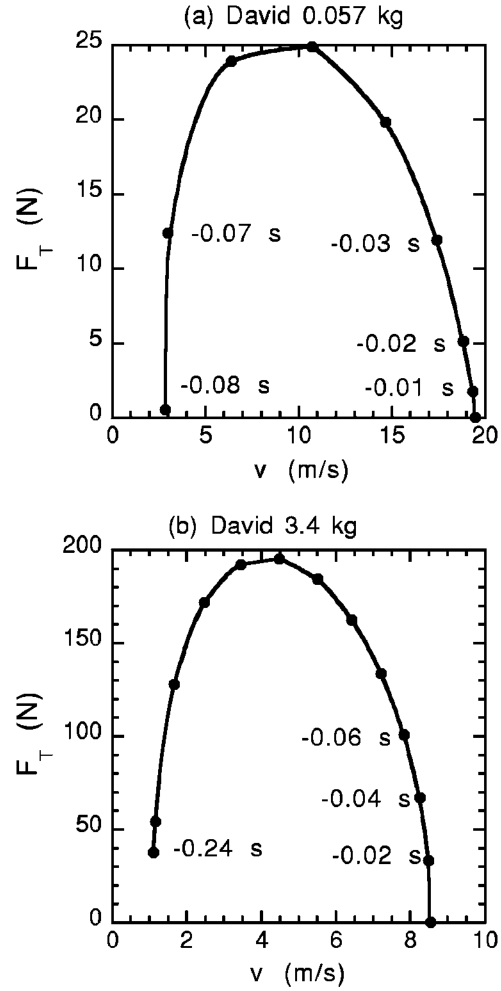

Fig. 5. Tangential force, F_T, acting on the lightest and heaviest masses as a function of the instantaneous speed during two of David's throws, at fixed time intervals prior to release.

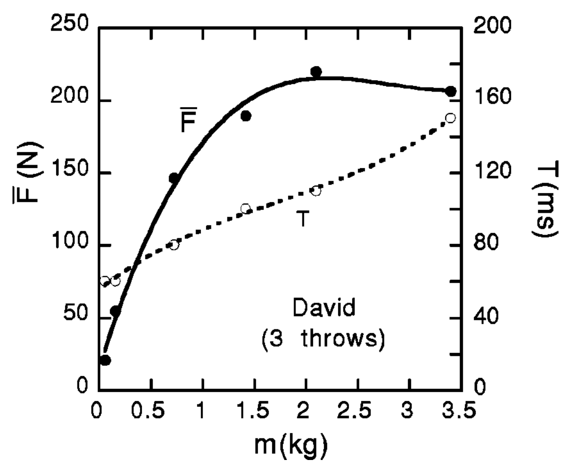

Fig. 6. Average force, $\bar{F}$, and throw duration, T, as a function of the thrown mass, averaged over three throws for each mass thrown by David.

over approximately the same distance, the kinetic energy of each mass increased approximately in proportion to the applied force.

VII. FORCE–VELOCITY RELATIONS

While researching this topic, I came across several articles in the physiology and sports science literature[11–13] that made the same surprising claim that heavy objects can be lifted or thrown only at low speed while light objects can be lifted or thrown more rapidly because muscles develop larger forces when contracting slowly and smaller forces when contracting rapidly. The latter effect is known as the force–velocity relation for muscles. My surprise was that this relation was apparently being used to account for the inertia of an object. Most physicists would probably assume that heavy objects can be lifted only at low speed because they are heavy. For any given applied force, light objects will accelerate faster than heavy objects. However, when lifting or throwing an object at maximum speed, the force acting on the object increases as the mass of the object increases or as the speed of the object decreases. If the force increased in direct proportion to the mass, and if each object were lifted or thrown through the same distance, then heavy objects could be lifted or thrown at the same speed as light objects. In fact, the force increases at a lower rate than the mass, which is the real reason that light objects can be lifted or thrown faster.

The results in Fig. 5 appear to contradict the force law for muscles. At the beginning of the throw period, the applied force increases as the speed of the object increases. It is only toward the end of the throw period that the applied force decreases as the speed of the object increases. An object is thrown not by a single muscle but by several different muscle groups acting in sequence. At the beginning of the throw period in Fig. 5, the object is accelerated primarily by rapid rotation of the upper arm while the forearm rotates at a lower speed. The muscles in the shoulder acting on the upper arm propel the upper arm and elbow forward, causing the forearm and the object in the hand to move forward. Presumably, the force on the upper arm decreases as the elbow accelerates, with the result that the force on the forearm and hence the force on the thrown object also will decrease as the elbow accelerates (at a decreasing rate with time). At least that would be the result if the forearm were completely passive and pulled along only by the fact that it is attached to the elbow. If the muscles acting on the forearm are activated soon after the upper arm muscles are activated, then there will be an increase rather than a decrease in the force acting on the thrown object. The initial increase in force shown in Fig. 5 presumably corresponds to increased activation of the muscles attached to the forearm. Once the muscles are fully activated, the applied force subsequently decreases as the object accelerates.

