

The sweet spots of a tennis racquet

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Abstract

Measurements are presented on the behaviour of a hand-held tennis racquet when it impacts with a tennis ball. It is shown that an impulse is transmitted through the racquet to the hand in about 1.5 ms, with the result that the hand and the forearm both have a strong influence on the behaviour of the racquet even while the ball is still in contact with the strings. Regardless of the impact point, the racquet head recoils as a result of the impact and an impulsive torque is applied to the hand, causing the hand to rotate about an axis through the wrist. The impulsive forces on the hand, arising from this torque, do not drop to zero for any impact point, even for an impact at either of the two sweet spots of the racquet. Forces on the hand arise from rotation, translation and vibration of the handle. For an impact at the vibration node, only the vibrational component is zero. For an impact at the centre of percussion, the net force on the hand or forearm is zero since the forces acting on the upper and lower parts of the hand are then equal and opposite.

Keywords: centre of percussion, hand, rotation, sweet spot, tennis

Introduction

The sweet spot of a tennis racquet is often identified, especially by manufacturers and their advertising agents, as the impact point that imparts maximum speed to the ball. This is not a well-defined point on the racquet. It can be located anywhere on the longitudinal axis between the tip and throat, depending on the incident speed of the ball (Brody 1997; Cross 1997). Alternatively, the sweet spot of a tennis racquet can be defined as the impact point that minimizes the impulsive forces transmitted to the hand. In 1981, Brody noted that there should be two such spots, one corresponding to a vibration node and one corresponding to the centre of percussion (COP). For a conjugate point near the end of the handle, both spots are close to

the centre of the strings, so it is difficult to distinguish one from the other in terms of the qualitative feel of the impact. To date, there have been no definitive experiments to distinguish the two points in terms of measured reaction forces on the hand. The sweet spots described by Brody were defined primarily in terms of a racquet that is freely suspended, with no restraining force acting on the handle. For a freely suspended racquet, the COP is not a unique point on the strings since there is no unique conjugate point (i.e. axis of rotation) in the handle. The present work examines the effects of the hand on the two sweet spots and provides a well-defined location for the COP in terms of the impulsive force acting on the forearm. As shown below, the relevant location is the end of the handle.

There has been debate for many years as to whether the hand plays a significant or a negligible role in determining the dynamics of the impact of a racquet and ball. The collision of a tennis racquet with a tennis ball can be modelled (Leigh and Lu

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1992; Brody 1995, 1997) by assuming that the hand plays no role during the impact, in which case the racquet can be regarded as being freely suspended. The main arguments presented to support this model are that (a) the hand has only a small effect on the vibration frequency of the racquet and (b) the ball will leave the strings before the impulse is transmitted along the racquet frame to the hand. Experimental studies of the effect of the hand are not entirely consistent with this model. Elliott (1982) and Watanabe *et al.* (1979) studied the effects of grip firmness on the coefficient of restitution (COR). In these studies, a ball was projected onto the strings of a racquet and the ball rebound speed was measured under various grip conditions and for impacts at several different locations on the strings. It was found that grip conditions have a negligible effect on the COR for impacts near the centre of the strings, even under extreme conditions where the racquet is either freely suspended or the handle is rigidly clamped. However, it was also found that the COR increased slightly with grip firmness for off-centre impacts.

Four new approaches have been adopted in this paper to examine the effect of the hand on the racquet (a) by measuring the propagation delay of the impulse along the racquet; (b) by comparing measured values of the handle velocity of a freely suspended racquet with those of a hand-held racquet; (c) by measuring the reaction forces on the hand and (d) by measuring the velocity of the forearm. It is shown below that an impulse is transmitted through the racquet to the hand in about 1.5 ms, with the result that the hand has a strong influence on the behaviour of the racquet, even while the ball is still in contact with the strings. The rotation axis and the vibration node in the handle are both shifted, from their locations in a freely suspended racquet, to points under or close to the hand. The reaction forces on the hand do not drop to zero for an impact at the vibration node, nor for an impact at the centre of percussion. The forces vary from one point to another under the hand, being negative at some locations and positive at others. This is because the racquet applies an impulsive torque to the hand, causing the hand to

rotate about an axis through the wrist. The net force acting on the hand is difficult to measure, but a good indication is provided by measuring the velocity of the forearm, at a point close to the wrist, during and after the impact.

