Cue and ball deflection (or "squirt") in billiards

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A billiard ball struck by a cue travels in the same direction as the cue unless the ball is struck toward one side in order to impart sidespin. In that case the ball deflects or "squirts" away from the line of approach of the cue, typically by a few degrees. Measurements and calculations are presented showing how a cue tip slides across the ball if it is unchalked, resulting in a large squirt angle, and how it grips the ball when it is chalked, resulting in a smaller squirt angle.

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I. INTRODUCTION

A simple problem in mechanics is the determination of the outcome of a head-on collision between two billiard balls. A slightly more complicated problem is to determine the outcome for an oblique collision, a situation commonly encountered in the game of billiards. If an incident ball strikes a stationary ball obliquely, then the two balls head off at an angle of about 90° to each other.[1–3] The coefficient of sliding friction between the billiard balls is very small, typically about 0.06, so the initially stationary ball moves in a direction almost parallel to the line joining the ball centers, corresponding to the direction of the normal reaction force.

A similar situation arises when a cue stick is used to strike a stationary billiard ball. Struck head-on, the ball heads off along the line of incidence of the cue. The ball can be given topspin or backspin by striking it above or below its center of percussion,[1,2] but the ball still moves along the line of the incident cue. The ball can be given sidespin or "English" by striking it to the left or right of center. The coefficient of friction between the cue tip and a ball is larger than that between the balls and can be enhanced by chalking the tip. The ball heads off along a path parallel to the resultant of the normal and friction forces. Surprisingly, the path followed by a ball struck with sidespin is generally within a few degrees of the line of incidence of the cue. If it wasn't, then the path of the ball would be more difficult to predict and players would be reluctant to employ sidespin.

There is a limit to the amount of sidespin that can be imparted to a ball. As the ball is struck further to the left or right of center, the amount of sidespin increases, but so does the deflection of the ball away from the line of the incident cue. The latter effect is commonly known by billiards players as "squirt" because the ball squirts away from its intended path. If the ball is struck too close to its edge, then a miscue will occur, where the tip slides around the side of the ball and the ball squirts away from its intended path by a large angle. The physics of squirt is not well understood, although it is a commonly discussed by pool players.[4,5] Anecdotal evidence[1,4,5] indicates that the squirt angle can be reduced by using a light cue tip, but the author has not seen any experimental data on this effect.

In this paper, experimental and theoretical results are presented showing how the friction force varies with the impact parameter in such a way that the squirt angle remains relatively small when the ball is struck not too far off center and when the tip is chalked. A large fraction of the elastic energy stored in the tip and the shaft is recovered during the latter stages of the impact, with the result that a cue tip has elastic properties similar to that of a superball.

II. NATURE OF THE PROBLEM

Figure 1 shows a situation where a cue stick approaches a billiard ball of radius R with impact parameter b, where b is the perpendicular distance from the center of the ball to the line of approach of the contact point. The tip of a cue has a radius of about 5 mm, so the contact point on the cue tip does not generally coincide with the central axis of the cue. The normal reaction force N acts along a line from the contact point to the center of the ball, while the friction force F acts at right angles to N. The resultant force T acts at an angle β to the radius vector, where $\tan\beta = F/N = \mu$ defines an effective coefficient of friction between the cue tip and the ball. If the tip slides on the ball, then $\mu = \mu_k$, the coefficient of sliding friction. If the tip grips the ball, then $\mu < \mu_k$. If the ball is initially stationary, it will exit from the cue in a direction parallel to T at an angle α to the line of approach of the contact point. The angle α, commonly known as the squirt angle, describes the undesirable deflection of the ball from the intended path.

From the geometry of Fig. 1(b) it can be seen that $\alpha + \beta + \theta = 90°$, where $\cos\theta = b/R$. The angle α is therefore a relatively simple function of the impact parameter and the coefficient of friction and is nominally independent of the mass of the ball and the mass or length of the cue stick. Figure 2 shows a graph of α versus μ for several values of b/R. The main physics question of interest is how the coefficient of friction varies in such a way that the squirt angle remains small regardless of the impact parameter. A related question is why some billiard cues generate smaller squirt angles than others. In principle, the squirt angle would be zero if $\theta + \beta = 90°$ or if $\mu = b/\sqrt{R^2 - b^2}$. The angle α is zero when $b = 0$ because then $F = 0$ and hence $\mu = 0$. If α were zero at say $b/R = 0.5$, then μ would need to be 0.577. In practice, α can be as large as 10° at $b/R = 0.5$, which suggests that a more abrasive chalk might act to reduce the squirt angle. However, the experimental data presented in Sec. IV shows that an increase in μ does not reduce the squirt angle.

III. EXPERIMENTAL METHODS

When a ball is struck with sidespin on a billiards table, the amount of sidespin is difficult to measure experimentally because the ball initially slides and then rolls forward. The ball therefore rotates about two different axes simultaneously.

