Experimental study of the gear effect in ball collisions

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The gear effect arises when a ball collides with an object and the object recoils with a component of its acceleration in a direction parallel to the two contacting surfaces. Two experiments are described that show how the surfaces of the ball and the object can engage without slip similar to two gears. The effect is used in golf to correct for off-center impacts and it plays a minor role in baseball.
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I. INTRODUCTION

A ball impacting off-axis on an object such as a bat or club or racquet will cause the object to rotate about an axis through its center of mass. In sports such as golf and tennis, this effect is usually undesirable because the ball will be projected at a speed and an angle that was not intended. Skilled players can avoid this problem by striking the ball in the center of the object or can intentionally strike the ball off-center to achieve another desired result. Equipment manufacturers have been able to minimize the effect by increasing the moment of inertia about the rotation axis and, in some sports, by making use of the gear effect. The rotation of the object in one direction can impart spin to the ball in the other direction, as if the object and the ball were engaged as mechanical gears. An application of the gear effect is employed in the design of golf clubs. The face of a driver is curved and the center of mass of the club head is located well back from the striking surface to optimize the gear effect. As a result, a ball struck off-center will go to one side of the fairway, but the resulting spin imparted to the ball will carry it back toward the center of the fairway.

Penner[1] provided a theoretical description of the gear effect to determine the optimum shape of the head of a golf club, assuming that the ball exits the club in a rolling mode. That is, the tangential speed of the spinning ball is equal to the tangential speed of the club face.

The objective of this paper is to provide experimental data on the gear effect to elucidate the physics of this phenomenon. For that purpose, two experiments were conducted. The first was a simple experiment in which a tennis ball was placed at rest on a horizontal surface, as shown in Fig. 1. The surface was then accelerated in a horizontal direction to measure the resulting speed and spin of the ball. In the second experiment, a golf ball was swung as a pendulum bob to impact a rectangular block of wood, as shown in Fig. 2. The spin acquired by the ball was measured for two values of the impact parameter and at various angles of incidence.

We found that the spin imparted to a ball by the gear effect arises from the static friction force that is generated when there is no slip between the contacting surfaces and when both surfaces are accelerating in a direction tangential to the two surfaces. A ball incident obliquely on a surface usually slides on the surface for a short time before bouncing. At large angles of incidence to the normal, the ball bounces off the surface while it is still sliding. In this case the spin imparted to the ball is due entirely to sliding friction, and the tangential motion of the surface itself can have no effect on the bounce parameters because the friction force is independent of the relative speed of the surfaces in contact. Penner[1] refers to spin generation in this case as an "angle effect." At small angles of incidence, the contact point on the ball slides to a stop relative to the surface, in which case the ball grips the surface before bouncing.[2] In this case, the tangential motion of the surface induced by the impact can affect the bounce in two ways. First, because sliding motion stops sooner when the surface is set in motion, sliding friction acts for a shorter duration and is therefore less effective in imparting spin to the ball. Second, the normal reaction force, N, continues to act during the grip phase and generates a tangential acceleration of the surface if N acts along a line that does not pass through the center of mass of the colliding object and the center of mass does not lie within the plane of the surface.[3] Depending on the direction of the tangential motion of the surface, the gear effect during the grip phase can either increase or decrease the spin of the ball acquired during the sliding phase. In the second experiment, the gear effect was especially evident by the strong asymmetry in the outgoing ball spin with respect to the sign of the angle of incidence.

II. THEORETICAL CONSIDERATIONS: ROLLING BALL EXPERIMENT

Suppose that a ball of mass m and radius r is at rest on a horizontal surface, as shown in Fig. 1. If the surface is initially at rest and then moves horizontally to the right with acceleration a_s, the ball will be subject to a horizontal friction force F acting to the right. If a is the acceleration of the ball center of mass and the ball rotates with angular acceleration α, then $F=ma$ and $Fr=I\alpha$, where $I=\beta mr^2$ is the moment of inertia of the ball about an axis through its center of mass. For a solid sphere, $\beta=0.4$. For a hollow tennis ball with a 6 mm thick wall, $\beta=0.55$. If the ball center of mass has velocity v and the ball has angular velocity ω, then a point P on the ball in contact with the surface will have velocity $v_P=v+r\omega$ and an acceleration $a_P=a+r\alpha$, assuming that the ball rotates in a counter-clockwise direction, as indicated in Fig. 1.

An interesting physics question is whether P has the same acceleration as the surface or whether there is some slip. Also of interest is the relation between F and the normal reaction force $N=mg$. If we define the coefficient of friction between the ball and the surface as $\mu=F/N$, then the ques-

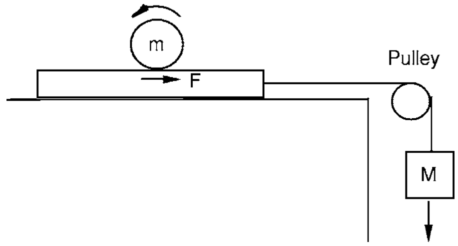

Fig. 1. Arrangement used for first experiment.

