Bounce of a spinning ball near normal incidence

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A spinning ball incident at right angles to a surface bounces at an oblique angle and with reduced spin. Consequently, a spinning ball struck head-on does not rebound along its incident path, which presents a control problem in bat and racquet sports. The rebound angle and spin depend in a nontrivial manner on the coefficient of friction between the ball and the surface and on the elastic properties of the ball and the surface. Values of the normal and tangential coefficients of restitution coefficients are presented for a tennis ball impacting a smooth and a rough surface and on strings of a tennis racquet. The implications for spin generation in tennis are briefly described.

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I. INTRODUCTION

The physics of a bouncing ball has been described by several authors.[1–10] One aspect that has escaped detailed attention is the case where a spinning ball is incident on a surface at angles near the normal. There is no theoretical and only sparse experimental data available that would allow us to predict the bounce angle or spin accurately. Available bounce models differ significantly in terms of their theoretical predictions. The model developed by Garwin[1] to describe superballs predicts that if a ball spinning about a horizontal axis is dropped vertically onto a horizontal surface, the ball will bounce forward (in the same direction as the motion of a point at the top of the ball), and the spin direction will be reversed by the bounce. The model developed by Brody[2] for a tennis ball predicts that the ball will bounce forward, spinning in the same direction as the incident ball but with reduced spin.

In this paper, experimental results are presented for a spinning tennis ball incident at angles near the normal on three different surfaces. The results are significant in a sporting context because a spinning ball struck head-on (or at normal incidence) does not rebound along its incident path. A player hitting such a ball needs to adjust for the spin of the incident ball accordingly to project the ball along its intended path. A skilled tennis player can control the magnitude and direction of spin after the bounce by varying the angle of incidence on the racquet and by varying the speed and trajectory of the racquet. This paper is concerned primarily with tennis, but there are close parallels in golf and baseball. Tennis and table tennis players tend to tilt the racquet head forward to impart topspin to a ball, while golf players use backward tilting clubs to impart backspin.

Backspin allows a ball to travel farther due to aerodynamic lift. A baseball bat cannot be tilted in this manner, but it can strike the ball below the center of the ball, in which case the bat effectively slopes backward at the impact point. The resulting ball spin in all cases depends on the coefficient of restitution in the tangential direction. The latter parameter is not as widely known and not as commonly measured as the coefficient of restitution in the perpendicular direction, but it has a strong influence on the rebound spin and bounce angle.

II. THEORETICAL BOUNCE MODELS

Consider the situation shown in Fig. 1 where a ball of radius R is incident at angle θ_1 on a horizontal surface with angular velocity ω_1. Let v_{y1} denote the vertical component of the incident ball velocity and v_{x1} denote the horizontal component. The ball rebounds with vertical velocity v_{y2}, horizontal velocity v_{x2}, and angular velocity ω_2. Each of the velocity components refers to the velocity of the center of mass of the ball. If we assume that the surface does not recoil, the bounce can be characterized in terms of the normal coefficient of restitution, e_y, and the tangential coefficient of restitution, e_x, defined by

$$e_y = v_{y2}/v_{y1}, \tag{1a}$$

$$e_x = -\frac{(v_{x2} - R\omega_2)}{(v_{x1} - R\omega_1)}. \tag{1b}$$

The two coefficients of restitution are defined, respectively, in terms of the normal and tangential velocities of the contact point on the ball (rather than its center of mass) immediately before and immediately after the bounce. As shown in Fig. 1, a point at the bottom of the ball approaches the surface at a tangential velocity $v_{x1} - R\omega_1$ and at a vertical velocity v_{y1}. Immediately after the bounce, a point at the bottom of the ball has a tangential velocity $v_{x2} - R\omega_2$ and a vertical velocity v_{y2}. The coefficient e_x can therefore be positive, zero, or negative depending on the amount of spin acquired during the bounce. To describe the bounce of a superball, Garwin[1] assumed that $e_x = e_y = +1$, regardless of the angle of incidence. To describe the bounce of a tennis ball, Brody assumed that a ball incident at right angles to a surface, or nearly so, would roll with $v_{x2} = R\omega_2$ by the end of the bounce, in which case $e_x = 0$.

The Garwin model assumes that the ball is perfectly elastic, so that any deformation of the ball normal or tangential to the surface results in complete recovery of the stored elastic energy. As a result, the velocity of the contact point is reversed by the bounce, in both the normal and tangential directions. The Brody model assumes in effect that there is no deformation of the ball and that the ball will commence to roll on the surface after an initial period of sliding. As a result, the velocity of a point at the bottom of the ball drops to zero in the tangential direction. In reality, there always is some deformation and some loss of elastically stored energy in the ball and/or the surface. Consequently, all balls bounce with $0 < e_y < 1$ and with $e_x < 1$ when incident obliquely on a

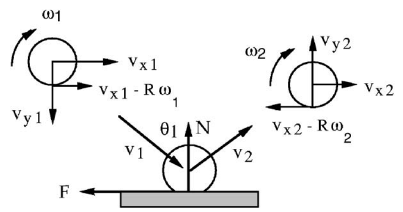

Fig. 1. Bounce geometry showing a ball incident with topspin and bouncing with topspin. If $e_x > 0$, the velocity of a point at the bottom of the ball reverses sign in both the horizontal and vertical directions, as indicated.

surface at angles within about 45° to the perpendicular. At larger angles of incidence, a ball can slide throughout the whole bounce period[3] in which case $e_x < 0$.