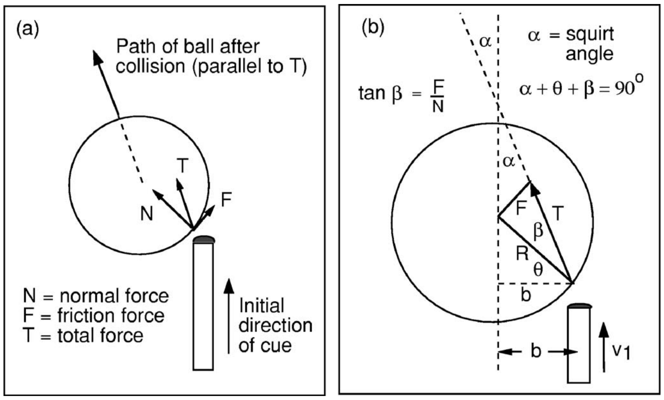

Fig. 1. View of ball and cue looking down onto the table. Sidespin is generated when the ball is struck off-center. The ball is deflected by an angle α away from the line of approach of the cue.

The ball also follows a curved path due to friction with the cloth, unless it is struck at its center of percussion to avoid sliding. Furthermore, the ball is normally struck on a billiards table with the handle end elevated, resulting in spin about the third orthogonal axis, pointing horizontally along the table. Due to friction with the table, the latter spin causes the ball to curve back toward the intended destination, thereby masking the true squirt angle to some extent.[1] All of these difficulties were avoided by mounting a billiard ball as a pendulum bob. The ball had a mass of 120 g and a diameter of 50.3 mm and was marked with a line around its circumference (from top to bottom) so that the rotation angle could be measured on film. The ball was suspended with light cotton thread using a small metal loop stuck to the top of the ball with double-sided tape. The pendulum length was 1.73 m. A video camera was mounted as close as possible to the upper support to view the ball from directly above, and a two-dimensional scale was placed on a table, 3 mm below the ball, to assist with the alignment. The distance scale on the film was calibrated in the same plane as the center of mass of the ball.

The ball was struck horizontally at low speed, typically about 0.7 m/s, with a 94 cm long cue of mass 186 g containing a screw-on tip. One tip was left unchalked, and another tip was well chalked with a high friction silicon chalk normally used in billiards. Blackboard chalk is a softer, low

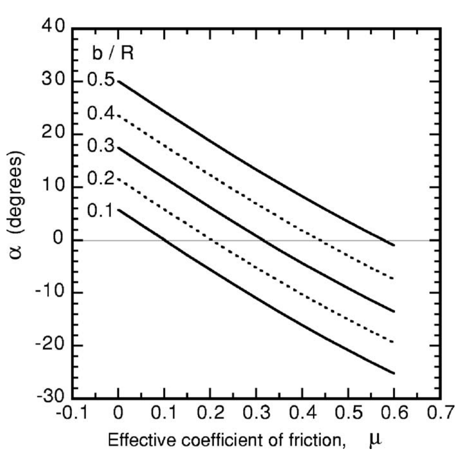

Fig. 2. Squirt angle α versus μ for different values of b/R.

friction, calcium carbonate compound. The cue was shorter than a conventional pool cue and not chosen specifically as a low squirt angle cue. It was a relatively inexpensive cue, suitable for the purpose of the experiment, and generated about twice the squirt angles of a low squirt cue.

The ball speed and spin were each determined to within 2% by analyzing each frame, recorded at 25 frames/s, using VideoPoint® software.[6] The angular deflection of the ball was measured to within $\pm 0.3°$. Measurements were made using both tips by striking the ball at various points along the equator, with an impact parameter between zero and 16 mm. Good billiards players like to chalk the cue tip after every shot to maintain maximum friction between the ball and the tip. To increase the friction force even further, I also took measurements with a 7 mm wide strip of P800 emery paper attached around the ball equator using double-sided tape. P800 emery paper contains fine grained grit, with about 800 grains each linear inch. That is, the diameter of each grain is about 1/800 in.

The impact parameter could not be determined from the video to better than about 2 mm. The location of the shaft could be measured to within 1 mm from the film, but the actual contact point on the tip could not be seen accurately. An alternative method of locating the impact point was used based on the following theoretical argument. The friction force on the ball acts to exert a torque $FR = I_{cm} d\Omega/dt$, where $I_{cm} = 0.4mR^2$ is the moment of inertia of the ball about an axis through its center of mass, and Ω is the measured angular velocity of the ball after the collision. Hence, $\int F\,dt = 0.4mR\Omega$. The velocity of the ball after the collision in the direction parallel to F is given by $v_F = (\int F\,dt)/m = 0.4R\Omega$. The squirt angle α and the absolute speed of the ball v were measured from the film and the angle β was calculated from the relation $\beta = \sin^{-1}(v_F/v)$. The angle θ was then calculated from the relation $\theta = 90 - \alpha - \beta$, giving $b = R\cos\theta$. The calculated value of b was taken as being more reliable than the value estimated from the video film, both values agreeing to within 1 mm in most cases after correcting the video cue position data to take the tip radius into account.