Table I. Results of first experiment. The units for a, a_s, and a_P are m/s^2; α is the angular acceleration of the ball in rad/s^2. M is shown in Fig. 1.

M (g)	a_s	a	α	a_P	μ	D (mm)
185	0.26	0.26	0	0.26	0.027	0.88
215	0.78	0.44	9.49	0.76	0.045	0.92
260	1.21	0.59	17.7	1.18	0.060	0.90
355	2.35	1.00	38.8	2.28	0.102	0.98
445	2.79	1.12	48.0	2.71	0.115	0.85
650	3.86	1.50	66.5	3.69	0.153	0.97
1000	5.56	2.11	98.2	5.35	0.215	1.09

tion of interest is whether μ represents a coefficient of sliding or rolling or static friction or whether F arises as a result of stick-slip. If there is no slip, then the ball and the surface engage like two gears. In that case $a_s = a + r\alpha$, and hence

$$F = \frac{ma_s}{(1+1/\beta)}. \tag{1}$$

For a tennis ball, $F = ma = 0.355ma_s = \mu mg$ and hence $a = 0.355a_s$ and $\mu = 0.355a_s/g$. This result is perhaps surprising because μ is proportional to a_s. Usually we would expect μ to be independent of the relative speed or acceleration of the two surfaces. However, F/N is not constrained for static friction, apart from the fact that F/N cannot exceed μ_s, the coefficient of static friction. Consequently, the condition of no slip implies that $a_s < \mu_s g/0.355$. A ball rolling on a stationary surface is a well-known special case where $a_s = 0$ and $F = 0$. If a ball rolls on an accelerating surface, then F is no longer zero. If $a_s > \mu_s g/0.355$, then slip will occur, in which case the horizontal friction force on the ball is given by $F = \mu_k mg$, where μ_k is the coefficient of sliding friction. The horizontal acceleration of the ball will then be given by $a = \mu_k g$, regardless of the value of a_s, and the angular acceleration will be given by $\alpha = \mu_k g/\beta r^2$. The gear effect will then cease to be evident, apart from the fact that motion of the surface to the right will cause the ball to rotate counterclockwise. If the ball were to bounce off the surface (for example, by impact with a small obstacle), it would not exit the surface in a rolling mode.

III. ROLLING BALL EXPERIMENT

A tennis ball of mass 57.5 g and diameter 66.0 mm was placed at rest on a 10 mm thick rectangular block with an upper surface of dimensions 400 mm × 80 mm. The block

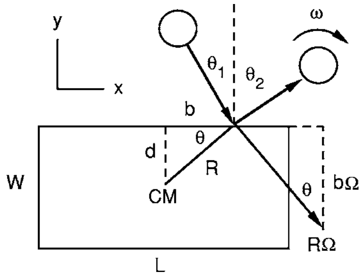

Fig. 2. Plan view of the gear effect experiment showing the positive directions for x, y, ω, θ_1, and θ_2. The gear effect is dominant at large b, large d, and small θ_1, resulting in a counter-clockwise spin of the ball ($\omega < 0$) when the block rotates clockwise ($\Omega > 0$).

was a sample of Rebound Ace, the court surface used for the Australian Open, with a mass of 302 g and a coefficient of friction of 0.70 ± 0.02 for a tennis ball sliding on the surface. The block was placed on a smooth table top and accelerated from rest in the horizontal direction by means of a string and pulley arrangement using a mass M from 185 to 1000 g tied to the end of the string. Simultaneous measurements of the displacement of the block and the ball and the rotation angle of the ball were made at intervals of 0.04 s by filming the block and ball motion with a digital video camera at 25 frames/s. Quadratic fits to the displacement data were used to calculate the relevant velocities and accelerations to within 2%. The accelerations a, a_s, and α remained constant to within 2% while the ball travelled the 400 mm distance from one end of the surface to the other.

The results are shown in Table I, including the calculated distance D between the line of action of the normal reaction force N and the ball center of mass. If N acts at a point to the left of the ball center of mass in Fig. 1, then the torque on the ball is given by $Fr - ND = I\alpha$. In all cases, it was found that D was ≈ 0.9 mm. For low values of a_s, the ball accelerated with $a = a_s$ and without rotation. Such a result can be explained if ND is equal and opposite to Fr. A positive value of D can be explained in terms of ball rotation. The leading edge of the ball rotates into the surface, thereby increasing the normal reaction force at the front edge. In an analogous fashion, the front end of a vehicle dips down when the brakes are applied. The increased reaction force on the front wheels prevents the whole vehicle rotating more than a few degrees.