An issue that is not directly addressed in this paper is whether the hand plays a significant role in determining the outgoing speed of the ball. If the leading edge of the pulse reflected from the hand arrives back at the ball just as the ball is leaving the strings, then the ball will be largely unaffected by the hand. Theoretical and experimental results recently obtained by the author support this hypothesis and the results will be described elsewhere.

Experimental techniques

All of the measurements presented in this paper were made using a 1990 vintage Wilson graphite composite racquet of mass 370 gm and length 685 mm. All measurements were made under conditions where the racquet was initially at rest, the ball was incident at low speed in a direction perpendicular to the strings and the ball impacted at a point on the central axis passing through the handle and the centre of the strings. These conditions are rarely encountered during normal play, but the physics of the collision between a ball and a racquet does not depend strongly on the speed of the ball or the racquet and is independent of the reference frame in which the collision is studied. The experimental conditions were therefore chosen to simplify the data collection process as far as possible and to ensure that the impact conditions were reproducible.

In order to measure the propagation delay and handle velocity, five piezoelectric disks were attached to the racquet: one in the centre of the strings, and one each at 24 cm, 17 cm, 12 cm and 1 cm from the end of the handle. The piezo elements, in the form of circular disks of diameter 19 mm, 0.3 mm thick, were extracted from piezo buzzers commonly available from electronics shops. The disk on the strings was glued with epoxy resin and three other disks were taped firmly

to the handle to avoid independent vibration of the elements themselves. The piezo at the far end of the handle was glued to a flat wall of the rectangular cross-section cavity inside the handle, in order to avoid the additional and variable response due to the pressure of the hand on a piezo element mounted on the outside of the handle. The piezo at 1 cm was therefore located under the base of the hand, and the piezo at 12 cm was located just beyond the index finger. The piezo disks were very light in weight (1.8 gm) and had no observable effect on the properties of the racquet, as evidenced by the fact that the signal observed from any one disk was not effected by adding or removing any or all of the other disks.

A brass electrode bonded to one side of each piezo disk and the silvered electrode on the other side were connected to 10 M Ω oscilloscope probes via very light connecting leads. It was necessary to tape the connecting leads to the racquet handle at points close to the piezo elements in order to avoid any spurious response as a result of independent motion of the leads. The piezo outputs were observed directly, in order to monitor the racquet acceleration, and were also integrated with a simple RC circuit, of time constant 100 ms ($R = 1 \text{ M}\Omega$, $C = 0.1 \text{ }\mu\text{F}$), in order to monitor the racquet velocity. The output of a piezo is directly proportional to the applied force and is therefore proportional to the acceleration of the disk. All piezos were connected to give a positive output when compressed, and all traces in this paper were recorded on a DC-coupled digital storage oscilloscope, pretriggered several ms prior to the impact in order to record the zero level of the corresponding acceleration or velocity waveforms. The outputs were not calibrated to determine the absolute acceleration or velocity since the only measurements of interest were the time delays between waveforms and the relative velocity at different points on the racquet. The technique of using an integrated piezo signal to measure racquet velocity has not previously been described as far as the author is aware. The validity of the technique was confirmed by an independent velocity measurement, obtained by differentiating the racquet dis-

placement waveform, as measured by the displacement of a small capacitor plate attached to the racquet frame, relative to a parallel fixed plate.

The racquet was suspended vertically by a 60-cm string tied to the handle, or held in the normal fashion by hand but with the strings in a horizontal plane. Tests with other lengths of string confirmed that the length chosen was adequate for the purpose of simulating the response of a completely free racquet, and that the restoring force of the string was negligible during and for at least 30 ms after the impact. A tennis ball was dropped or thrown at low speed, from a distance of about 10 cm, onto the strings near the tip or throat of the racquet or directly onto the piezo disk in the centre of the strings. The results of these measurements are shown in Figs 1–4.