The most critical factor in this experiment is the behavior of the friction force acting on the ball during the collision. It was possible to measure the time integrated value of the friction force in terms of the change in ball momentum or in terms of its spin, but it was not possible to measure the friction force as a function of time during the collision. As an alternative, the time variation of the friction force was measured when the cue impacted on a 210 g, rectangular glass block similar in hardness and surface texture to a billiard ball. The block was placed on a horizontal surface coated with silicone to minimize friction between the block and the surface, and struck horizontally at various angles of incidence. The block was struck slightly off-center, on a trial and error basis, until there was no torque about its center of mass, and hence the block translated on the horizontal surface without rotation. An accelerometer on one face of the block was used to measure the normal reaction force N, and an accelerometer on an adjacent, perpendicular face was used to measure the friction force F. The purpose of the experiment was to determine for an oblique impact whether a cue tip begins sliding on a surface at the beginning of the impact and subsequently grips the surface, or whether the cue tip grips the surface at the beginning of the impact, as suggested by Alciatore[1] and by Shepard.[5] In theory, when two rigid objects

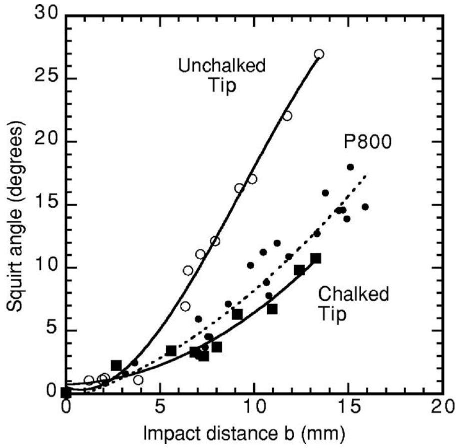

Fig. 3. Measured values of the squirt angle α versus b.

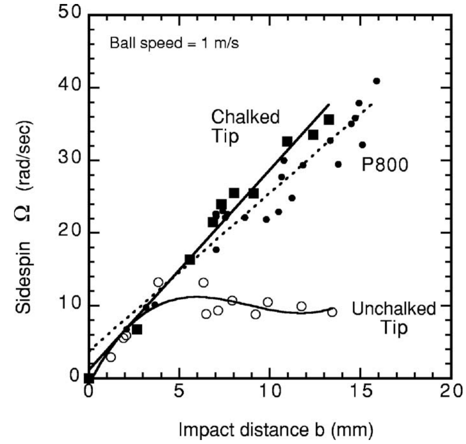

Fig. 4. Measured values of the ball spin Ω versus b, normalized to a ball speed of 1 m/s.

collide obliquely, the contact surfaces will commence sliding and subsequently grip if the relative tangential speed of the two surfaces drops to zero during the collision. However, if the surfaces approach almost at right angles, then the initial sliding period may be negligible, especially if one or both of the surfaces distort elastically in the vicinity of the impact point in a direction parallel to the two surfaces. If the mass of the elastic region is a negligible fraction of the total mass of the colliding objects, then the relative speed of the two surfaces can drop to zero almost immediately on contact.[7] The result is a large reduction in the initial friction force between the surfaces, compared with the result expected if the surfaces slide.

A measurement of the tip acceleration was also made, but not using the wood cue because the cue tip was too small to conveniently mount an accelerometer. Instead, a cue of similar cross-sectional area was constructed using a 1.0 m long aluminum bar of cross section 20 mm × 6 mm, fitted with a rubber tip. A 15 mm diameter piezo disk was mounted on the 20 mm wide face, 25 mm from the tip end. The output voltage provided a measure of the acceleration of the tip in a direction perpendicular to the plane of the disk (that is, transverse to the initial direction of motion of the cue). The cue was used to strike the billiard ball when suspended as a pendulum bob, in the same manner as described previously, resulting in similar squirt angles to those obtained with the chalked tip cue.

IV. EXPERIMENTAL RESULTS

Figure 3 shows the squirt angle α versus the impact distance b for an unchalked and a well chalked tip, as well as for a chalked tip impacting on emery paper taped to the ball. Each point represents a single collision, and the smooth curves are best fit polynomials used to guide the eye. The scatter in the data is the combined result of measurement errors and random variations in the friction force between the cue tip and the ball. Corresponding values of the ball spin Ω and the effective coefficient of friction are shown in Figs. 4 and 5, respectively. The ball spin was normalized to a ball speed of 1.0 m/s, assuming that the spin is linear function of the cue speed and hence to the resulting ball speed. The effective coefficient of friction is given by $\mu=\tan\beta=\int F\,dt/\int N\,dt$. The coefficient μ is less than μ_k if the cue tip grips the ball. The friction force is then due to static friction, is determined by the dynamics of the collision and can even reverse direction during the collision.[8–10] The results in Figs. 4 and 5 have the same functional form because the spin depends linearly on $\int F\,dt$ and because the spin was normalized to the ball speed which is proportional to $\int N\,dt$.

As expected, the unchalked tip has a lower coefficient of friction than the chalked tip, resulting in a generally lower ball spin and a generally larger squirt angle. At small impact parameters, where the cue approaches the ball surface almost at right angles, even the unchalked tip grips the ball. The resulting squirt angle and ball spin are the same, regardless of whether the tip is chalked or not. Surprisingly, there is not a large difference between a chalked tip impacting directly on the smooth ball or on emery paper taped to the ball. The implication is that the squirt angle and the ball spin are almost the same, regardless of the coefficient of sliding friction, provided only that the tip grips the ball during the collision.