In Table I the quantity $a_P = a + r\alpha$ is the acceleration of a point on the circumference of the ball. If the ball were to roll without sliding, then $a_P = a_s$. In fact, a_P was 3–5% smaller than a_s in all cases, apart from the one case shown in Table I where $\alpha = 0$. This result implies either that there was a slight slip as the ball rolled along the surface or that the ball bounced slightly off the surface as it rolled from one end to the other. There was no visual evidence of bouncing, but the ball was not perfectly spherical and had a cloth cover. Any local thickening of the cloth might cause the ball to rise or bounce imperceptibly off the surface. Apart from this slight departure from the no slip condition, the ball rolled on the surface. Gross sliding did not occur because a_s remained less than $\mu_s g/0.355 = 19.3\ m/s^2$ for all conditions studied, with the result that μ remained less than μ_s.

IV. SIMPLIFIED THEORY FOR GEAR EFFECT EXPERIMENT

Consider a ball of mass m and radius r impacting a rectangular block of mass M with impact parameter b, as shown in Fig. 2. In the experiment we will describe, the ball was allowed to impact at an oblique angle, but we will restrict the discussion in this section to the simple case where the ball is incident at right angles to the block. Oblique impacts are described in the Appendix. An estimate of the rotation speed and rebound angle of the ball after impact can be obtained as follows. Assume that the ball is incident at speed v_{y1} and exits with velocity components v_{x2} and v_{y2}, where x and y are, respectively, parallel and perpendicular to the surface of the block in its initial position at rest. The rebound angle of the ball is given by $\theta_2=\tan^{-1}(v_{x2}/v_{y2})$. The normal reaction force N generates a change in v_y given by

$$\int N\,dt = m(v_{y1}+v_{y2}) = MV_y, \tag{2}$$

where V_y is the velocity of the center of mass of the block after impact. N also generates a torque on the block, causing it to rotate after impact at the angular velocity Ω given by $b\int N\,dt=I_B\Omega$, where I_B is the moment of inertia of the block for rotation about an axis through its center of mass. For simplicity, the small friction force acting parallel to the block has been ignored in this calculation of block rotation. Consequently,

$$\Omega = \frac{b}{I_B}\int N\,dt = \frac{mb}{I_B}(v_{y1}+v_{y2}). \tag{3}$$

Because the impact point P on the block rotates clockwise about the center of mass, we take the positive directions of Ω and ω to be clockwise, as shown in Fig. 2. The impact point P rotates at linear velocity $R\Omega$, where R is the radial distance from P to the block center of mass. We define the angle θ as in Fig. 2 and find that $R\Omega$ has an x component $R\Omega\sin\theta=d\Omega$, where d is the perpendicular distance from the center of mass to the front surface of the block; the y component is $R\Omega\cos\theta=b\Omega$. Consequently, the velocity of P in the x direction is $V_{px}=d\Omega$ and the net velocity of P in the y direction is $V_{py}=V_y+b\Omega$.

The coefficient of restitution e for the collision is given by

$$e = \frac{v_{y2}+V_{py}}{v_{y1}}. \tag{4}$$

We combine Eqs. (2)–(4) and obtain the result that

$$\Omega = \frac{(1+e)v_{y1}}{b(1+A)}, \tag{5}$$

where $A=I_B(1+m/M)/(mb^2)$. If the block and ball engage as gears without slip so that the ball rotates at angular velocity ω, then $v_x-r\omega=V_{px}=d\Omega$, where v_x is the horizontal speed of the center of mass of the ball. In Fig. 2 we define the positive direction of ω to be clockwise. The gear effect results in counter-clockwise rotation of the ball, with $\omega<0$. The friction force F acting on the ball is given by $F=mdv_x/dt$ and the torque on the ball is given by $Fr=-I_{\rm cm}d\omega/dt$ where $I_{\rm cm}=0.4mr^2$ for a solid sphere. Consequently, $\int F\,dt=mv_{x2}=-I_{\rm cm}\omega/r$. Hence we find that $1.4r\omega=-d\Omega$ and

$$\omega = -\frac{(1+e)dbv_{y1}}{1.4r[b^2+I_B(1/m+1/M)]}, \tag{6}$$

indicating that there is no gear effect when $d=0$ or when $b=0$. For the conditions of the golf ball impact experiment described in the following, Eq. (6) indicates that $\omega=0$ for an impact in the middle of the block and $\omega=-7.6$ rad/s for an impact with $b=50$ mm; ω was observed to be zero for $b=0$ and -9.0 rad/s for $b=50$ mm, in reasonably good agreement with the estimated theoretical values. At $b=50$ mm, the theoretical rebound angle $\theta_2=\tan^{-1}(v_{x2}/v_{y2})=25.1°$, consistent with the experimental value.

For a ball incident normally without spin on a very heavy object with $M\gg m$, the ball rebounds in a direction normal to the surface and without spin. The gear effect, resulting from rotation of a finite mass block, has a relatively large effect on the rebound spin and angle even under the conditions of the present experiment for which $M=4.5m$. Even though the block rotates during the impact, the angle of rotation of the block is relatively small. In the present experiment the impact duration was about 1.0 ms. During this time the block rotated by only about 0.2° for the off-axis impacts, while the ball itself was deflected by an angle of 25°. The gear effect therefore arises not from simple reflection off an inclined surface, but from the tangential static friction force on the ball arising from tangential acceleration of the surface. The essential feature of the gear effect is therefore the effect demonstrated in the first experiment.

