Increase in friction force with sliding speed

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A block sliding down an inclined plane normally accelerates. However, if the friction force increases with speed, then the block can slide at a constant terminal speed in a manner similar to the fall of an object through a fluid. Measurements of the increase in the coefficient of friction for tennis ball cloth sliding on a smooth surface are described over speeds varying by a factor of 9000. For the low speed measurements, the ball cloth was attached to the bottom of a weighted box and pulled along a horizontal surface by a constant horizontal force. Results at higher speeds were obtained by bouncing a tennis ball off the surface.
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I. INTRODUCTION

The coefficient of sliding friction (COF) between two surfaces is commonly assumed to be independent of surface area, normal reaction force, and sliding speed, as originally determined by da Vinci, Amonton, and Coulomb.[1,2] Although these approximations may sometimes be appropriate for metals and other elastically hard materials, they are not generally applicable for elastically soft materials such as rubber or textiles.[3,4] Dry, unlubricated metals also have a velocity-dependent value of the COF at low sliding speeds, as evidenced by slip-stick phenomena.[3–6] Nevertheless, departures from Amontons' or Coulomb's original laws of friction are still regarded as unusual. An interesting observation of such a departure[7] is examined in more detail here. The experiment is easy to set up and may be already done in some undergraduate laboratories. The apparatus consists of a mass M_1 that slides horizontally and a second mass M_2 attached to the first by a string over a pulley. In this way, a known horizontal force $M_2 g$ is applied to M_1. The COF between M_1 and the surface on which it slides is determined from a measurement of the acceleration of M_1.

A surprising result is obtained with smooth surfaces if cloth or rubber is attached underneath the sliding mass. For relatively small values of M_2, M_1 slides at constant speed after a brief initial acceleration. If M_2 is increased, then M_1 slides at a higher constant speed. The explanation is that for any given value of M_2, the COF increases with sliding speed until the net horizontal force on M_1 is zero. Not all materials exhibit such behavior, but it is common with easily deformable materials, even for paper sliding on paper.[6] The effect is examined in this paper for over four orders of magnitude of sliding speed for tennis ball cloth sliding on a smooth table and on a rougher surface. Other materials and surfaces also were tested in an attempt to understand why the COF might increase with sliding speed. Stick-slip phenomena are associated with a decrease in the COF as the sliding speed increases. The transition from static to sliding friction also can be regarded as a case where the COF decreases as sliding speed increases.

Measurements of the COF at high sliding speeds were obtained by bouncing a tennis ball obliquely off a surface. The experiments were conducted in part to extend the range of sliding speeds, but also to determine whether the COF for a short impact is the same as that found when sliding is maintained over longer time intervals.

II. EXPERIMENTAL PROCEDURES

A. Measurements of COF at low sliding speeds

For sliding speeds from about 0.002 to about 1 m/s, the COF between tennis ball cloth and two different surfaces was measured using the apparatus shown in Fig. 1. This cloth is not much different from other cloth in terms of its frictional properties, but was chosen because it is strongly resistant to abrasion and so that measurements could be made at higher sliding speeds by bouncing a tennis ball off each surface. For the low speed measurements, an aluminum die-cast box was used to support various masses on top of a rectangular piece of ball cloth glued to the bottom of the box. The cloth was 110 mm long in the sliding direction, 57 mm wide, and 3 mm thick.

The mass of the box and cloth was 121 g when the box was empty. Masses up to 2.1 kg could be placed inside the box, and measurements also were made with a 3.4 kg lead brick resting on top of the box. The box was pulled horizontally by means of a string attached to the box. The string passed over a 12 g pulley and was hooked at the bottom end to a 10 g plastic bucket used to support various masses. Alternatively, large masses could be attached directly to the hook. The pulley spun freely under light or heavy loads and offered negligible frictional resistance on its own. An unbalanced load of only 0.4 g on either side of the pulley was enough to generate rotation of the pulley.

For each surface, the mass M_1 was pulled over a horizontal distance of about 40 cm before coming to rest, either when the mass M_2 hit soft padding on the floor or when the mass M_1 impacted soft padding located near the pulley. The apparatus was mounted on a laminated desk top, which provided a suitably hard and smooth surface on which to conduct low friction measurements. Alternatively, other surfaces such as paper or emery paper could be taped firmly to the desk top. Speed and acceleration measurements were made by filming the motion of M_1 at 25 frames/s with a digital video camera and transferring short clips to a computer for subsequent analysis. Very low sliding speeds were measured more conveniently with a ruler and stopwatch. The desk top was cleaned and dried regularly to ensure that no dust or finger marks influenced the results. As a result, constant sliding speeds could be maintained for periods up to several minutes at the lowest sliding speeds.