Results of measuring the friction and normal reaction forces on the glass block are shown in Fig. 6. At small angles of incidence, the cue tip was found to grip the glass block without an initial sliding stage, and it continued to grip the block throughout the collision. Had the tip commenced sliding on the block, then the F/N ratio would initially be equal to the coefficient of sliding friction. The F/N ratio would

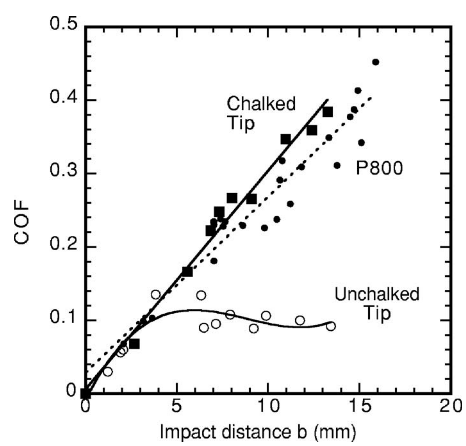

Fig. 5. Measured values of the effective coefficient of friction (COF) versus b.

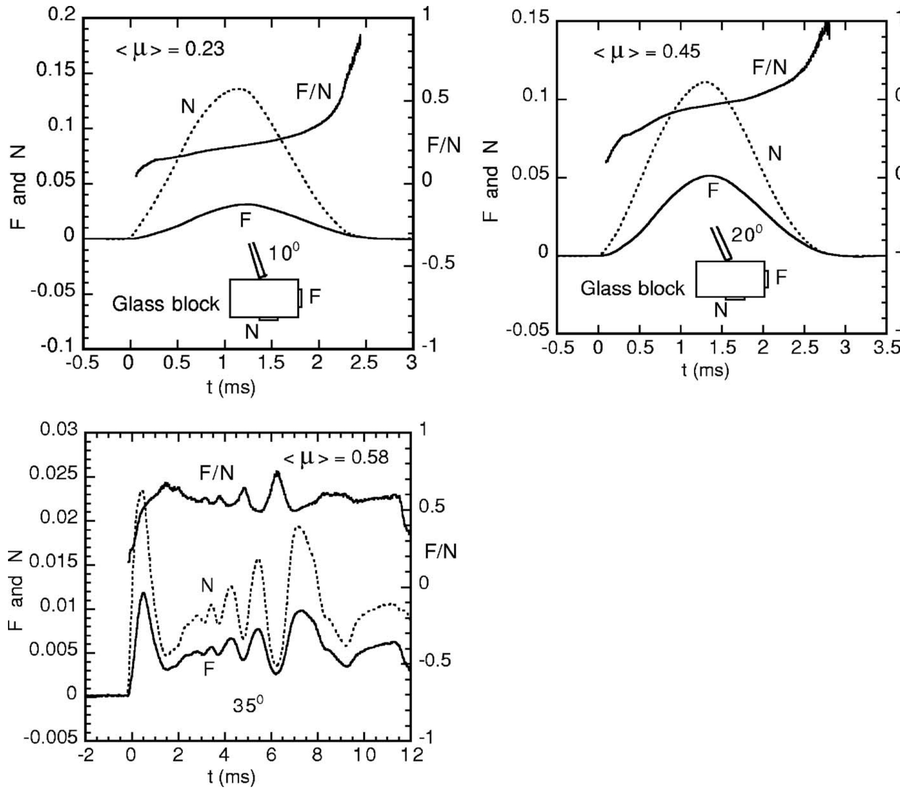

Fig. 6. F, N, and F/N versus time for the impact of a chalked cue on a rectangular block of glass at three angles of incidence. The time average value of $\mu=F/N$, i.e., $\langle\mu\rangle$, increases with the angle of incidence.

have then decreased to a lower value if the tip subsequently gripped the block. Instead, it was found that the F/N ratio was relatively small at the beginning of the impact and increased during the collision. The average value of F/N, integrated over the collision time, increased with the angle of incidence of the cue. At angles of incidence exceeding about 30°, the tip gripped the glass at the beginning of the collision, but then slid on the glass when F/N increased to about 0.7. The sliding stage extended the impact duration to $\approx$12 ms. During the sliding stage the tip commenced vibrating and generated an oscillating force on the block. The oscillation corresponded to a transverse bending mode of the cue, as evidenced by the fact that this mode is dispersive, high frequency components having a higher phase velocity than low frequency components. An accelerometer located at the handle end of the cue revealed that the high frequency components of the wave arrived at the handle about 2 ms after the initial impact at the tip, and the low frequency components arrived at the handle about 4 ms after the initial impact.

The transverse acceleration of the aluminum bar cue, measured 25 mm from the tip of the cue, is shown in Fig. 7. The impact duration was 3.0 ms with this cue. The tip accelerated initially in the expected direction, consistent with the tip gripping the ball and rotating in the same direction as the ball, but the acceleration changed sign before the end of the impact with a consequent decrease in the transverse velocity of the tip. A transverse bending wave reflected off the far end of the cue can be seen in Fig. 7 arriving at the tip after the collision with the ball. Similar behavior of the tip was observed at all impact points on the ball (and on the glass block) provided there was no miscue. As described in Sec. VI, the sudden change in the sign of the acceleration can be attributed to a release of the elastic energy stored in the cue tip.