V. GEAR EFFECT EXPERIMENT

The arrangement for the second experiment is shown in Fig. 2. A golf ball of mass $m=45.4$ g and diameter 42.7 mm was suspended as a pendulum bob on the end of a 1.52 m length of cotton thread using a small eye hook inserted in the ball to attach the thread. The combined mass of the ball and hook was 46.1 g. The ball was translated horizontally by 30 cm and allowed to impact a 210 g rectangular wood block of dimensions $L=120$ mm, $W=69$ mm, and $H=44$ mm. The block rested on its larger face on a smooth table and the ball impacted on its 120 mm×44 mm face, either in the center of the block or 50 mm off-center. The ball was incident without spin and impacted at an angle of incidence θ_1 with respect to the normal that was varied from −30° to +30°. Positive values of θ_1 and θ_2 are as indicated in Fig. 2. Each impact was recorded at 25 frames/s using a digital video camera located 1 m above the block.

Markings on the ball were used to measure its rotation angle as a function of time so that its angular velocity could be determined together with the incident and rebound speeds and angles. For this purpose, selected video clips were transferred to a computer. Each position of the ball was recorded on a frame by frame basis with VideoPoint software.[4] Separate polynomial fits were made for the incident and rebound trajectories so that the time of impact and the relevant speeds immediately before and after impact could be determined to within ±2%. No measurements were made of the speed or rotation of the block because the block quickly came to rest after sliding a short distance on the table. The friction force on the block was much smaller than the impact force, so that reliable estimates of block speed immediately after impact could be obtained by assuming conservation of linear and angular momentum.

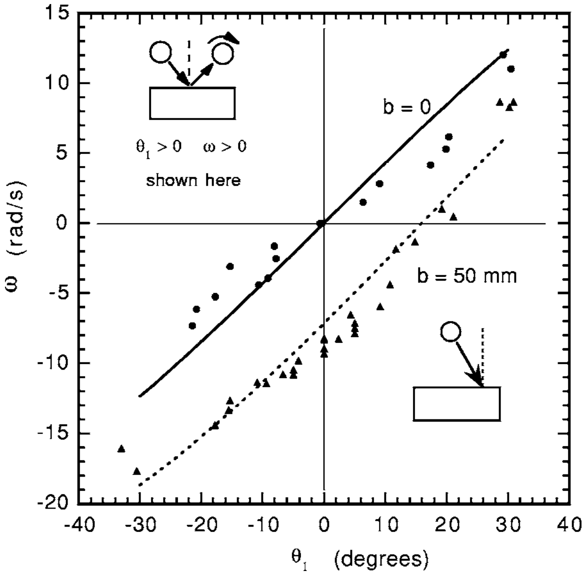

Fig. 3. Experimental values of the ball spin, ω versus the angle of incidence θ_1 for a golf ball impacting a 210 g wood block with impact parameter $b=0$ (dots) and $b=50$ mm (triangles). Also shown are the theoretical estimates based on Eq. (A15) with $e_x=0$ and $e=0.83$ for $b=0$ and $e=0.89$ for $b=50$ mm.

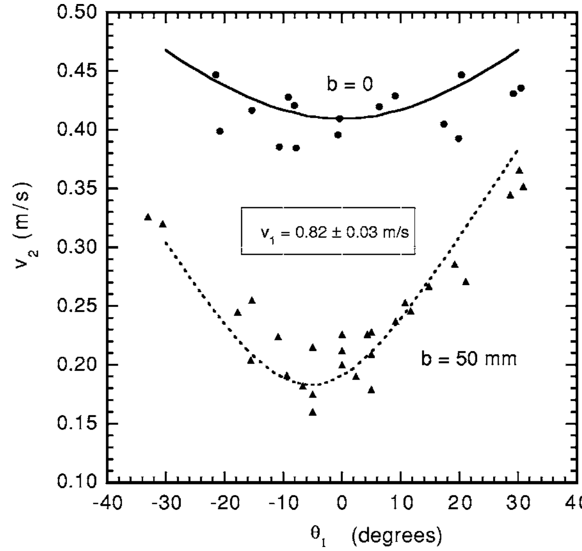

Fig. 5. Experimental values of the rebound speed v_2 versus the angle of incidence θ_1 for a golf ball impacting a 210 g wood block with $b=0$ (dots) and $b=50$ mm (triangles). Also shown are the theoretical estimates derived in the Appendix for the same parameters as those indicated in Fig. 3.