An apparent force–velocity relation for muscles results if one plots the maximum tangential force, F_T, or the calculated value of $\bar{F}$, as a function of the speed of each thrown object. Such a plot is shown in Fig. 7 for David's throws. Figure 7 shows that the maximum force that can be applied to an object decreases as the speed of the object increases. However, the plot in Fig. 7 does not provide a valid test of the force–velocity relation for muscles and gives rise to an apparent inconsistency between the results in Figs. 5(a) and 5(b). Figure 5(a) shows that the maximum force exerted on an object when throwing a tennis ball at a speed of about 6 m/s is about 25 N, whereas Fig. 5(b) shows that a force of about 180 N can be exerted on the lead brick when it is moving at 6 m/s.

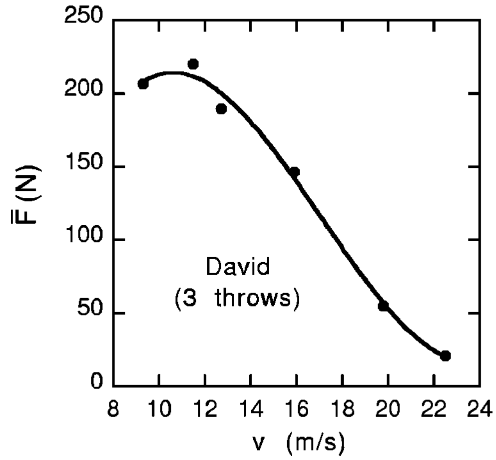

Fig. 7. $\bar{F}$ as a function of the speed of each thrown object averaged over three throws for each mass thrown by David.

The force exerted by the muscles is not the same as the force exerted on the object. Consider the situation shown in Fig. 8 where a muscle force F_m is exerted on the forearm and where F_m exerts a torque $\tau = F_m d = I d\omega/dt$ about an axis through the elbow; I is the combined moment of inertia of the forearm, the hand, and the mass m located in the hand. If the object–elbow distance is L, then the mass will move at speed $v = L\omega$ with respect to the elbow. If we assume for simplicity that the elbow remains at rest and the mass m moves at speed v, then the force acting on m is given by

$$F = m\frac{dv}{dt} = mL\frac{d\omega}{dt}, \tag{8}$$

and hence

$$F = (mLd/I)F_m. \tag{9}$$

The force exerted on the object must therefore be divided by mLd/I to estimate the muscle force. If m were much larger than the mass of the forearm, then $I = mL^2$ and $F = dF_m/L$ would be directly proportional to F_m, F would be independent of m and would be significantly less than F_m. In the present experiment, m was either much less than the mass of the forearm or comparable to the mass of the forearm. When

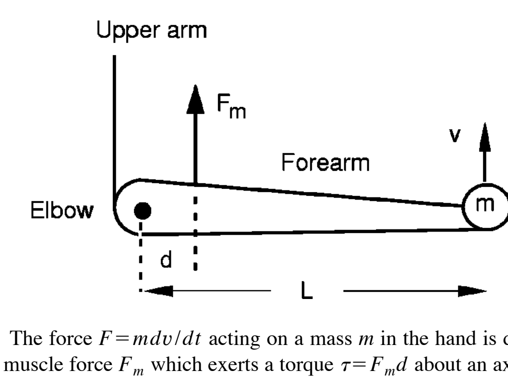

Fig. 8. The force $F = mdv/dt$ acting on a mass m in the hand is determined by the muscle force F_m which exerts a torque $\tau = F_m d$ about an axis through the elbow.

m is small, the rotation speed of the forearm is limited only by its moment of inertia, in which case the force applied to m is directly proportional to m and to the force exerted by the muscles.