V. THEORETICAL MODEL

The impact of a cue stick with an initially stationary billiard ball is shown in Fig. 8. The cue stick is modeled as a uniform, rigid rod of mass m and length L and is incident at speed v_1 and angle θ with impact parameter b. A real cue stick is moderately flexible and contains a flexible tip, but we

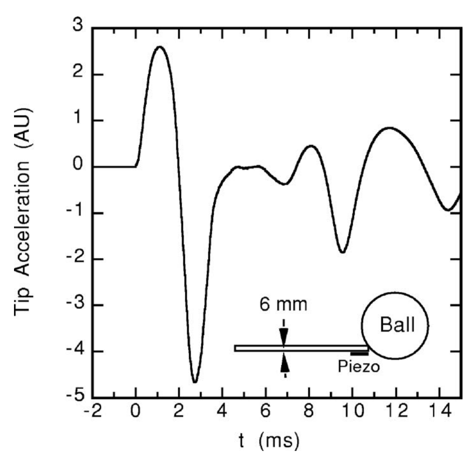

Fig. 7. Transverse acceleration of the tip of a 6 mm $\times$ 20 mm cross section aluminum cue, measured using a piezo disk attached to the tip. The ball was struck 8 mm off center, in the manner shown by the inset.

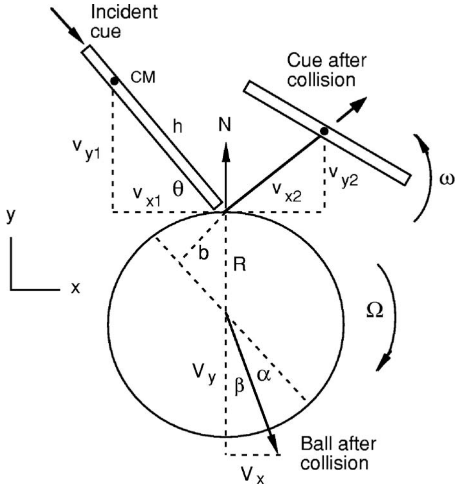

Fig. 8. Geometry of the collision between a cue and ball. The deflection of the cue is exaggerated in this drawing, at least when the cue is supported by hand near the impact point. Without this support, the cue would deflect approximately as shown. The positive directions of ω and Ω are indicated.

assume here that the stick is sufficiently rigid that it has a well-defined center of mass speed and angular velocity both before and after the collision. However, the cue stick is not regarded as being perfectly rigid. The elastic properties of the cue can be described in terms of its coefficient of restitution in the normal and tangential directions. These coefficients describe energy losses in both the cue and the ball, but the ball can be assumed to be perfectly rigid in comparison with the much greater flexibility of the cue and its tip. In this respect, the leather tip of a cue stick can be regarded as a flexible link of negligible mass between the rigid ball and the rigid cue shaft, in a manner described in detail by Stronge.[7] The last few inches of the thin end of the shaft might also be regarded as part of this flexible link, given that its mass and stiffness is much smaller than the rest of the cue.

For a uniform rod, the distance from the tip of the rod to its center of mass is $h = L/2$. For a real cue, h is typically about $0.7L$. The ball has mass M and radius R. The moments of inertia of the rod and the ball about their centers of mass are, respectively, $I_r = mh^2/3$ (or about $0.8mL^2/12$ for a real cue) and $I_b = 0.4MR^2$. The normal reaction force is N and the friction force tangential to the contact surfaces is F.

An x, y coordinate system is chosen so that y is parallel to N at the impact point and x is parallel to F. F acts in the negative x direction on the rod and in the positive x direction on the ball. The components of v_1 are $v_{x1} = v_1 \cos\theta$ and $v_{y1} = v_1 \sin\theta$. If the ball is struck in the normal manner without a miscue, the rod rebounds at angle θ_2, speed v_2, and angular velocity ω with velocity components $v_{x2} = v_2 \cos\theta_2$ and $v_{y2} = v_2 \sin\theta_2$. The ball is set in motion with velocity components V_x and V_y and rotates clockwise at angular velocity Ω after the collision. All quantities are taken to be positive in the following calculations. The coefficient of restitution e_y, in the normal direction is given by

$$e_y = (v_{y2} + V_y)/v_{y1}. \tag{1}$$

The time integrals of F and N over the duration of the impact are given by

$$\int F\,dt = m(v_{x1} - v_{x2}) = MV_x, \tag{2}$$

$$\int N\,dt = m(v_{y1} + v_{y2}) = MV_y. \tag{3}$$

From Eqs. (1)–(3) we find that

$$v_{y2} = \left(\frac{Me_y - m}{m + M}\right) v_{y1}, \tag{4}$$

$$V_y = \frac{m(1 + e_y)v_{y1}}{(m + M)}. \tag{5}$$

The torques acting on the rod and the ball are, respectively, $Nh\cos\theta - Fh\sin\theta$ and FR. Integration over the duration of the impact yields

$$I_r\omega = h\left(\cos\theta \int N\,dt - \sin\theta \int F\,dt\right), \tag{6}$$

$$I_b\Omega = RMV_x. \tag{7}$$

Equation (7) reduces to $R\Omega = 2.5V_x$, indicating that a measurement of ball speed and spin does not provide any information that is not already contained in a measurement of ball speed and squirt angle. Both sets of data depend only on V_x, V_y, and the ball radius. The results shown in Figs. 3 and 4 therefore contain the same information.