All impacts on the block were at an incident ball speed of 0.82 ± 0.03 m/s. A head-on impact in the middle of the block resulted in a perpendicular rebound of the ball without spin of 0.41 m/s. Conservation of linear momentum implies a block rebound speed of 0.27 m/s and a normal coefficient of restitution $e=0.68/0.82=0.83$. Results at other angles of incidence and for an impact 50 mm off-center are shown in Figs. 3–5. For $b=50$ mm, the coefficient of restitution could not be measured directly, but a good fit to the experimental data was obtained with $e=0.89\pm0.02$. The increase in e in this case was unexpected but is plausible. The effective mass of the block for an off-axis impact is less than its actual mass because the recoil speed of the block at the point of impact is larger due to rotation of the block. The rebound speed of the ball and the impact force were correspondingly smaller, as was the impact duration. It is well known that the coefficient of restitution for any type of collision decreases as the impact speed and impact force increase. Consequently, the increase in e at $b=50$ mm could possibly be attributed to the fact that the impact force was reduced. Reductions in the impact duration and in block vibration energy losses may also have contributed to the increase in e.

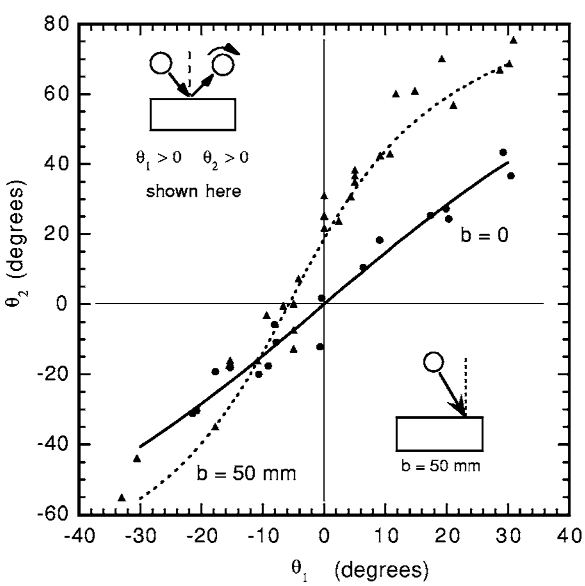

Fig. 4. Experimental values of the rebound angle θ_2 versus the angle of incidence θ_1 for a golf ball impacting a 210 g wood block with impact parameter $b=0$ (dots) and $b=50$ mm (triangles). Also shown are the theoretical estimates derived in the Appendix for the same parameters as those in Fig. 3.

The most obvious indication of the gear effect is the result shown in Fig. 3 where the rebound ball spin ω is plotted as a function of the angle of incidence. For an impact in the middle of the block the ball rebounds with top spin, in the same qualitative manner as it would had it bounced off a wood floor or another very heavy surface. The magnitude of the spin is symmetrical with respect to the sign of the angle of incidence as expected. For off-axis impacts, there is a large asymmetrical offset in the magnitude of the spin, resulting from rotation of the block during the impact. For an impact at normal incidence, clockwise rotation of the block results in a counter-clockwise rotation of the ball of magnitude 9 ± 0.5 rad/s. As the angle of incidence is varied, the resulting ball spin is essentially the same as that for an impact in the middle of the block but offset by about 9 rad/s. Consequently, the ball rotates counter-clockwise for all angles of incidence with $\theta_1 < 15°$ and does not rotate clockwise until θ_1 exceeds about 18°.

The gear effect also results in a change in the rebound angle of the ball, as shown in Fig. 4. For an impact at $b=0$, the rebound angle is symmetrical with respect to a change in sign of the angle of incidence. The rebound angle is asymmetrical for an off-axis impact. The tangential static friction force on the ball that acts to change the ball spin also changes the horizontal speed of the ball. As a result, a ball incident normally at $b=50$ mm was found to rebound at 25° to the normal, in agreement with the theoretical prediction in

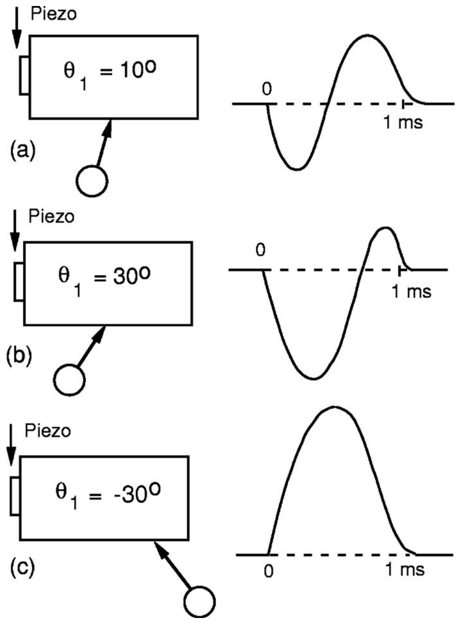

Fig. 6. Output signals from a piezo disk attached to one end of the block. A positive (negative) output indicates acceleration of the block to the left (right). The dashed line in each trace is the zero acceleration level. In each case, the ball was incident without spin and commenced sliding along the block. Reversal of the signal indicates a grip phase followed by a reversal in the sliding direction. The grip phase commenced earlier in (a) than in (b), resulting in a smaller ball spin.