The equations describing motion of M_1 and M_2 are given by

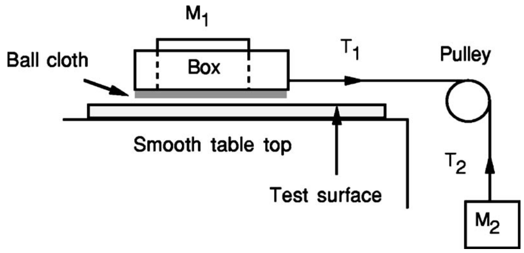

Fig. 1. Arrangement used to measure the coefficient of friction (COF) at low sliding speeds.

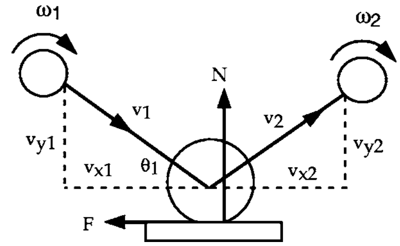

Fig. 2. Oblique impact of a ball on a horizontal surface. F is the friction force and N is the normal reaction force. N does not necessarily act through the center of the ball, and F is not necessarily equal to μN.

$$T_1 - \mu M_1 g = M_1 a, \tag{1}$$

$$M_2 g - T_2 = M_2 a, \tag{2}$$

where T_1 and T_2 are the string tensions, a is the acceleration of M_1 and M_2, and μ is the COF. If the pulley has a radius R and a moment of inertia I_0 about an axis through the center of the pulley, the net torque on the pulley is given by

$$T_2 R - T_1 R = (I_0/R)a, \tag{3}$$

where we have ignored any frictional torque on the pulley axle. For a pulley of mass M_p, $I_0 = M_p R^2/2$ to a good approximation. A more precise estimate of I_0 was not warranted in this experiment because M_p was much smaller than $M_1 + M_2$. The equation of motion is therefore given by

$$(M_1 + M_2 + M_p/2)a = (M_2 - \mu M_1)g. \tag{4}$$

The COF can be determined by measuring a.

Alternatively, μ can be measured by finding the value of M_2 that allows the masses to move at constant speed, in which case $\mu = M_2/M_1$. For sufficiently small values of M_2, there is no sliding at all because the horizontal force is too small to overcome the static friction force. For large values of M_2, both masses accelerate rapidly as soon as the load is applied. For intermediate values of M_2, the masses moved at a constant low speed after accelerating for a short period. The constant speed condition $\mu = M_2/M_1$ could be maintained up to a maximum speed of about 0.1 m/s. For larger values of M_2, the increase in μ was insufficient to satisfy the condition $\mu = M_2/M_1$, in which case M_1 continued to accelerate until it reached the end of its travel. The implication is that μ increases with speed, but is not proportional to the speed or to the ratio M_2/M_1.

B. Measurements of COF at high sliding speeds

Consider a ball incident at speed v_1 and angle θ_1 on a horizontal surface, as shown in Fig. 2. The ball rebounds at speed v_2 and angle θ_2. If the ball slides throughout the bounce, the horizontal (x) and vertical (y) components of the ball speed are given by $e = v_{y2}/v_{y1}$ and[8]

$$v_{x2}/v_{x1} = 1 - \mu(1+e)\tan\theta_1, \tag{5}$$

where e is the coefficient of restitution and $\mu = F/N$, which is assumed to remain constant throughout the bounce. In fact μ is likely to vary during the bounce in which case a measurement of the relevant speeds and angles yields a time-averaged value of μ.