If we assume that the normal reaction force on the ball acts through its center of mass, then conservation of angular momentum about a point at the bottom of the ball is described by

$$I\omega_1 + mRv_{x1} = I\omega_2 + mRv_{x2}, \tag{2}$$

where m is the ball mass and $I = \alpha mR^2$ is the moment of inertia of the ball about its center of mass. For a solid sphere, $\alpha = 2/5$. For a thin spherical shell, $\alpha = 2/3$. For a 6 mm wall thickness tennis ball of outer radius $R = 33$ mm, $\alpha = 0.55$. In one of the experiments that we will describe, a spinning ball impacted the strings of a hand-held racquet. As a result, the racquet did not remain at rest but recoiled away from the ball as a result of the collision. It was assumed for convenience in Eqs. (1) and (2) that the impact surface remains at rest, but such an assumption is not necessary in the present context. Regardless of whether the surface recoils or not, the friction force F acting on the ball results in a change in both its horizontal velocity v_x and its angular speed ω given, respectively, by $F = -m dv_x/dt$ and $FR = I d\omega/dt$. If R remains constant, then $\int F dt = m(v_{x1} - v_{x2}) = m\alpha R(\omega_2 - \omega_1)$, which yields Eq. (2). Equation (1) also is unaffected by recoil of the surface, although e_x should then be regarded as an apparent tangential coefficient of restitution due to the neglect of tangential motion of the impact surface. In the same way, the ratio v_{y2}/v_{y1} is commonly regarded as an apparent coefficient of restitution for an impact on a surface that recoils, due to the neglect of the normal velocity component of the surface after the impact. The coefficient of restitution for two colliding objects is defined as the ratio of the relative velocity of the objects after the collision to the relative velocity before the collision. Apparent coefficient of restitution values are easier to measure for a surface that recoils than actual coefficient of restitution values, and the information so obtained is just as useful if not more so. Actual coefficient of restitution values for a surface that recoils can be determined from measurements of apparent coefficient of restitution values using conservation of linear momentum in the normal and tangential directions to estimate the recoil velocity of the surface.[4] Equations (1) and (2) can be solved to show that

$$\frac{v_{x2}}{v_{x1}} = \frac{(1 - \alpha e_x)}{(1 + \alpha)} + \frac{\alpha(1 + e_x)}{(1 + \alpha)} \left(\frac{R\omega_1}{v_{x1}} \right), \tag{3}$$

and

$$\frac{\omega_2}{\omega_1} = \frac{(\alpha - e_x)}{(1 + \alpha)} + \frac{(1 + e_x)}{(1 + \alpha)} \left(\frac{v_{x1}}{R\omega_1} \right). \tag{4}$$

It can be seen that the vertical bounce velocity of any given ball is determined by e_y, and spin and horizontal bounce velocity are determined by the two parameters e_x and $R\omega_1/v_{x1}$. Depending on the magnitude and sign of the latter parameter, v_{x2} and ω_2 can each be positive, zero, or negative. Qualitative features of the bounce are easily understood. A ball incident at an oblique angle without spin will bounce forward with topspin. At angles of incidence less than about 45°, the spin decreases as the angle of incidence decreases, becoming zero at normal incidence. The ratio v_{x2}/v_{x1} is independent of the angle of incidence if the ball is incident without spin and if e_x is independent of the angle of incidence. The effect of backspin, that is, $\omega_1 < 0$, is to reduce both the forward velocity, v_{x2}, and the forward spin, ω_2. Topspin increases both the forward bounce velocity and the forward spin. Effects of changes in e_x are more subtle. If a ball is incident obliquely without spin, then an increase in e_x acts to increase the forward spin. If a spinning ball is incident at right angles on a surface, then an increase in e_x acts to decrease the forward spin, to the extent that the spin direction is reversed by the bounce when e_x is greater than about 0.4 (as it is for a superball). The coefficient of sliding friction does not enter directly into Eqs. (3) and (4). If the angle of incidence is less than about 45°, a ball that initially slides along the surface will quickly come to rest and grip the surface before bouncing off the surface. Consequently, we might expect that the bounce parameters would not be especially sensitive to the coefficient of sliding friction at small angles of incidence. The experimental results we will describe are consistent with this expectation. For a solid ball at normal incidence where $v_{x1} = 0$, and in the limit $e_x = 1$, $v_{x2} = 4R\omega_1/7$ and $\omega_2 = -3\omega_1/7$. If $v_{x1} = 0$ and $e_x = 0$, then $v_{x2} = 2R\omega_1/7$ and $\omega_2 = 2\omega_1/7$. In the latter case the ball rolls along the surface toward the end of the bounce period and therefore exits with $v_{x2} = R\omega_2$. In the former case ($e_x = 1$) the ball exits with $v_{x2} = -4R\omega_2/3$ and therefore slides along the surface just prior to bouncing off the surface. The implication for a real solid ball incident normally is that it will bounce forward at a horizontal velocity $v_{x2} \approx 3R\omega_1/7$ and with reduced spin compared with the incident spin. The limiting cases just considered indicate that the rebound velocity and angle of the ball do not depend strongly on e_x, but they do depend strongly on the spin of the incident ball. The dependence of ω_2 on e_x is sufficiently strong that we cannot predict (without knowing e_x) the direction in which a spinning ball will spin after it bounces. Conversely, the magnitude and direction of spin after the bounce provides a sensitive measure of the tangential coefficient of restitution.