In order to measure the reaction forces on the hand, a separate experiment was performed using a 9-mm diameter piezo disk, of thickness 0.3 mm, located at a point on the handle underneath the hand. A second 9-mm diameter disk was located on the strings to provide a reference signal for timing purposes and was attached to the strings by means of re-usable adhesive putty so that it could be easily relocated to several different points on the strings. To minimize bending of the piezo on the strings, it was bonded with epoxy to a 0.5-mm thick, 10-mm square sheet of epoxy fibreglass. The piezo under the hand was taped to the handle with clear adhesive tape so that it could easily be relocated to different points under the hand on relatively flat parts of the handle. The observed signals were found to be accurately reproducible even after relocating the piezo many times. However, the area under the tip of the little finger was too close to the knob on the end of the handle to generate reliable results.

Measurements of hand forces using force sensing resistors have previously been made by Knudson and White (1989). They reported considerable variability in the magnitude of the observed impulsive forces, which they attributed to variations in impact location and racquet velocity. In the present experiment, these variations were minimized since

the initial racquet velocity was zero and the impulsive forces were measured at selected impact locations.

Despite the low speed of the impacts studied, qualitatively similar results can be expected during high speed impacts. The phenomena described in this paper are almost entirely linear up to the elastic limits of the racquet, strings and ball. The only nonlinear process of any significance during the collision of a ball with a racquet relates to the effects of hysteresis in the ball. The coefficient of restitution and the duration of the impact varies slightly with ball speed (Brody 1979), but this will have no effect on the transmission time of a pulse along the handle or on the resulting effect of the impact along the handle or on the resulting effect of the impact. The vibration amplitude of the fundamental mode remains zero for an impact at the node, regardless of the ball speed.

Transit time of an impulse along the handle

The transit time of an impulse from the impact point to the hand has not previously been measured for a tennis racquet. It can be obtained from the time delay between the signal recorded on the strings and the signal recorded at a point on the handle close to the hand. Results are shown in Fig. 1 for an impact at the centre of the strings, and in Fig. 2 for an impact on the strings near the tip of the racquet.

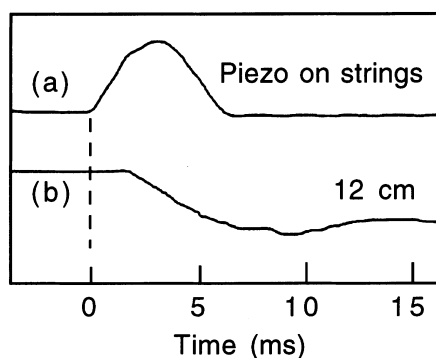


Figure 1 Measurement of handle velocity for a hand-held racquet when a ball is dropped onto the centre of the strings. Traces show (a) the direct output of a piezo disk at the centre of the strings and (b) the handle velocity waveform at a point 12 cm from the end of the handle.

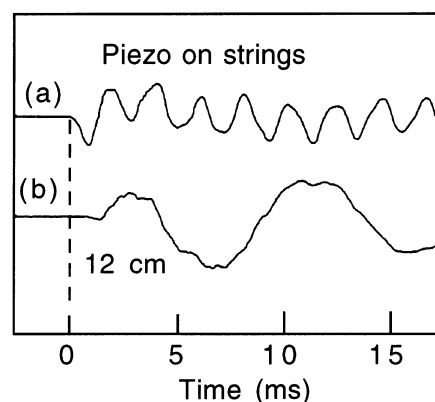


Figure 2 Measurement of pulse propagation for a hand-held racquet when a ball is dropped on the strings near the tip. Traces show the direct outputs of piezo disks located at (a) the centre of the strings and (b) 12 cm from the end of the handle. The time integral of waveform (b) is shown in Fig. 4.

In both cases, the racquet was hand-held and stationary prior to the impact. In Fig. 1, the fundamental vibration mode of the frame is excited with very low amplitude since the impact occurs close to a node for this mode. Motion of the handle is therefore due almost entirely to rotation and translation of the racquet, the vibrational component being negligible.