If the tip slides on the ball throughout the impact, then $F = \mu_k N$, in which case we find from Eqs. (2) and (3) that

$$v_{x2} = v_{x1} - \frac{\mu_k(1 + e_y)Mv_{y1}}{(m + M)}, \tag{8}$$

and Eq. (6) reduces to

$$I_r\omega = mh(\cos\theta/\mu_k - \sin\theta)(v_{x1} - v_{x2}). \tag{9}$$

The contact end of the rod translates in the x direction at speed $v_{px} = v_{x2} + h\omega \sin\theta$ at the end of the collision, while the contact point on the ball translates in the x direction at speed $V_{px} = V_x + R\Omega = 3.5V_x$. At least, these are the speeds determined from the rigid body model of the cue and the ball. Because the leather tip of a cue is relatively flexible, it is possible for the contact area of the tip to remain stationary with respect to the ball even if the rest of the cue shaft translates at finite speed in the x direction with respect to the ball.

Given that the friction force acts to decrease the x component of the tip velocity from v_{x1} to v_{px} during the collision while the contact point on the ball increases in speed to V_{px}, the tip will slide throughout the collision if v_{px} is significantly larger than V_{px}. The relevant value of μ in this case is the coefficient of sliding friction. However, if $v_{px} < V_{px}$, then Eqs. (8) and (9) are no longer valid because the two surfaces will stop sliding at some time during the collision. When the relative speed of the two surfaces drops to zero, the surfaces will stick or grip as a result of static friction. The experimental evidence obtained by colliding the cue with the glass block indicates that the grip stage is essentially instantaneous, presumably due to the low mass and flexibility of the leather tip. That is, the contact area of the leather tip rapidly came to rest with respect to the block even if the heavy shaft of the cue did not.

There is no simple relation between F and N when static friction is active. Alciatore[1] and Shepard[5] describe similar models to deal with this problem, assuming that the cue grips the ball throughout the collision and that the cue and ball

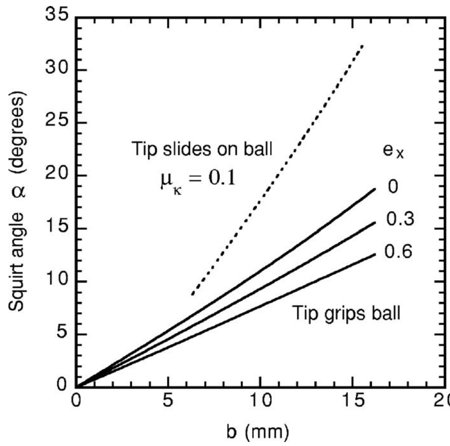

Fig. 9. Squirt angle α versus b for an unchalked cue tip with μ_k=0.1 and for a chalked cue tip with e_x=0, 0.3, and 0.6.

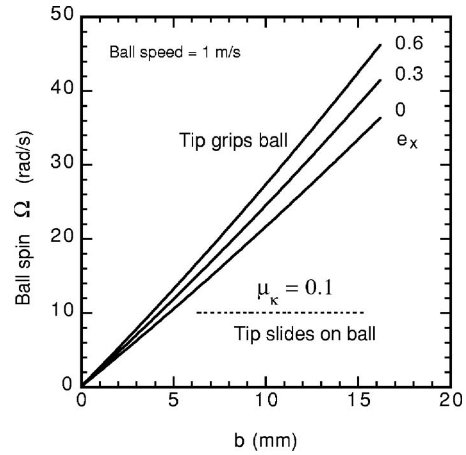

Fig. 10. Ball spin Ω, normalized to a ball speed of 1 m/s, versus b for an unchalked cue tip with μ_k=0.1 and for a chalked cue tip with e_x=0, 0.3, and 0.6.

have a common velocity at the end of the collision in the direction perpendicular to the line of incidence of the cue. The latter assumption is essentially a rigid body approximation, with no allowance for flexibility of the cue tip. Solutions allowing for a flexible tip can be obtained in terms of the tangential coefficient of restitution e_x, defined as the ratio of the relative tangential speed after the collision to that before the collision.[8] In the present case,

$$e_x = -\frac{(v_{px} - V_{px})}{v_{x1}} = -\frac{(v_{x2} + h\omega \sin\theta - 3.5V_x)}{v_{x1}}. \qquad (10)$$

Like e_y, e_x is a quantity that is best determined experimentally. Experimental results obtained with various balls colliding obliquely on different surfaces show that F usually reverses direction during the grip phase, in which case e_x is typically about 0.2.[8–10] In the present case, there was no reversal in the direction of F, at least for a cue tip impacting on a glass block. From Eqs. (2), (3), and (6) we find that

$$h\omega = 3[(M/m)V_y \cos\theta - (v_{x1} - v_{x2})\sin\theta]. \qquad (11)$$

Substitution of Eqs. (2) and (11) in Eq. (10) yields

$$v_{x2} = \frac{(A - e_x)v_{x1} - B}{1 + A}, \qquad (12)$$

where $A=3\sin^2\theta+3.5m/M$, $B=3(M/m)V_y\sin\theta\cos\theta$, and V_y is given in terms of v_{y1} by Eq. (5). Equations (8) and (9) are more directly relevant if the cue tip slides on the ball during the entire collision. Equations (11) and (12) are more relevant if the surfaces grip.