III. QUALITATIVE FEATURES OF BOUNCE PROCESS

A description of ball bounce in terms of e_x and e_y allows us to relate the bounce velocity, spin, and rebound angle to conditions prior to the bounce, but it gives no information about the behavior of the ball during the bounce. A simple qualitative description is illustrated in Figs. 2 and 3 for a tennis ball incident obliquely on a surface without initial spin and with $e_x = 0.2$. Initially the ball slides along the surface and the friction force, F, acts backward at the bottom of the

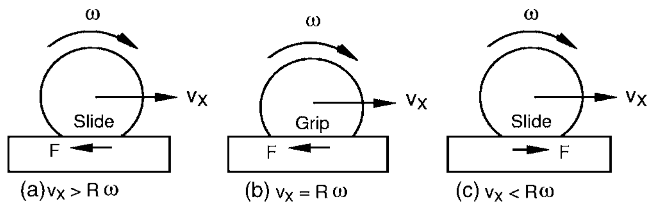

Fig. 2. A ball incident obliquely on a surface without initial spin slides until $v_x = R\omega$ and then grips the surface. When the ball releases its grip, the ball slides backward and the friction force reverses direction.

ball. As a result, the horizontal velocity, v_x, decreases with time and the angular velocity, ω, increases due to the clockwise torque exerted on the ball. When $v_x = R\omega$, the bottom of the ball comes to rest on the surface and the large normal reaction force generated during the bounce causes the ball to grip the surface. The upper part of the ball continues to move forward and rotate and therefore exerts a force on the lower part of the ball. A static friction force acting backward on the ball allows the ball to maintain its grip until the normal reaction force drops sufficiently or until the force exerted by the upper part of the ball causes the ball to release its grip.

While the static friction force acts backward, v_x continues to decrease and ω continues to increase. Consequently, $R\omega$ exceeds v_x during the grip phase, leading to a build up of stress in the contact region. When the ball releases its grip, the bottom of the ball suddenly slides backward on the surface and hence the friction force reverses. Consequently, v_x increases and ω decreases during the latter stage of the bounce. The ball can then exit the surface with $R\omega_2 > v_{x2}$, in which case $e_x > 0$, or it can repeat the process in the reverse direction and exit with $R\omega_2 < v_{x2}$, in which case $e_x < 0$. This discussion is somewhat simplified because the normal reaction force is not uniform over the contact region and hence some parts of the contact region slide while others grip, as described in detail by Maw *et al.*[5] and by Stronge.[6]

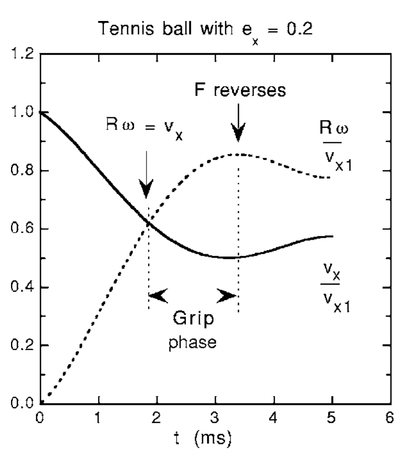

Fig. 3. The quantities v_x/v_{x1} and $R\omega/v_{x1}$ vs time for a tennis ball incident obliquely on a surface without initial spin. The impact duration is typically about 5 ms. The ball bounces with $v_{x2}/v_{x1} = 0.574$ and $R\omega_2 = 1.348 v_{x2}$ if $e_x = 0.2$. Note that v_x increases with time and ω decreases after F reverses direction.

IV. EXPERIMENTAL PROCEDURES

A. Bounce off wooden floor

Measurements of the bounce of a new tennis ball on a smooth wood floor were made by filming the bounce with a JVC DVL9600 digital video camera operating at 100 frames/s and with an exposure time of 2 ms. Satisfactory results also could have been obtained with a standard 25 or 30 frames/s camera, but the higher speed was useful in avoiding ambiguous measurements of the ball orientation (that is, an uncertainty of 180° or 360°). A line drawn around the ball with a felt pen, together with a dot on one side of the line, indicated its orientation so that ball spin could be measured, in addition to the incident and rebound velocities and angles.