Sec. IV. At other angles of incidence, good agreement was obtained with the theoretical analysis presented in the Appendix.

Measurements of the rebound speed of the ball are shown in Fig. 5, together with our theoretical calculations. The agreement is good, but this plot shows slightly more scatter than the other plots because the rebound speed is sensitive to the precise impact point. Because the ball was launched by hand, it sometimes impacted the block at a point a few mm away from the intended spot. The impact point was monitored when analyzing the film, and impacts more than 3 mm from the intended impact spot were not included in the analysis. The scatter in the data is partly the result of the remaining scatter in the impact parameter, and partly a result of imperfections in the ball itself. A golf ball is covered in dimples spaced about 10° apart around its circumference. The irregular nature of the surface results in slight irregularities in the rebound angle, to the extent that a golf ball dropped vertically onto a perfectly flat, horizontal surface can bounce a few degrees away from the vertical.

VI. MEASUREMENTS OF FRICTION FORCE

A measurement of the friction force acting on the ball was obtained by attaching a small piezo disk to one end of the wood block, as shown in Fig. 6. The disk acts as an accelerometer, because its output is linearly proportional to the acceleration of the block in a direction perpendicular to the disk. Locating the disk precisely in the middle of the end face ensured that there was no response due to rotation of the block. The disk responded only to acceleration of the block in the x direction, arising from friction between the ball and

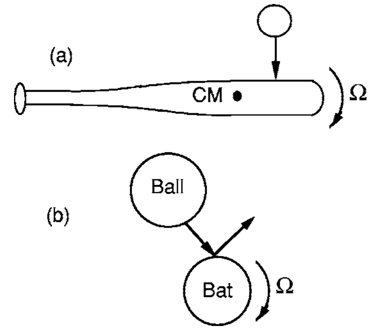

Fig. 7. A ball impacting on a baseball bat can result in bat rotation about two different axes, as shown in (a) and (b).

the block. For impacts in the middle of the block (where $b=0$), the friction force waveforms shown in Fig. 6 were qualitatively similar to those obtained for impacts on an infinitely massive block.[2] A reversal in the direction of the friction force corresponds to a grip phase followed by a backward sliding phase. Impacts toward one end of the block (at $b=50$ mm) resulted in force waveforms consistent with expectations. For example, there is no reversal in the F waveform in Fig. 6(c) because counter-clockwise rotation of the block generated a static friction force on the ball in the same direction as the initial sliding friction force. For an impact with $b=50$ mm and $\theta_1=+30°$, the F waveform was similar to that shown in Fig. 6(b). That is, a reversal in the direction of F was observed because the static friction force arising from rotation of the block acted in the direction opposite to the initial sliding friction force.

VII. DISCUSSION

Our experimental results indicate that the gear effect arises as a result of static friction between a ball and an object with which it collides. Minor discrepancies between our experimental results and theoretical predictions do exist and can be resolved in part in terms of two additional effects described previously.[5] One is the fact that the normal reaction force does not necessarily act through the center of mass of the ball, as indicated in the first experiment. The other is due to the storage of elastic energy in the colliding surface as a result of the local deformation in a direction parallel to the surface.[6] The latter effect results in a finite value for e_x. Satisfactory fits to the experimental results in Figs. 3–5 can be obtained with $-0.2 < e_x < 0.2$, indicating that the primary source of the discrepancies is the offset in the normal reaction force. Of greater significance in the second experiment is the fact that the gear effect has a strong influence on the ball's rebound spin, speed, and angle for impacts resulting in block rotation, consistent with the models described in Sec. IV and in the Appendix.

Although the gear effect is known to be important in the design of golf clubs, it has not previously been examined in relation to baseball collisions. There are two circumstances where the gear effect might play a role. For the situation shown in Fig. 7(a) we can apply the formalism of the Appendix. We consider the collision of a baseball ($m=5.1$ oz) with two bats, one wood and one aluminum, each having the shape of an R161 wood bat, a length of 34 in., and a weight of 31.5 oz. Each bat is impacted 6 in. from the tip, where the radius is 1.313 in., corresponding to $b=5.2$ and 7.8 in. for wood and aluminum, respectively. We consider a typical ball-bat head-on collision ($v_{x1}=0$), with a pitch speed of 90 mph, no initial spin, and a bat speed of 70 mph at the impact location and $e=0.50$. If we transform to the bat rest frame, $v_{y1}=160$ mph. For the wood bat we find $v_{y2}=37$ mph, $v_{x2}=0.78$ mph, and $\omega=226$ rpm. The ball leaves the bat at an angle of 1.2°. If we transform back to the original frame, the ball-bat scattering angle is only 0.4°. For the aluminum bat we find $v_{y2}=30$ mph, $v_{x2}=0.95$ mph, $\omega=276$ rpm, and a ball-bat scattering angle of 0.5°. We conclude that the gear effect results in a negligible scattering angle and a modest spin that is larger for aluminum than for wood.