Solutions of Eqs. (2)–(12) are shown in Figs. 9–11 for parameters matching or fitting the experimental data, with M=0.12 kg, R=25.17 mm, m=0.19 kg, h=0.47 m, e_y=0.2, e_x=0.6, and μ_k=0.1. Solutions with e_x=0 and e_x=0.3 are shown for comparison. The value e_y=0.2 was estimated by measuring the bounce height of the cue stick when dropped vertically onto a hard surface. The solution with μ_k=0.1 is shown only for the pure sliding phase. The latter solution provides a good fit to the experimental data for the unchalked tip, at least when $b>6.3$ mm. When μ_k=0.1 and $b<6.3$ mm, the tip is predicted to grip the ball, so the sliding solution was terminated at b=6.3 mm. The alternative grip phase solution with e_x=0.6 provides a good fit to the chalked tip and the P800 data, and also provides a good fit to the unchalked tip data when $b<6$ mm. Similar good fits result when $e_x=0.6\pm0.1$. The observed squirt angles at low b are slightly less than predicted by the theoretical fit, a result that can be matched by decreasing e_y, decreasing m, and increasing e_x. However, such a fit would be inconsistent with the good fit at larger values of b and also inconsistent with the good fits to the ball spin and coefficient of friction results.

VI. DISCUSSION

The essential features of the interaction between a cue stick and a billiard ball are well described by the model developed in Sec. V. The modeled behavior of the friction force acting on the ball as shown in Fig. 11 is consistent with the experimental data shown in Fig. 5. The force arises from static friction when the cue grips the ball. The dynamic response is therefore similar to that occurring when a cue is pushed obliquely at an angle δ onto a heavy, horizontal surface. The cue can be supported in an equilibrium position by static friction, without sliding, if $\tan\delta=F/N$. The dynamic response in the present case is similar, in that F increases as θ increases, with F being slightly smaller in the dynamic case due to rotation of the cue. The resultant force on the cue

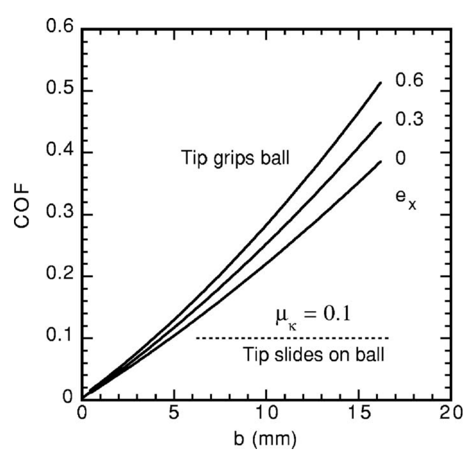

Fig. 11. The effective coefficient of friction given by $\int F\,dt/\int N\,dt=V_x/V_y$ versus b for an unchalked cue tip with μ_k=0.1 and for a chalked cue tip with e_x=0, 0.3, and 0.6.

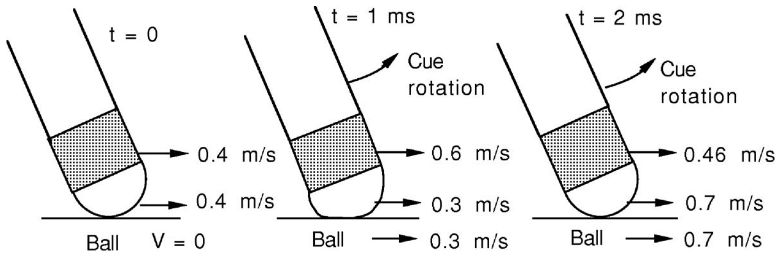

Fig. 12. Inferred behavior of the cue tip and shaft. The tangential speeds are representative, not calculated values. During the first half of the collision, the leather tip decelerates tangentially as it grips the ball, while the shaft accelerates tangentially due to the counter-clockwise torque on the cue. During the second half of the collision, elastic energy stored in the tip is released, accelerating the ball and decelerating the shaft.

does not act along the axis of the cue through its center of mass, but is slightly offset and causes the cue to rotate. If the resultant force did act along the cue in the dynamic situation, then the effective coefficient of friction would be equal to tan δ, the cue would not rotate, the resultant force on the ball would also act along the axis of the cue, and the squirt angle would be zero.

Of particular interest is the fact that e_x is relatively large, indicating that a large fraction of the elastic energy stored in the cue tip is recovered. In this respect, the cue tip has elastic properties similar to that of a superball which also has an e_x value of about 0.6. In contrast, the value of e_x for other spherical balls impacting on a rigid surface is typically about 0.2. The leather tip appears to be relatively inelastic, given that the coefficient of restitution of the cue in the normal direction was found to be only about 0.2. The large tangential elasticity of the cue may therefore be due in part to high flexibility and low energy loss in the thin wood shaft at the end of the cue to which the leather tip is attached.

A plausible representation of the interaction between the cue tip and the ball, consistent with both the model and the experimental data, is shown in Fig. 12. At time $t=0$, when the tip first contacts the stationary ball, both the leather tip and the shaft approach the surface of the ball at a tangential speed of, say, 0.4 m/s. The tip immediately grips the ball, so that after a delay of, say, 1 ms, the tip and the ball are moving at a common speed of about 0.3 m/s, while the shaft has accelerated to about 0.6 m/s due to counter-clockwise rotation of the cue about its center of mass. The latter result is consistent with the model which indicates that the torque arising from the normal reaction force on the cue is slightly larger than that due to the friction force on the cue. Because the ball surface and the leather tip move tangentially at a lower speed than the shaft, the leather tip stretches elastically in the tangential direction and causes the shaft to bend slightly as well. High speed video film of the interaction between a cue tip and a billiard ball, showing that the leather tip stretches in the manner indicated in Fig. 12, can be seen at Ref. 1.