A reliable measurement of μ is possible only if θ_1 is relatively small, typically less than 30°. At larger values of θ_1 the ball will stop sliding during the bounce and grip the surface, in which case a measurement of μ based on Eq. (5) will underestimate the true COF because the friction force reverses direction during the grip phase. The critical angle at which the ball stops sliding depends on μ and on the spin of the incident ball. If the ball is incident without spin, the critical angle is given by[8]

$$\tan\theta_1 = \frac{1}{\mu(1+e)(1+1/\alpha)}. \tag{6}$$

The parameter, α, is defined by the relation $I = \alpha m R^2$, where I is the moment of inertia of a ball of mass m and outer radius R; $\alpha = \frac{2}{5}$ for a solid sphere, $\frac{2}{3}$ for a thin spherical shell, and $\alpha = 0.55$ for a tennis ball. Because e is ≈ 0.75 for all angles of incidence, the critical angle for a tennis ball is given by $\tan\theta_1 = 0.203/\mu$. For example, if $\mu \approx 0.75$ and if the ball is incident without spin, then θ_1 needs to be less than about 15° to prevent the ball from gripping the surface. A simple indication of whether the ball grips the surface, based on the measured ball spin, is that $R\omega_2 < v_{x2}$ for pure sliding, but $R\omega_2 > v_{x2}$ if the ball grips.[8]

III. EXPERIMENTAL RESULTS

A. Low sliding speeds

Measurements of μ made by sliding ball cloth on the smooth, laminated desk top are shown in Fig. 3. All results in Fig. 3 were obtained using the apparatus shown in Fig. 1. The COF increases with sliding speed, v, over a relatively wide range of v, and decreases as M_1 increases. Under conditions for which M_1 slides at constant speed, μ could be determined to within 0.5% from the relation $\mu = M_2/M_1$. The higher values of μ were determined to within ±3%, the main error arising from the difficulty of obtaining accurate acceleration measurements from position versus time data. Because the sliding speed increases while the box accelerates, the velocities plotted in Fig. 3 are time-averaged velocities measured over a fixed 20 cm sliding distance. Each data point in Figs. 3–5 represents the average of three measurements.

The nominal surface area of the cloth used to obtain the results in Fig. 3 was 64 cm². The experiment was repeated using two narrow strips of cloth glued to the bottom of a second die-cast box, with a total surface area of 34 cm². The two sets of results are compared in Fig. 4, showing that μ increases with nominal surface area. The small difference in friction coefficients in Figs. 3 and 4, each determined a few days apart, can be attributed to small differences in the alignment of the cloth fibers after repeated sliding. A more sig-

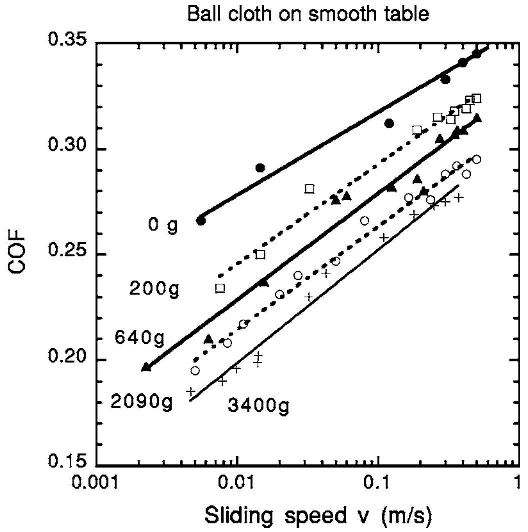

Fig. 3. Results at low sliding speeds. The mass values show the mass added to the 121 g box and the straight lines are best linear fits to the experimental data.

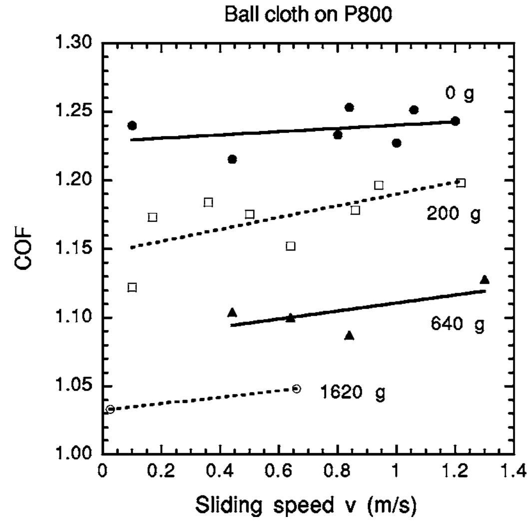

Fig. 5. Low speed results for ball cloth sliding on P800 emery paper.

nificant result was obtained a few weeks later after the cloth had been used on rougher surfaces. The rough surfaces acted to align and twist together individual cloth fibers in a direction transverse to the sliding direction. In that condition, the COF on the smooth table was essentially independent of the load on the cloth, but it still increased with sliding speed at the same rate as in Fig. 3. In effect the cloth became compacted by repeated sliding under heavy loads so that any subsequent or additional load on the cloth had a negligible effect on μ. The overall thickness of the cloth was unaffected, but the bottom surface of the cloth was visibly matted by tight twisting of adjacent fibers in random, slightly raised patches over the whole surface.