Two sets of bounce measurements were made to examine the effects of spin. The ball was first projected by hand without spin at various angles from 3° to 15° away from the normal and at an incident velocities from 5.60 to 6.03 m/s. Nine bounces were analyzed both on the smooth floor and with fine grain emery paper (P800) taped to the floor to provide a surface with a larger coefficient of sliding friction. Previous experiments indicated that the coefficients of sliding friction were 0.4 ± 0.1 on the smooth wood floor and 0.85 ± 0.05 on P800. The second set of measurements was made by projecting a spinning ball onto each surface. Spin was imparted to the ball by dropping the ball onto the strings of a racquet while swinging the racquet across the bottom of the ball with a slight upward motion of the racquet to project it vertically. The spinning ball was then allowed to fall onto the floor at angles of incidence up to 2° away from the vertical. Nine bounces on each surface (P800 or smooth wood) were analyzed with initial spins varying from 61 to 116 rad/s and incident velocities varying from 5.13 to 6.78 m/s.

At least three and up to five images of the ball before and after each bounce were used to determine the horizontal and vertical velocities of the ball just before and after each bounce. The velocities were determined to within 2% using a linear fit to the horizontal position data and a quadratic fit to the vertical position data, assuming a vertical acceleration of 9.8 m/s^2. The angular velocity of the ball also was determined to within 2%. The main source of error in this experiment was the fact that the ball was not perfectly spherical because it had a slightly indented seam joining the two halves of the cloth cover. As a result, the ball could bounce up to 1° away from the vertical even when dropped vertically without spin. Consequently it was not possible to obtain reliable measurements of the tangential coefficient of restitution when the ball was incident without spin at angles within 3° of the vertical. The same problem did not arise when the ball was incident with spin because the ball bounced at an angle about 10° away from the vertical, in which case a random error of 1° in the bounce angle was much less significant.

B. Bounce off tennis strings

Measurements of the bounce of a spinning tennis ball also were made by allowing the ball to fall back onto the strings of the hand-held 334 g racquet used to impart spin to the ball. String tension was not measured, but was estimated to be about 250 N, which is medium tension for a racquet. The ball was allowed to fall almost vertically onto the strings

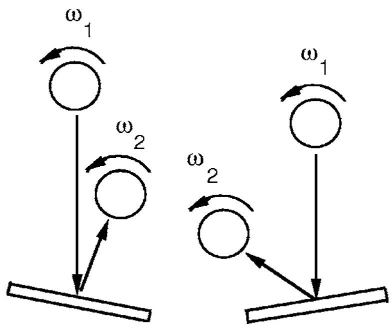

Fig. 4. A ball spinning counterclockwise when dropped vertically onto the strings of a horizontal racquet will bounce to the left. If the racquet is tilted to the right, the ball will bounce almost vertically. If the racquet is tilted to the left, the ball will bounce even further to the left.

while the strings were held in an approximately horizontal plane. Many other measurements have been made of ball bouncing off a racquet, but always under controlled conditions where the racquet is initially at rest and usually firmly clamped. The advantages of these conditions are that there is no need to measure the racquet speed and the impact point on the strings can be more precisely controlled. A hand-held racquet was chosen for the experiment because it provided a convenient method of imparting spin to the ball and because a hand-held racquet is of greater practical significance than a clamped racquet. About 80 such impacts were filmed and 39 of the "best" impacts were analyzed based on the fact that the ball was incident within 10° of the normal, the line drawn on the ball remained centered so that the ball spin could be measured accurately, and the ball landed near the middle of the strings.

Two series of measurements were made for bounces off the strings, one where the racquet was deliberately tilted to the right at angles up to 10° away from the horizontal, and one where the racquet was deliberately tilted to the left, as shown in Fig. 4. The direction of the ball spin was not changed, but in the first case the ball was incident with backspin and in the second case the ball was incident with topspin. In the first case the ball bounced almost vertically. In the second case the ball bounced at angles up to about 40° away from the vertical. The bounce angle is determined by the combined effects due to spin and tilt. In both cases the ball tended to deflect to the left due to its spin, but in the first case the tilt of the racquet acted to deflect the ball back to the right so that the resulting bounce was almost vertical, while in the second case the tilt exaggerated the deflection to the left. The situation represented by Fig. 4 (when rotated by 90°) can present a difficult problem for an inexperienced tennis player because a player who tilts the racquet in the wrong direction is likely to make a large error. Even an experienced player can misjudge the required tilt angle.