VIII. THROW WITH FOREARM ONLY

The muscle forces exerted when throwing an object are difficult to determine from kinematic data due to the large number of muscles and linkages involved. Even if one were to measure the net force and torque acting on each segment, that information would be sufficient to determine only the magnitude and the line of action of the net force acting on each segment. Given that the upper arm exerts a force on the forearm through the elbow joint and several different muscle groups exert a torque on the forearm, there is no unique solution for the individual forces involved.[14] Furthermore, most limb muscles run over two or more joints so the contraction of any one muscle affects more than one segment. The problem is further compounded by the fact that single muscles rarely act alone and by the fact that motion of any one segment affects other segments linked to it. A separate experiment to determine the muscle force on the forearm was therefore conducted, by throwing a tennis ball and a bocce ball with the hand and forearm only. The upper arm was held at rest in a horizontal position on a table and each ball was thrown as fast as possible in a horizontal direction by the author. A video was used to determine the throw speed and the throw duration T. The forearm was initially at rest and inclined at an angle $\theta = 20°$ to the horizontal. The throw duration was taken as the time to accelerate the forearm from $\theta = 25°$ to the ball release ($\theta \sim 90°$) position.

The tennis ball was accelerated to 9.5 m/s in 0.20 s, and the bocce ball was accelerated to 6.3 m/s in 0.25 s (averaged over five throws in both cases). The throw speeds were each about half the throw speeds shown in Fig. 1. The average tangential force on the tennis ball, mv/T, was 2.71 N and the average tangential force on the bocce ball was 18.4 N. The moment of inertia of the hand and forearm alone was approximately $0.077\ \mathrm{kg\,m^2}$. The elbow–ball distance was $L = 0.37$ m, and hence $I = 0.085\ \mathrm{kg\,m^2}$ with a tennis ball in the hand, and $I = 0.177\ \mathrm{kg\,m^2}$ with the bocce ball in the hand. If we use Eq. (9) and the assumed value of $d = 5$ cm, we find that the average muscle force was 218 N when throwing the tennis ball, and 241 N when throwing the bocce ball. The average muscle force was only slightly larger when throwing the bocce ball, despite the fact that the tangential force on the bocce ball was 6.8 times larger than the tangential force on the tennis ball. The additional tangential force on the heavier ball was therefore due primarily to its inertia, rather than the increased muscle force arising from the lower contraction speed. The total kinetic energy of the forearm and the ball was similar in each case (16.5 and 21.0 J for the tennis and bocce balls, respectively), indicating that a similar amount of work was done by the muscles.

The angular position and acceleration of the forearm when throwing the tennis ball is shown in Fig. 9. The applied torque $\tau = I d\omega/dt$ is not maximized when the muscles acting on the forearm are first activated, but is a maximum just before the ball is released. One can infer from this result that the muscles acting on the forearm take about 0.15 s to become fully activated, as assumed in Sec. VII.

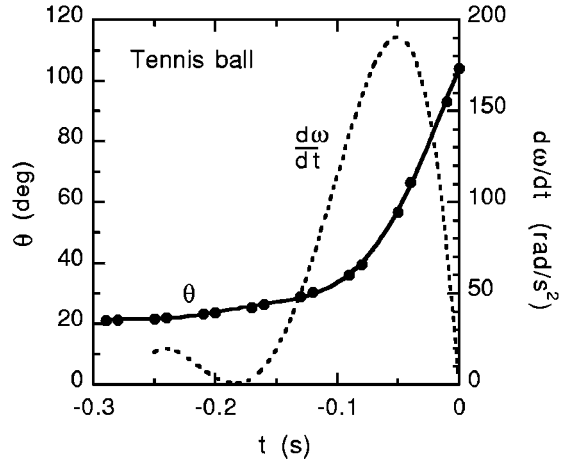

Fig. 9. Angular displacement, θ, and acceleration, $\alpha = d\omega/dt$, of the forearm vs time when only the forearm is used to throw a tennis ball. The solid curve is sixth-order polynomial fitted to the experimental data (closed dots) to calculate the acceleration, assuming that $\alpha = 0$ at $t = 0$ (the time at which the ball was released).

IX. THROWING VERSUS SWINGING

The original motivation for this study was related to a problem in ball sports such as baseball, golf, cricket, or tennis, where the implement (bat, club, or racquet) is swung rather than thrown. Should players choose to use a light or a heavy implement in order to impart maximum speed to the ball? A light implement can be swung faster, but a heavy implement is more effective in transferring energy into the ball. By "effective" we mean that if two implements of different mass are swung at the same speed toward the ball, the ball speed will be greater when it is struck by the heavier implement. Even though the heavier implement is more effective, it is less efficient because it retains a larger fraction of its initial energy after the collision. When swing speed and energy transfer are considered together, it is not obvious how the resulting ball speed will depend on the mass of the implement. Some insights can be gained by considering the physics of the problem,[7,8] but the relation between swing speed and implement mass is not well known. If the total kinetic energy of the implement and the arms remains constant, as in the present experiment, then a small increase in ball speed can be expected as the mass of the implement is increased. This increase needs to balanced by the fact that heavy implements are less maneuverable, so the choice for a player is one of optimizing the power of the implement and the control that the player can exert in directing the implement toward its intended target at the desired speed and angle of approach.