Figure 1(a) shows the direct piezo signal detected when the ball is dropped onto the piezo in the centre of the strings, and Fig. 1(b) shows the waveform of the handle velocity (i.e. the integrated acceleration waveform) measured simultaneously at a point 12 cm from the end of the handle. The traces in Fig. 1 were triggered 4 ms before the ball impacted the strings. The ball exerts a force on the strings that is approximately a half-sine pulse of duration 7.4 ms, at least for this low impact speed test. The negative polarity waveform from the piezo located on the upper surface of the handle, indicates that the handle deflected downwards, in the same direction as the incident ball. From the relative magnitude of the velocity waveforms at other positions along the handle, it was concluded that the racquet pivoted about a point near the end of the handle, immediately on arrival of the impulse at the hand. The transit time of a pulse from the centre of the strings to the point 12 cm from the

end of the handle was 1.5 ms, as indicated by the time delay between the two corresponding waveforms in Fig. 1. The racquet handle therefore begins to move well before the ball leaves the strings and reaches a maximum velocity just after the ball leaves the strings.

The pulse propagation time is much faster than previously estimated (Brody 1997). Brody estimated the propagation time from an analysis of the fundamental vibration period. In Fig. 1, vibrational motion of the racquet frame is not readily apparent since the impact occurred close to a node of the fundamental mode and since higher frequency modes are excited with relatively small amplitude and attenuate more rapidly than the fundamental mode. Nevertheless, the impact excites a broad spectrum of frequency components and the resultant motion of the handle represents a superposition of all transverse waves excited by the impact. The fundamental mode of vibration of the hand-held racquet had a measured frequency of 102 Hz and a wavelength of about 0.8 m, with nodes about 15 cm from each end of the racquet. The wave speed of this mode is therefore about 80 m s^{-1} . The propagation time from the centre of the strings to the end of the handle, a distance of 0.53 m, is therefore about 6.5 ms for the fundamental mode. The next vibration mode has a theoretically predicted frequency of 276 Hz, a wavelength of about 0.52 m and a velocity of about 143 m s^{-1} . For this frequency component, the transit time from the centre of the strings to the end of the handle is 3.6 ms. The initial motion of the handle, 1.5 ms after the ball first contacts the strings, cannot simply be explained in terms of the first few vibration modes, nor purely in terms of rigid body rotation. For an infinitely stiff racquet, one would expect zero delay between the initial impact and motion of the handle. The short delay must therefore represent the combined effects of all high frequency transverse waves generated by the impact. Because of the increased stiffness of a racquet for short wavelength vibrations, high frequency transverse waves in a racquet, or any other solid beam, propagate faster than low frequency transverse waves (Cross 1997, 1998). The high wave speed through the strings also contributes to the

short delay time. The velocity is approximately $2fL \approx 300 \text{ m s}^{-1}$ where $f \approx 500 \text{ Hz}$ is the vibration frequency of the strings and $L \approx 0.3 \text{ m}$ is the string length. The propagation delay from the centre of the strings to the frame is therefore about 0.5 ms, accounting for 1/3 of the observed delay and about 1/3 of the transit distance.

The above interpretation is supported by the results shown in Fig. 2, where the ball impacted the strings near the tip of the racket. The piezo on the centre of the strings responds mainly to vibrations of the strings at 500 Hz, and the piezo on the handle generates a waveform representing the acceleration of the handle at that point. The string vibrations are not seen in Fig. 1 since the force on the piezo due to compression of the ball is much larger than the force due to the string vibrations. In Fig. 2, the fundamental mode at 102 Hz is seen clearly, but higher frequency components appear at the beginning of the handle acceleration trace, after a propagation delay of 1.5 ms. A similar effect has also been observed with a baseball bat. An estimate of the propagation time along a baseball bat, based on the fundamental mode frequency, indicates that the ball should leave the bat well before the impulse arrives at the hand. In fact, measurements show that the impulse arrives at the hand before the ball leaves the bat (Cross 1998).

Measurements of handle velocity

Given that an impulse propagates to the hand well before the ball leaves the strings, one would expect that the reaction force from the hand should have a significant effect on the motion of the racquet even while the ball is still in contact with the strings. This effect was investigated by comparing the handle velocity for a freely suspended racquet with that of the same racquet when it was hand-held. The results are shown in Figs 3 and 4, respectively.