To position the strings underneath the falling ball, the racquet was maneuvered slowly in a horizontal plane and at about 0.8 m/s vertically upward. The vertical motion was unintentional, but it had the effect of imparting additional vertical velocity to the ball, above that due to the bounce itself. If a ball is falling vertically at velocity v_{y1} and impacts at normal incidence on a racquet moving vertically up at velocity v_R, then the ball will be projected upward at a velocity v_{y2} given by[4]

$$v_{y2} = (1 + e_A)v_R + e_A v_{y1}, \tag{5}$$

where e_A is the apparent coefficient of restitution, defined as the ratio v_{y2}/v_{y1} under conditions where $v_R=0$; e_A differs from e_y, in that no account is taken of recoil of the racquet when defining e_A. Measurements of e_A have been made previously and values around 0.4 are found for most tennis racquets for impacts near the middle of the strings.[4] The coefficient e_A is easier to measure than e_y because there is no need for a simultaneous measurement of the racquet speed. However, if the impact is filmed with a video camera, measurements of both ball and racquet speed are possible, provided that the racquet does not rotate too far out of alignment between video frames. In the experiment it was possible to measure v_R accurately before the impact, but the racquet recoiled too rapidly after the impact to obtain a reliable racquet speed measurement. All bounce data obtained in the laboratory frame of reference were subsequently processed to determine bounce parameters in a reference frame where the racquet was initially at rest and in a horizontal plane.

V. FLOOR BOUNCE RESULTS

When a tennis ball was thrown obliquely onto the floor without spin, the ball bounced forward at reduced horizontal velocity and with topspin. On the bare wood floor it was found that $v_{x2}/v_{x1}=0.35\pm0.05$ averaged over nine bounces (±standard deviation). On the P800 surface, $v_{x2}/v_{x1}=0.51\pm0.02$. The ball therefore slowed down more on the low friction surface. The magnitude of the ball spin generated by the bounce decreased as θ_1 decreased, decreasing to zero at $\theta_1=0°$, as expected. Ball spin was essentially the same off both surfaces, with $R\omega_2/v_{x1}=0.66\pm0.05$ on the bare wood floor, and 0.65 ± 0.04 on the P800 surface. The corresponding values of $S_2=R\omega_2/v_{x2}$ were 1.91 ± 0.28 on the bare wood floor and 1.27 ± 0.09 on P800. The ball bounced in an "overspinning" mode off both surfaces, with $R\omega_2>v_{x2}$. In the Brody bounce model, a ball incident at angles near the normal bounces in a rolling mode, with $R\omega_2=v_{x2}$. The present results indicate that the ball slides forward on each surface at the start of the bounce (because $R\omega_1<v_{x1}$) and slides backward on each surface by the end of the bounce. Such a result indicates that the ball grips the surface during the bounce and then slides backward when the ball releases its grip due to the build up of elastic stress in that direction.

When a spinning tennis ball was incident on the wood floor at angles close to the normal (within 3°), the ball continued to spin in the same direction after bouncing but with reduced spin. On the wood floor, $\omega_2/\omega_1=0.29\pm0.03$. For P800 taped to the floor, $\omega_2/\omega_1=0.30\pm0.04$. There was no significant difference in the effect of the two different surfaces on ball spin, as found when the ball was incident without spin at larger angles to the normal. Only a minor difference in the two surfaces was found for the horizontal bounce velocity, v_{x2}. In both cases, the ball was incident vertically at $v_{y1}\approx6.0$ m/s with $v_{x1}\approx0.15$ m/s. The ball was incident with backspin in all cases, with $\omega_1\approx-80$ rad/s, and bounced backward with $v_{x2}\approx-1.1$ m/s on the bare wood floor and at $v_{x2}\approx-0.95$ m/s on the P800 surface. A summary of the main bounce parameters, averaged over nine bounces in each case, is included in Table I. For a ball incident with spin, the ball bounced in an "underspinning" mode with $R\omega_2<\omega_2$ and with a smaller value of e_x than observed when the ball was incident without spin.

Table I. The bounce of a tennis ball off a wooden floor. The results are averaged over nine bounces and the errors are the standard deviations.

Surface	v_1(m/s)	θ_1(deg)	ω_1(rad/s)	e_y	e_x	$S_2=R\omega_2/v_{x2}$
P800	5.70±0.12	13±3	<1	0.79±0.007	0.13±0.05	1.27±0.09
Wood	5.76±0.14	10±4	<1	0.79±0.009	0.32±0.07	1.91±0.28
Wood	6.09±0.44	2±0.5	−61 to −116	0.77±0.008	0.08±0.04	0.79±0.08
P800	5.93±0.46	1±0.5	−65 to −90	0.78±0.011	0.055±0.04	0.84±0.11

VI. RACQUET BOUNCE RESULTS

A summary of results obtained when the ball was incident with spin on the racquet strings is given in Table II. The results are given in the racquet frame of reference and also are corrected by a coordinate rotation as if the strings were initially at rest in a horizontal plane, as in Fig. 1. The ball was incident at speeds varying from 4.14 to 6.98 m/s in the racquet reference frame and with varying spins and angles of incidence as shown in Table II. Angles of incidence smaller than 0° are given, despite the potential for ambiguity, to emphasize the fact that the actual direction of ball spin remained the same while the angle of incidence was varied. In the present context, a ball incident at −5° with backspin is taken to mean that it spins in the same physical direction as a ball incident at 5° with backspin, but it behaves in essentially the same manner as a ball incident at 5° with topspin. The only difference is that it travels in the opposite horizontal direction.