A number of studies[13,15–17] have been undertaken to determine the relation between swing speed and the mass of the implement that is swung. However, in none of these studies has a clear picture emerged regarding the factors that determine the swing speed. The problem is complicated by the fact that the swing speed of an extended object such as a bat or a racquet depends on both its mass and its moment of inertia. The present experiment on throwing was devised to eliminate one of these variables.

X. CONCLUSIONS

When an object of mass m is thrown by hand, the tangential force acting on the object is given by $F=ma$, where a is its tangential acceleration. If m is much smaller than the mass of the hand, then a will be independent of m and will be limited only by the moment of inertia of the hand and forearm and by the forces that can be developed by the muscles involved in throwing. Under these conditions, F will be directly proportional to m. If m is comparable to or larger than the mass of the hand, then a will decrease as m increases due to the increased inertia at the end of the forearm. If one assumes that the torque applied to the various arm segments is unaffected by an increase in m, then one can use a simple one or two segment model of the arm to estimate the change in F as a function of m. Experimentally, it was found that the muscle torque acting on the forearm was almost independent of the thrown mass, and that the total kinetic energy of the hand, forearm, and mass in the hand was essentially independent of the mass of the thrown object. Throwing can therefore be regarded for physics teaching purposes as a relatively straightforward problem in undergraduate mechanics, without being too concerned about subtle or unfamiliar biomechanical or physiological effects.

[a]Electronic mail: cross@physics.usyd.edu.aw

[1]N. P. Linthorne, "Optimum release angle in the shot put," J. Sports Sci. **19**, 359–372 (2001).
[2]E. Kreighbaum and K. M. Barthels, *Biomechanics* (Allyn and Bacon, Boston, 1996), 4th ed., p. 578.
[3]C. A. Putnam, "Sequential motions of body segments in striking and throwing skills: Descriptions and explanations," J. Biomech. **26**, Suppl 1, 125–135 (1993).
[4]R. McN. Alexander, "Optimum timing of muscle activation for simple models of throwing," J. Theor. Biol. **150**, 349–372 (1991).
[5]H. J. J. Joris, A. J. Edwards Van Muyen, G. J. Van Ingen Schenau, and H. C.-G. Kemper, "Force, velocity and energy flow during the overarm throw in female handball players," J. Biomech. **18**, 409–414 (1985).
[6]D. Southard, "Mass and velocity: Control parameters for throwing patterns," Res. Q. Exerc Sport **69**, 355–367 (1998).
[7]H. Brody, R. Cross, and C. Lindsey, *The Physics and Technology of Tennis* (RacquetTech, Solana Beach, CA, 2002), pp. 163–165. Available from ⟨www.racquettech.com⟩.
[8]A. M. Nathan, "Characterizing the performance of baseball bats," Am. J. Phys. **71**, 134–143 (2003).
[9]R. Baugh, "Dynamics of spear throwing," Am. J. Phys. **71**, 345–350 (2003).
[10]We used Kaleidagraph, Synergy Software, ⟨www.synergy.com⟩.
[11]B. R. MacIntosh and R. J. Holash, "Power output and force-velocity properties of muscle," in *Biomechanics and Biology of Movement* (Human Kinetics, Champaign, IL, 2000), pp. 193–210.
[12]R. F. Schmidt and G. Thews, *Human Physiology* (Springer-Verlag, New York, 1989), 2nd ed., p. 76.
[13]G. Watts and A. T. Bahill, *Keep Your Eye on the Ball: The Science and Folklore of Baseball* (Freeman, New York, 1990).
[14]R. Bartlett, *Sports Biomechanics: Reducing Injury and Improving Performance* (E and FN Spon, London, 1999), pp. 109–145.
[15]C. B. Daish, *The Physics of Ball Games* (English Universities Press, London, 1972).
[16]S. R. Mitchell, R. Jones, and M. King, "Head speed vs. racket inertia in the tennis serve," Sports Eng. **3**, 99–110 (2000).
[17]G. S. Fleisig, N. Zheng, D. F. Stodden, and J. R. Andrews, "Relationship between bat mass properties and bat velocity," Sports Eng. **5**, 1–14 (2000).