Results similar to those in Fig. 3 were obtained using a variety of other materials glued or taped to the bottom of the die-cast box. In general, however, smooth sliding at constant speed was observed only with elastically soft materials such as rubber, a variety of textiles of various thicknesses, nylon or polyester string, and steel wool. A change in thickness or number of layers of a particular textile material had no significant effect on the COF. Elastically hard materials tend to slide at variable speed or come to a complete stop at low values of M_2. At higher values of M_2, elastically hard materials accelerate uniformly with an essentially constant value of the COF, at least at sliding speeds between about 0.1 and 1.0 m/s. The COF of fine nylon strands on a tennis ball and even the soft nylon strings of a racquet increases with sliding speed, but the COF of hard nylon or steel washers does not. It therefore appears that compliance of the mating surfaces, or possibly the ability of a material to stretch in the sliding direction, contributes to the increase in COF with sliding speed. Using a transparent weight such as glass or lucite and viewing the surface with a magnifying lens, there was no visible sticking of fibers to the surface under steady sliding conditions, nor was there any relative movement between adjacent fibers.

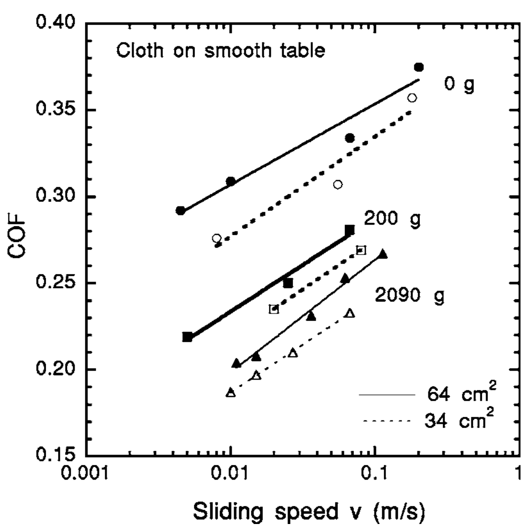

Fig. 4. Effect of apparent surface area on the COF.

The apparatus shown in Fig. 1 also was used to obtain the results shown in Fig. 5. These results were obtained with fine-grained emery paper (P800) taped to the smooth table top to measure the COF between ball cloth and a rough surface. The roughness of P800 is similar to or slightly higher than that of hardcourt and clay tennis courts. Smooth sliding at constant speed was observed under some conditions, but generally the sliding speed was variable at low values of M_2 and even at high values of M_2, where masses placed in the box tended to rattle as the box accelerated, indicating stick-slip behavior. The COF values shown in Fig. 5 are plotted from measurements of the average speed and acceleration over a fixed 20 cm sliding distance. Compared with the results in Fig. 3, the COF on the rougher surface is significantly higher and relatively independent of sliding speed. There also is a smaller decrease in μ as M_1 is increased, both in a relative and absolute sense, compared with the results in Fig. 3. For heavy loading conditions the friction force was sufficiently large to leave yellow tracks consisting of fine particles of polished nylon and even whole strands of broken fibers on the emery paper.

B. High sliding speeds

Results showing an increase in COF with sliding speed at high speeds are shown in Fig. 6. These results were obtained by filming a tennis ball incident obliquely at $\theta_1 = (17 \pm 1)°$

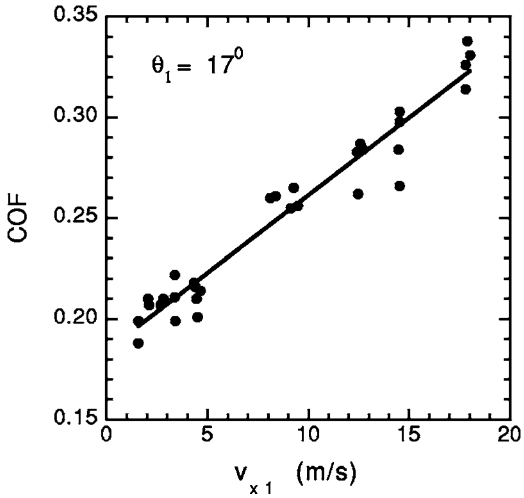

Fig. 6. High speed results for a tennis ball sliding on a smooth table top.