In all cases the ball bounced with reduced spin, spinning in the same direction as the incident ball. The ball bounced with apparent coefficient of restitution values varying from 0.35 to 0.52, the lower values corresponding to a bounce about 6 cm away from the middle of the strings toward the tip, and the higher values corresponding to a bounce about 6 cm away from the middle of the strings toward the handle end.

At angles of incidence between 0° and 7° the ball was incident with backspin and bounced backward with $v_{x2}<0$. At larger angles of incidence the ball bounced forward in most cases, but it sometimes bounced backward if the initial spin rate was sufficiently high. Under these conditions the magnitude of $S_2=R\omega_2/v_{x2}$ can approach infinity because v_{x2} can approach zero. Consequently, the parameter S_2 is of no real value in characterizing the bounce when a ball is incident with backspin. The parameter e_x is a more generally useful parameter and was −0.11±0.06 averaged over all bounces, as determined from Eq. (1), regardless of whether the ball was incident with backspin or topspin.

Plots of v_{x2}/v_{x1} versus $R\omega_1/v_{x1}$ and ω_2/ω_1 versus $v_{x1}/R\omega_1$ are shown for all 39 bounces in Figs. 5 and 6, respectively. A linear best fit to each data set to Eqs. (3) and (4), respectively, gives

Table II. The bounce of a tennis ball off a racquet. The results are averaged over 39 bounces.

v_1(m/s)	θ_1(deg)	ω_1(rad/s)	e_A	e_x
4.14 to 6.98	−6 to 12	−44 to −131	0.42±0.04	−0.11±0.06

$$\frac{v_{x2}}{v_{x1}} = 0.648 + 0.300\left(\frac{R\omega_1}{v_{x1}}\right), \tag{6}$$

$$\frac{\omega_2}{\omega_1} = 0.400 + 0.583\left(\frac{v_{x1}}{R\omega_1}\right), \tag{7}$$

whereas Eqs. (3) and (4) indicate that

$$\frac{v_{x2}}{v_{x1}} = 0.684 + 0.316\left(\frac{R\omega_1}{v_{x1}}\right), \tag{8}$$

$$\frac{\omega_2}{\omega_1} = 0.426 + 0.574\left(\frac{v_{x1}}{R\omega_1}\right), \tag{9}$$

for $\alpha=0.55$ and $e_x=-0.11$. The good agreement between Eqs. (6) and (7) and Eqs. (8) and (9) indicates that Eqs. (1) and (2) provide a good description of the bounce. The assumption in Eq. (2) that the normal reaction force acts through the center of mass is therefore justified, at least for the low speed bounces in the present experiment. The results in Figs. 5 and 6 show that the apparent tangential coefficient of restitution provides a useful measure of the bounce process and that it is not necessary to measure the racquet recoil speed or the actual coefficient of restitution to quantify the bounce parameters.

We conclude that Eqs. (3)–(5) provide a good description of the bounce off a racquet even if the actual coefficient of restitution values are unknown. In principle, we could measure or calculate the racquet velocity after the collision, but

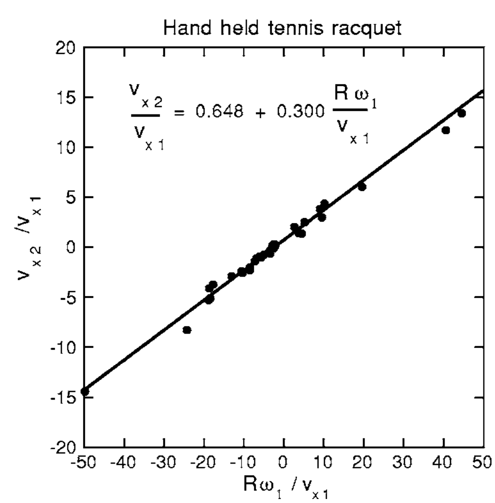

Fig. 5. Experimental data for 39 individual bounces of a tennis ball off the strings of a hand-held racquet, showing v_{x2}/v_{x1} vs $R\omega_1/v_{x1}$. The data can be fit by a straight line of slope 0.300 and intercept 0.648.