Figure 3 shows the handle velocity at several points along the handle for a freely suspended racquet and for an impact 8 cm from the tip of the racquet. The absolute values of the handle velocity were not calibrated, but the relative velocities were preserved by recording and displaying all signals at

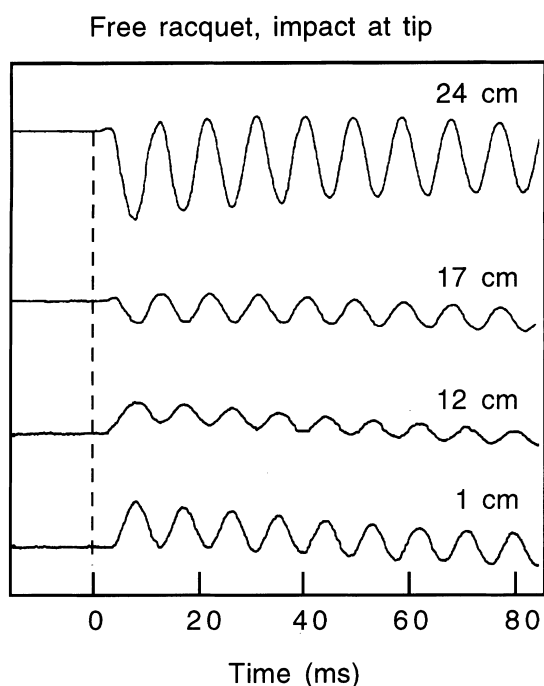


Figure 3 Measurements of the handle velocity for a freely suspended racquet initially at rest. The ball impacted the strings at low speed 8 cm from the tip of the racquet, and the handle velocity was measured at points 24, 17, 12 and 1 cm from the end of the handle, as indicated. The velocity is negative if the handle moves in the same direction as the incident ball.

the same sensitivity. The vibrational components of the velocity traces at the 24 cm and 17 cm locations are in phase, but the traces at the 17 cm and 12 cm locations are 180° out of phase, indicating that a vibration node exists at a point about 15 cm from the end of the handle, as expected for the fundamental mode of oscillation of a beam that is free at both ends (Cross 1997).

All of the traces in Fig. 3 have an obvious DC as well as an AC component, except for the trace at 17 cm where the DC component is close to zero. It can be inferred from these results that the DC component is zero at about 16 cm, at which point the velocity due to rotation is equal and opposite to the velocity due to translation. The racquet therefore rotates about an axis located about 16 cm from the end of the handle. The racquet has a measured moment of inertia $I_{cm} = 0.017 \text{ kg m}^2$ for rotation

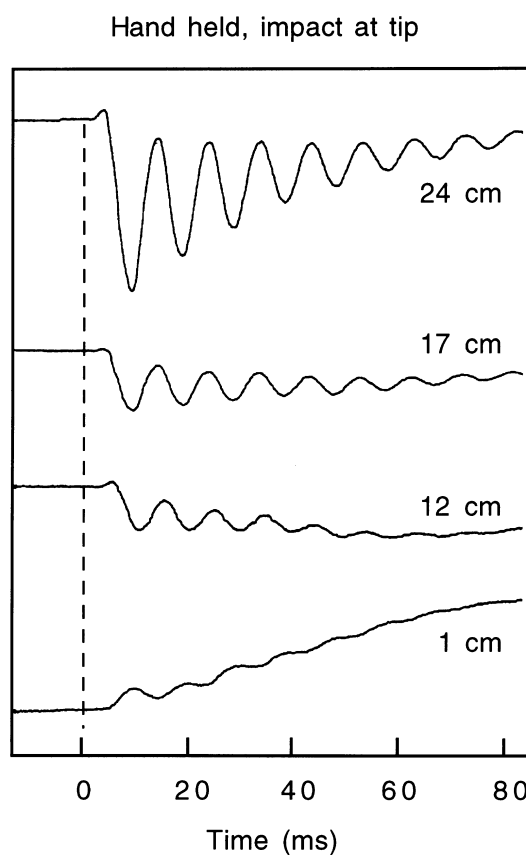


Figure 4 Measurements of the handle velocity for a hand-held, horizontal racquet that is initially at rest. The ball was dropped on the strings 8 cm from the tip of the racquet and the handle velocity was measured at points 24, 17, 12 and 1 cm from the end of the handle, as indicated. The velocity is negative if the handle moves in the same direction as the incident ball (downwards).

about the CM. For an impact 8 cm from the tip of the racquet, the conjugate point (i.e. the actual axis of rotation) is expected to be located 16 cm from the end of the handle, as observed. At least, that is the case immediately after the arrival of the impulse at the handle and for a short period after the ball leaves the strings. On a longer time scale, the DC component of the traces in Fig. 3 drifts slowly, partly as a result of the weak restoring force due to the string suspension and partly due to the 100 ms time constant of the integrator.