on the smooth table top. Data at speeds $v_{x1}<6$ m/s were obtained by dropping the ball without spin from various heights to impact the table top which was inclined at 17° to the vertical. The higher speed data were obtained with the table top horizontal by projecting the ball downward with zero spin from a ball launcher. Measurements of ball spin after each impact confirmed that the ball did not grip the surface. The COF was obtained from Eq. (5) using the measured ball trajectories, correcting for gravitational acceleration before and after each bounce to determine the incident and rebound speeds and angles immediately before and just after each bounce. The main source of error in this experiment was the variability in bounce speed and angle from one bounce to the next, as indicated by the scatter in the data in Fig. 6. Each data point in Fig. 6 corresponds to a separate bounce.

The data in Fig. 6 are consistent with the results in Figs. 3 and 4, but a direct comparison cannot be made because the nominal surface area of the cloth was different and because the normal reaction force on the ball cloth also was different. When a 66 mm diameter tennis ball contacts a surface at high speed, the maximum possible contact area is about 35 cm^2. For low speed impacts the nominal contact area might be as small as 4 cm^2. Given that the impact time[8] is about 5 ms and the change in momentum in the y direction is about $1.75mv_{y1}=0.53mv_{x1}$, where $m=57$ g is the ball mass, the time-averaged normal reaction force on the ball varied from about 9 N to about 109 N for the results in Fig. 6. The effective load on the ball therefore varied from about 0.9 kg at the lowest impact speeds to about 11.0 kg at the highest impact speeds. Over a range of speeds from $v_{x1}=1.5$ to 18 m/s, the nominal contact surface area would therefore have increased by a factor of about 5 and the load increased by a factor of about 12. If these factors are taken into account, the results in Fig. 6 are not inconsistent with those in Figs. 3 and 4, but the uncertainty in contact area is too large to claim that the two sets of results are in complete agreement.

The results in Fig. 3 are plotted on a $\log(v)$ scale and the results in Fig. 6 are plotted on a linear v scale. It may therefore appear that the two sets of results are inconsistent. However, the results in Fig. 3 were obtained by keeping the normal load and apparent surface area fixed, while the results in Fig. 6 were obtained by allowing the load, surface area, and sliding speed to vary simultaneously. We can conclude from the results in Fig. 6 only that the COF increased with sliding speed, despite the increased load on the ball, partly because of the increase in the nominal contact area and partly because the COF increases with sliding speed, even when the load and nominal contact area remain constant. From a practical rather than a physics point of view, these three separate effects are of no particular interest. In a sporting context the main feature of interest is that the "speed" or COF of a sporting surface is likely to depend on the speed of the ball, given that the balls and/or surfaces used in most ball sports are elastically soft. An increase in COF with speed has previously been described for billiard balls sliding on cloth.[9]

Measurements of the COF for a ball impacting on P800 emery paper also were attempted, but the required angle of incidence was too low to obtain a reliable value of μ. According to Eq. (6) and the results in Fig. 5, θ_1 must be less than 10° when $\mu\approx1.1$. Such low grazing angles of incidence are difficult to achieve experimentally and are difficult to measure with appropriate accuracy. Bounce results obtained at $\theta_1>10°$ indicated that μ was at least 0.85 for a 5 ms impact on P800, as expected from the results in Fig. 5. I note that the official rules of tennis regarding the measurement of court speed should be modified. As they currently stand, the rules specify that court surfaces must be tested by bouncing a ball at an angle of incidence of 16°.