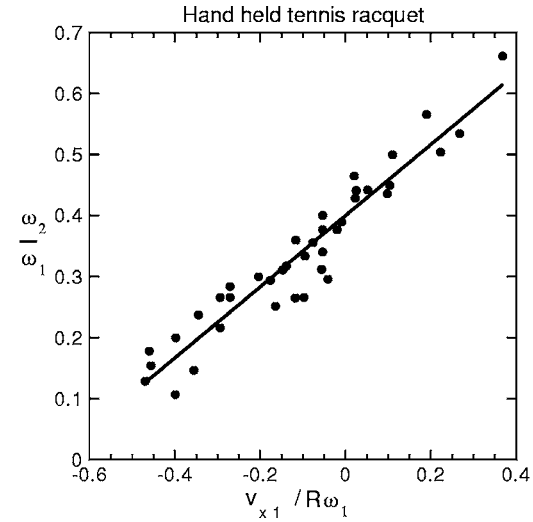

Fig. 6. Experimental data for the same 39 bounces as in Fig. 5 showing ω_2/ω_1 vs $v_{x1}/R\omega_1$. The data can be fit by a straight line of slope 0.583 and intercept 0.400.

such a measurement was not possible in the present experiment and was not necessary for the calculations presented in Sec. VIII. Sufficient information is provided by Eqs. (5)–(7) to predict the bounce speed, spin, and angle for all incident angles up to about 45° away from the normal. At larger angles, the ball may slide throughout the bounce, in which case the coefficient of sliding friction determines the bounce spin and angle.[7]

VII. DISCUSSION

The values of e_y for a bounce on the floor and the values of e_A for a bounce on a hand-held racquet are consistent with previous measurements. Very few measurements have previously been made of e_x for any ball type apart from those published by the author for relatively large angles of incidence.[3,4,7,9,10] An extensive series of measurements of the bounce of a high speed tennis ball off the strings of a head-clamped racquet was reported by Goodwill and Haake,[8] including measurements of ball spin, but they did not evaluate e_x and the angle of incidence was fixed at 40°. Their data indicate that e_x was $\approx +0.05$ (ranging from 0 to 0.1) regardless of ball speed (from 20 to 30 m/s), incident spin (from +50 to −500 rad/s), string tension, or string type.

The author has previously observed[3,9,10] that e_x for tennis balls and golf balls increases as the angle of incidence decreases, up to a maximum value of ≈ 0.2 at an angle of incidence of $\approx 50°$. At large angles of incidence, around 75°, $e_x < 0$ because the ball slides throughout the bounce. As the angle of incidence decreases, e_x increases until the ball commences to grip the surface at which point e_x becomes positive. Tennis ball results were obtained on various court surfaces as well as on the strings of a head-clamped racquet. However, measurements of e_x have not previously been made at angles of incidence less than 40°. The present results extend the previous observations and show that e_x decreases to about 0.06 for a tennis ball impacting at angles near normal incidence on a rigid, very heavy surface, and it decreases to about −0.1 on the strings of a hand-held racquet for angles of incidence near the normal.

A sophisticated model of the bounce mechanism would be required to explain these data. The negative value of e_x obtained for a hand-held racquet may be associated with the lower rebound speed and the associated decrease in the normal reaction force during a bounce (compared with the bounce on a heavy surface). A ball will therefore tend to release its grip on the strings at an earlier stage of the bounce and may have sufficient time to grip a second time after the friction force reverses. Such behavior would be consistent with the fact that $R\omega_2 < v_{x2}$ when $e_x < 0$. The negative value of e_x observed for an impact on the strings also is due in part to the neglect of x-directed motion of the racquet resulting from the collision. A simple estimate of the horizontal racquet velocity, V_{x2}, after the collision, based on conservation of linear momentum in the x direction, indicates that e_x would be ≈ -0.02 rather than −0.11 if the numerator in Eq. (1) were replaced by $v_{x2} - R\omega_2 - V_{x2}$. The latter expression denotes the horizontal velocity of a contact point at the bottom of the ball relative to the racquet, immediately after the bounce.

VIII. SPIN GENERATION IN TENNIS

The current game of tennis at an elite level differs from the game played before 1970 in that players generate more spin on almost every shot, because they hit the ball harder and because of the larger head size of present-day racquets. The larger head allows a player to tilt the head further forward or to swing the head upward at a larger angle to the incoming path of the ball while maintaining the same collision cross section as an old wooden racquet. In the racquet frame of reference, the ball may therefore approach the strings at angles further away from the normal, which has the effect of increasing the spin of the ball as it bounces off the strings. For example, consider the four situations shown in Fig. 7, where a ball approaches a player along a horizontal path at 15 m/s and is spinning clockwise at 400 rad/s as a result of its bounce off the court. If the racquet approaches the ball along the same horizontal path with the strings in a vertical plane, the ball will be deflected upward with reduced spin, and will therefore travel back to the opponent with backspin, as shown in Fig. 7(a). To hit the ball back with topspin, the player needs to reverse the direction of the spin after the bounce. One solution would be to use a superball instead of a tennis ball. Alternatively, the player can swing the racquet in an upward path, as in Fig. 7(b), or tilt the racquet head forward, as in Fig. 7(c). Elite players use both techniques simultaneously, as shown in Fig. 7(d).