The elastic energy stored in the leather tip and the shaft, arising from displacement in the tangential direction, is recovered during the latter half of the impact while the normal reaction force drops back to zero. An analogous action occurs when a person flicks the middle finger by pressing and gripping the nail against the thumb to store elastic energy in the finger and then releases the stored energy by moving the thumb away or by pushing harder with the finger until the thumb releases its grip. In Fig. 12 the tangential speed of the ball and leather tip at the end of the impact ($t=2$ ms) is estimated as 0.7 m/s, while the speed of the shaft is estimated as 0.46 m/s to give the observed result that $e_x=(0.7-0.46)/0.4=0.6$.

Additional evidence supporting the behavior shown in Fig. 12 was observed on film taken at 100 frames/s. The evidence is shown in Fig. 13, which includes two frames before a particular collision, one frame during the collision, and one frame 10 ms after the collision. The impact duration was 2.1 ms, as measured by a piezoelectric disk accelerometer attached to the end of the handle. The tip deflected slightly to the left, while the ball deflected to the right at a squirt angle of 6°. The main feature of interest in Fig. 13 is that in the 10 ms interval following the collision, the contact point P on the ball moved significantly further in the tangential direction than did the tip of the cue. Such a result can be obtained artificially by decelerating the tip rapidly after contacting the ball by "jabbing" at the ball. But in this case the tip was pushed forward at approximately constant speed, as evidenced by the position of the tip in the following frames. At the end of the impact, the tip was therefore moving backward with respect to the ball. At least the last few inches of the wood shaft moved backward with respect to the ball.

VII. CONCLUSION

The impact of a cue stick and a billiard ball was found to be well described by a model in which the cue is allowed to grip the ball throughout the collision, provided the impact parameter is sufficiently small. The effective value of the coefficient of friction, given by the time integral of the friction force divided by the time integral of the normal reaction force, is equal to the coefficient of sliding friction μ_k only if the cue slides on the ball for the entire impact duration. Otherwise, the coefficient of friction is less than μ_k by an amount that depends on the impact parameter, falling to zero for a head-on collision where the impact parameter is zero. The deflection angle of the ball with respect to the line of incidence of the cue increases with the impact parameter, but remains less than about 8° for impacts up to half the ball diameter off axis. The impact model shows that the deflection angle does not depend on μ_k, provided that μ_k is large enough for the tip to grip the ball. The experimental data shows that elasticity of the cue tip plays a dominant role in the collision process and suggests that cues with thin shafts might generate lower squirt angles as a result of their greater flexibility rather than their lower mass. Further theoretical and experimental work will be needed to determine why some cues generate lower squirt angles than others.

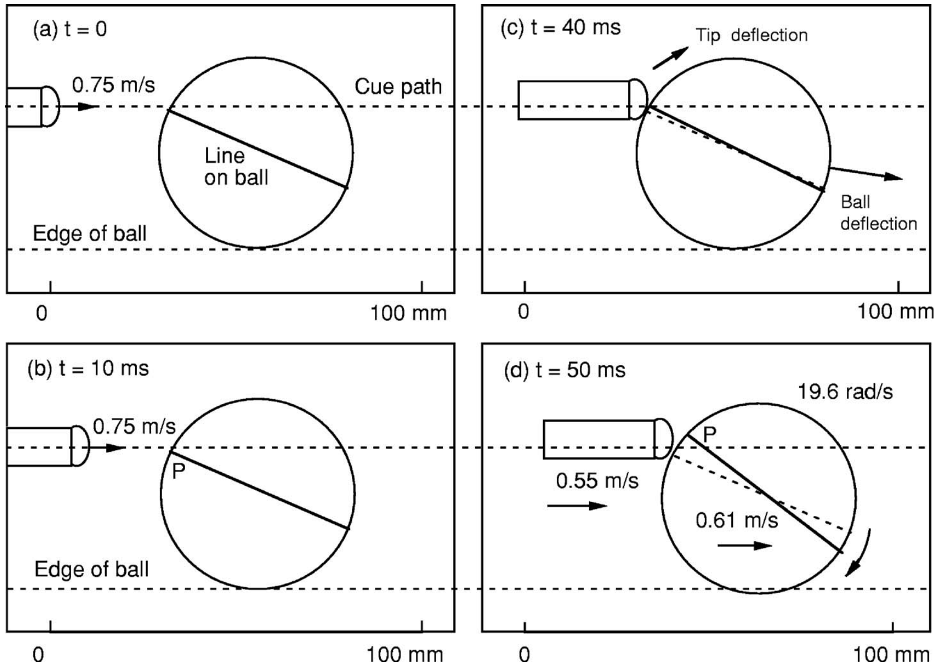

Fig. 13. Impact of a chalked cue tip with a billiard ball as seen on 100 frames/s film, viewed from above. The four rectangular borders and the two dashed horizontal lines define a fixed reference frame and fixed reference positions for the ball and the cue. P is the contact point on the ball located at $b/R=0.4$. The ball rotated through 11.2° from position b to position d, and rotated through about 1.2° during the 2.1 ms impact.

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