The situation shown in Fig. 7(b) was considered previously,[5] but can be regarded as being equivalent to an impact with $b=0$ in the second experiment. We find from Eq. (A15) that the ball spin depends on the moment of inertia of the block or the bat, even when $b=0$. The spin is enhanced for a bat with a large moment of inertia, but the effect is not due to the gear effect because there is no tangential acceleration of the surface during the grip phase either by the friction force (which is zero if the ball rolls with $e_x=0$) or by the normal reaction force (which acts through the bat center of mass). The effect of the moment of inertia on the ball spin arises from the effect of the moment of inertia on the bat surface speed during the sliding phase. As described in Sec. I, sliding friction is less effective in inducing ball spin when the bat surface accelerates because the sliding phase is truncated. Aluminum bats generally have a higher moment of inertia than wood bats and are therefore more effective than wood bats in imparting backspin to the outgoing ball.[5]

VIII. CONCLUSION

The experiments discussed in this paper provide a clear picture of the gear effect, where a ball collides with an object in such a way that the ball and object engage like gears. The effect is observed when there is no slip and when both surfaces accelerate in a direction tangential to the two surfaces. For near-normal, off-axis collisions of a ball with a block, it is primarily the normal force that is responsible for the tangential acceleration of the block. Tangential acceleration at the impact point arises when the center of mass of the block is located behind the plane of the impact surface. Static friction is responsible for tangential acceleration of the ball after an initial sliding phase. For conditions where the collision of a ball with an object induces tangential acceleration of the object, the static friction force on the ball can act in the same or opposite direction as the initial sliding friction force. Consequently, the rebound spin and angle of the ball are not symmetrical with respect to the angle of incidence.

A practical application of the gear effect is found in the construction of golf clubs, which can be designed to apply a self-correcting spin to the ball when a player miss-hits the ball. The gear effect plays a small role in baseball and an impact on the barrel of the bat will cause the bat to rotate about its center of mass, thereby inducing accelerated motion of the cylindrical surface of the bat with a component parallel to the surface. However, the geometry of a bat is such that the resulting gear effect spin and rebound angle are negligible.

APPENDIX: DERIVATION OF COLLISION EQUATIONS

The geometry of the collision is shown in Fig. 2. A spherical ball of radius r and mass m is incident with velocity components (v_{x1}, v_{y1}) and no spin on a block of mass M, initially at rest. After the collision, the ball has velocity components (v_{x2}, v_{y2}) and spin ω, while the block recoils with velocity components (V_x, V_y) and angular velocity Ω. The moments of inertia of the block and ball about its center of mass are I_B and I, respectively, where $I=\beta mr^2$ and $\beta=0.4$ for a uniform sphere. It is convenient to define the post-collision surface velocities of the ball and block at the impact point as $v_{px2}=v_{x2}-r\omega$, $V_{py}=V_y+b\Omega$, and $V_{px}=V_x+d\Omega$. The block exerts a normal force N and a tangential force F on the ball. The reaction forces act on the block resulting in

$$MV_y = \int N\,dt, \tag{A1a}$$

$$MV_x = -\int F\,dt, \tag{A1b}$$

$$I_B\Omega = b\int N\,dt - d\int F\,dt, \tag{A1c}$$

from which we find

$$V_{py} = \left(\frac{1}{M} + \frac{b^2}{I_B}\right)\int N\,dt - \frac{bd}{I_B}\int F\,dt, \tag{A2a}$$

$$V_{px} = -\left(\frac{1}{M} + \frac{d^2}{I_B}\right)\int F\,dt + \frac{bd}{I_B}\int N\,dt. \tag{A2b}$$

If we apply a similar procedure to the ball, we find

$$\int N\,dt = m(v_{y2} + v_{y1}), \tag{A3}$$

$$\int F\,dt = \frac{m\beta}{1+\beta}(v_{px2} - v_{x1}). \tag{A4}$$

The equations for the block and ball can be combined to eliminate the forces:

$$V_{py} = \left(\frac{m}{M} + \frac{mb^2}{I_B}\right)(v_{y2} + v_{y1}) - \frac{\beta}{1+\beta}\frac{mbd}{I_B}(v_{px2} - v_{x1}), \tag{A5a}$$

$$V_{px} = -\frac{\beta}{1+\beta}\left(\frac{m}{M} + \frac{md^2}{I_B}\right)(v_{px2} - v_{x1}) + \frac{mbd(v_{y2} + v_{y1})}{I_B}. \tag{A5b}$$