IV. DISCUSSION

Observations of the increase in COF with speed were made using several techniques other than those described so far to establish that the effect depends on the sliding surfaces and not on the technique itself. The simplest method, and one that would be suited to a classroom demonstration, is to slide a blackboard or whiteboard eraser down a smooth incline. The felt side slides at constant speed for a range of incline angles (provided dust or chalk is removed first), but the wood or plastic side accelerates down the incline. Another simple technique is to slide string, tape, or cloth over a fixed smooth cylinder with unbalanced masses hanging on each end. If a string is wound n times around the cylinder (n can be fractional) and slides at constant speed, then it is easy to show that $T_2=T_1e^{2\pi n\mu}$, where μ is the COF between the string and the cylinder, and T_1 and T_2 are the tensions at each end of the string. Observations at higher speeds can be made by launching a mass along a long table, by hand or by a spring mechanism, and measuring its deceleration. The latter method is not reliable at large values of the COF, because the torque generated by the friction force tends to flip over the mass.

The actual area of contact between any two surfaces is usually much less than the nominal surface area. The friction force is proportional to the actual area of contact, but for polymers the actual area of contact is not directly proportional to the load. Solids normally contact over many individual, microscopic points of contact. At high loads and for smooth surfaces, polymers and rubbers can contact over much larger contact regions, in which case the actual contact area might be almost as large as the nominal surface area. Alternatively, if a polymer sphere contacts a flat plate, the area of contact A is not proportional to the load N, but is proportional[3] to $N^{2/3}$. For these conditions we would expect that μ would be proportional to A/N or to $1/N^{1/3}$. Note that Amonton's laws also can be explained in terms of the relation $\mu = F/N$, using the fact that F is proportional to the actual contact area A. However, if A is proportional to N, then μ is independent of N.

Tennis ball cloth contains a felted mixture of some wool but mostly nylon fibers. As the load increases, the number of fibers in contact with a flat surface will increase, while the actual contact area of each fiber might obey a power law similar to that for a sphere. The results at high v in Fig. 3 indicate that μ is proportional to $N^{-0.054}$ for fresh samples of cloth, but it also was found that μ is essentially independent of N when the cloth fibers became tightly twisted. It was found that μ is essentially independent of N when the ball cloth was replaced by several strands of 1.3 mm diameter nylon tennis string. Consequently, we cannot attribute the decrease in μ with increased load to the theoretically expected $\frac{2}{3}$ power law. Rather, it appears that for a fresh sample of cloth, the number of fibers in contact with the surface increases with N in such a way that A/N decreases as N increases where A is the actual contact area. The data indicates that when N is doubled, A increases by a factor of $2^{0.946} = 1.93$.

An increase in μ with sliding speed has been observed previously, especially for textiles,[10] but the underlying mechanism remains unclear. Rabinowitz[5] and also Persson[11] indicate that slow creep of one or both sliding surfaces can explain the effect at extremely low speeds, but the rate of creep of nylon is much too slow to account for the fact that μ increases with speed even at speeds above 1 m/s. In fact, creep was used in Ref. 6 to explain a decrease in μ with sliding speed that also can be observed at extremely low speeds (around 10^{-6} m/s). Studies of the latter type are commonly undertaken to investigate earthquake or glacier dynamics.

In the absence of a well-established mechanism to explain an increase in μ with sliding speed, we can speculate that the mechanism might be modeled microscopically by a block sliding on a frictionless surface on which many small balls are resting and which hinder the progress of the block. As the block collides with the balls, it experiences an average retarding force that is proportional to the square of the speed of the block, because the momentum transferred to each ball is proportional to the speed of the block and because the collision frequency also is proportional to the speed of the block. Alternatively, if the collision frequency is determined by the speed of the balls rather than the speed of the block, then the average retarding force will be directly proportional to the speed of the block. The latter situation has been modeled at a much more sophisticated level in terms of lattice vibrations induced by sliding. Provided that the actual contact area is not altered by a change in the sliding speed, it is found that the electronic and photonic contributions to the friction force are both proportional to sliding speed.[11,12] These contributions in the present experiment were not investigated directly.

V. CONCLUSIONS

Experimental results have been presented showing that the coefficient of sliding friction increases with speed, over a wide range of sliding speeds, when elastically soft materials slide on smooth surfaces. A number of different materials and surfaces were examined in coming to this conclusion, but the emphasis in this paper was to measure the COF of tennis ball cloth. It was found that the COF is larger and less dependent on speed if cloth slides on a rough rather than a smooth surface. A decrease in the COF with increasing speed is a well-known cause of stick-slip phenomena, but the increase in COF with speed has not been as extensively studied. The latter effect is likely to play an important role in ball sports, because variations in the COF between a ball and the striking implement or the playing surface will alter the bounce speed and angle and will therefore challenge even the best of players.

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