The results in Fig. 7 were evaluated by first changing to the racquet frame of reference to apply Eqs. (6) and (7), and then changing back to the court frame of reference. In the racquet frame of reference the ball approaches the strings with backspin in Figs. 7(b)–7(d). If the angle of incidence is sufficiently large in the racquet frame, the ball will bounce with topspin. In practice, the racquet needs to rise upward at an angle of about 30° to the horizontal (in the court frame of reference) to achieve such a result. The same effect can be achieved by tilting the racquet head forward while swinging the racquet horizontally. In the racquet frame of reference the ball approaches the strings at an oblique angle, with backspin if the head is tilted forward, and will be deflected downward with topspin if the incident angle is large enough. In Fig. 7(c) the forward tilt angle is only 5°, which was not sufficient to reverse the direction of spin. There is, however, a difference

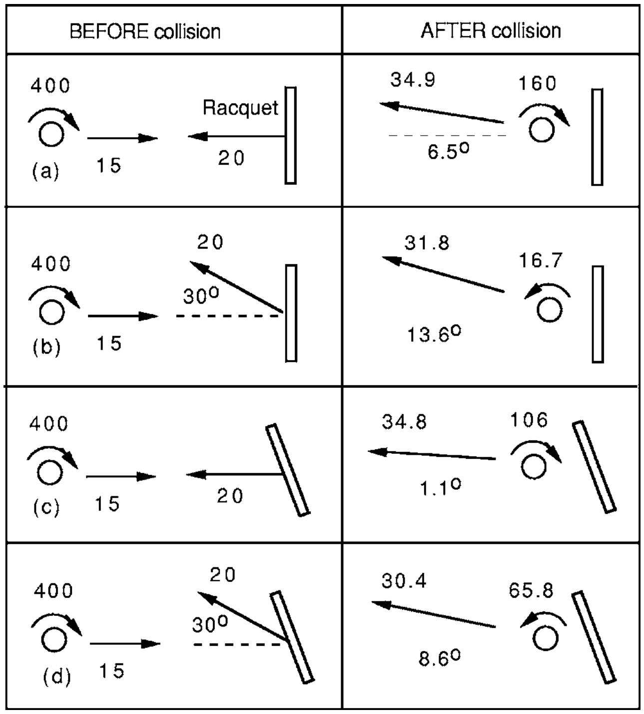

Fig. 7. Impact of a tennis ball with the strings of a racquet. Before the collision, the ball moves horizontally to the right at 15 m/s spinning at 400 rad/s. The racquet approaches the ball at 20 m/s along a horizontal path in (a) and (c), or rises upward at 30° to the horizontal in (b) and (d). The racquet is tilted forward at 5° to the vertical in (c) and (d). The resulting velocity, spin, and rebound angle of the ball after the collision is shown in the diagrams on the right, assuming e_y=0.42 and the bounce parameters given by Eqs. (6) and (7).

in the two techniques in that the outgoing ball trajectories are different. A desirable trajectory for a groundstroke is one where the ball is projected slightly upward in the court frame, in which case a player will usually use both techniques simultaneously to achieve the desired result, as shown in Fig. 7(d). The technique shown in Fig. 7(d) results in a slightly lower ball speed, but the spin is enhanced considerably and the rebound angle is closer to that required for a successful trajectory over the net and into the opposite court. The extra one inch width of modern racquets has allowed modern players to generate considerably more spin than was possible when players used nine-inch wide wood racquets. The extra topspin allows players to hit the ball faster, which generates even more spin. As a consequence of this positive feedback effect, the game is now played at a much faster pace, using a Western grip to tilt the racquet head, and it is played largely from the baseline.[11]

IX. CONCLUSION

Despite the fact that tennis has been played for several hundred years, there has been no previous attempt to predict the rebound spin and angle of a ball off a racquet under realistic playing conditions. Measurements of the oblique bounce of a tennis ball have previously been made for a clamped racquet but not for a hand-held racquet. Similarly, accurate theoretical studies of the generation of topspin have not been possible because there has been insufficient data for the tangential coefficient of restitution. The results presented in this paper are the first to consider a case of practical interest where a spinning ball is incident on a hand-held racquet at angles near normal incidence. The normal component of the rebound velocity agrees with previous measurements, and the (apparent) tangential coefficient of restitution was found to be -0.11 ± 0.06. This new measurement is significant because it provides the information needed to calculate the rebound spin and angle of a tennis ball off a hand-held tennis racquet. The results were obtained at moderately low ball speeds and spin rates, but there are indications, obtained from high speed data obtained by others using clamped racquets, that a similar result would be found at high ball speeds and spin rates.[8] Calculations of spin generation at a high racquet speed, assuming $e_x=-0.11$, are consistent with the known swing styles of elite players, but accurate measurements of ball spin generated in the game of tennis are yet to be made. Additional measurements for a tennis ball bouncing off a solid wooden floor indicate that a spinning ball has a lower e_x (about 0.07) than a non-spinning ball (about 0.2) and that the frictional properties of the surface have a stronger effect on the tangential ball speed than on the rebound spin. Further theoretical work would be required to account for these results.

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