We next define the coefficients of restitution,

$$e_x = -\frac{v_{px2} - V_{px}}{v_{x1}}, \tag{A6a}$$

$$e = \frac{v_{y2} + V_{py}}{v_{y1}}, \tag{A6b}$$

and use them to eliminate the block velocity to find relations between the final and initial velocities of the ball,

$$(1+r_y)v_{y2} - r_{xy}v_{px2} = (e-r_y)v_{y1} - r_{yx}v_{x1}, \quad \text{(A7a)}$$

$$(1+r_x)v_{px2} - \frac{1+\beta}{\beta}r_{xy}v_{y2} = \frac{1+\beta}{\beta}r_{xy}v_{y1} - (e_x - r_x)v_{x1}, \quad \text{(A7b)}$$

where we have defined the dimensionless quantities

$$r_y = \left(\frac{m}{M} + \frac{mb^2}{I_B}\right), \quad \text{(A8a)}$$

$$r_x = \frac{\beta}{1+\beta}\left(\frac{m}{M} + \frac{md^2}{I_B}\right), \quad \text{(A8b)}$$

$$r_{xy} = \frac{\beta}{1+\beta}\frac{mbd}{I_B}. \quad \text{(A8c)}$$

The rolling condition is equivalent to $e_x=0$, although we will continue to keep the explicit factors of e_x in our formulas. We next define three additional dimensionless quantities:

$$k_y = r_y - \frac{1+\beta}{\beta}\left[\frac{r_{xy}^2}{1+r_x}\right], \quad \text{(A9a)}$$

$$k_x = r_x - \frac{1+\beta}{\beta}\left[\frac{r_{xy}^2}{1+r_y}\right], \quad \text{(A9b)}$$

$$k_{xy} = \frac{r_{xy}}{(1+r_y)(1+r_x) - [(1+\beta)/\beta]r_{xy}^2}. \quad \text{(A9c)}$$

With these definitions, we solve Eq. (A7) to find

$$v_{y2} = \left[\frac{e-k_y}{1+k_y}\right]v_{y1} - (1+e_x)k_{xy}v_{x1}, \quad \text{(A10a)}$$

$$v_{px2} = -\left[\frac{e_x-k_x}{1+k_x}\right]v_{x1} + \frac{1+\beta}{\beta}(1+e)k_{xy}v_{y1}. \quad \text{(A10b)}$$

Equation (A10) can be written in a compact form by defining the apparent coefficients of restitution,

$$e_T = \frac{e_x-k_x}{1+k_x}, \quad \text{(A11a)}$$

$$e_A = \frac{e-k_y}{1+k_y}. \quad \text{(A11b)}$$

If we substitute Eq. (A11) into Eq. (A10), we find

$$v_{y2} = e_A v_{y1} - (1+e_x)k_{xy}v_{x1}, \quad \text{(A12a)}$$

$$v_{px2} = -e_T v_{x1} + \frac{1+\beta}{\beta}(1+e)k_{xy}v_{y1}. \quad \text{(A12b)}$$

Finally, we apply angular momentum conservation of the ball about the contact point,

$$\beta m r^2 \omega + m r v_{x2} = m r v_{x1}, \quad \text{(A13)}$$

and use Eq. (A12b) to derive that

$$v_{x2} = \left(\frac{1-\beta e_T}{1+\beta}\right)v_{x1} + (1+e)k_{xy}v_{y1}, \quad \text{(A14)}$$

$$r\omega = \left(\frac{1+e_T}{1+\beta}\right)v_{x1} - \frac{(1+e)k_{xy}}{\beta}v_{y1}. \quad \text{(A15)}$$

Equations (A12a), (A14), and (A15) are the final expressions we use to compare with data.

We note that for normal collisions ($v_{x1}=0$), Eq. (A15) reduces to Eq. (6) if the terms in r_x and r_{xy}^2 are neglected in Eq. (A9c). We further note that our expressions are equivalent to those derived by Penner[1] if the rolling condition is assumed ($e_x=0$), once transformed to the frame in which the ball is initially at rest.

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[1]A. R. Penner, "The physics of golf: The convex face of a driver," Am. J. Phys. **69**, 1073–1081 (2001).
[2]R. Cross, "Grip-slip behavior of a bouncing ball," Am. J. Phys. **70**, 1093–1102 (2002).
[3]Baseball bats, golf clubs, and tennis racquets all differ in this respect. For an off-axis impact, *N* can act through the center of mass of a bat [as in Fig. 7(b)], but it does not normally act through the center of mass of a club or racquet. For a tennis racquet, the center of mass lies in the plane of the impact surface, and the center of mass of a bat or a club lies behind the plane of the impact surface.
[4]⟨www.vpfundamentals.com⟩.
[5]R. Cross and A. Nathan, "Scattering of a baseball by a bat," Am. J. Phys. **74**, 896–904 (2006).
[6]A ball impacting on a soft surface such as rubber would cause the surface to accelerate locally in the contact region due to the finite tangential stiffness of the surface. Acceleration of the surface will also result in a gear effect bounce if the ball grips the surface, in the sense that the static friction force on the ball arises from local acceleration of the surface, possibly as a result of elastic deformation in both the ball and the contact surface.