The corresponding handle velocity traces for a hand-held racquet are shown in Fig. 4. The gain settings and drop heights were held constant to

compare the relative magnitudes of the handle velocity, and the traces were all triggered at the same fixed time before the arrival of the pulse at the centre of the strings. However, the gain for the 1 cm position is double the gain at other locations in order to show the oscillations more clearly. The vibrational mode of the racquet is similar to that of a freely suspended racquet, but the frequency is slightly lower, the vibrations are more strongly damped and the vibration node shifts to a point somewhere under the hand, judging from the fact that the oscillations at 1 cm and 12 cm are about 180° out of phase. The vibration amplitude decreases towards the end of the handle with a slight phase shift along the handle. The axis of rotation, for an impact near the tip of the racquet, shifts from the 16 cm position for a free racquet to a position about 5 cm from the end of the handle. The axis of rotation is established immediately on arrival of the impulse at the handle, 1.5 ms after the ball first contacts the strings, as evidenced by the traces in Figs 1–4. For an impact between the centre of the strings and the throat of the racquet, the axis of rotation was observed to be close to the end of the handle, regardless of whether the racquet was free or hand held, as was expected since the impact is near the centre of percussion.

Figure 5 (and also Fig. 8) shows schematically the results of the above measurements. The shift in location of the node in the handle can be explained qualitatively by the fact that the vibration amplitude of the handle is reduced when it is hand-held, thereby approximating the behaviour of a racquet that is pivoted or clamped at the handle end. The shift of the axis of rotation is explained by the fact that the end of the handle does not translate freely but is constrained by the inertia of the hand and forearm. Both of these effects have also been observed with a hand-held baseball bat (Cross 1998).

A shift in the location of the vibration node and the lowering of the frequency can be roughly modelled if one assumes that the hand acts as an additional mass loading the end of the handle. The vibration frequencies in Figs 3 and 4 are, respectively, 109 Hz (free) and 102 Hz (hand held).

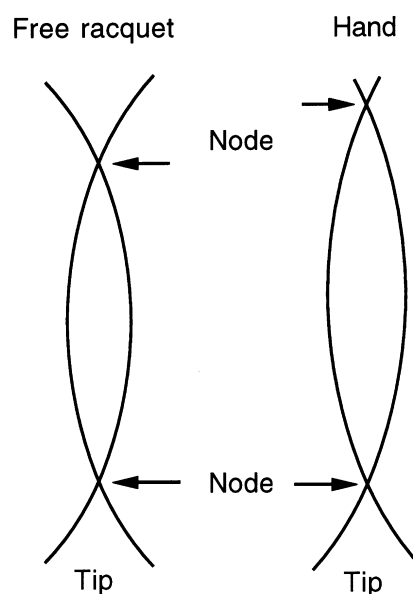


Figure 5 Vibration modes of a free and a hand-held racquet.

Figure 6 shows the effects of adding 40 g and 80 g masses to the end of a freely suspended racquet. The vibration frequency drops from 109 Hz with no additional mass to 103 Hz with an additional 40 g mass and to 100 Hz with an 80 g mass. The observed frequency shift when the racquet is hand held can therefore be modelled by the additional 40 g, as noted previously by Brody (1995), but the shift in the node location is not correctly simulated by the additional 40 g mass. The vibration node shifts from 15 cm to a point 12 cm from the end of the handle when a 40 g mass is added, and it shifts even further towards the end of the handle when an 80 g mass is added, since the vibrations at 17 cm and 12 cm are then in phase. It is therefore possible to simulate the shift in node location and the lowering of the frequency by additional masses, but both effects cannot be simulated simultaneously with the same additional mass.

The axis of rotation also shifts towards the end of the handle as additional mass is added to a freely suspended racquet, but the actual shift observed when the racquet is hand-held can only be simulated by adding a mass in excess of 80 g to the end of the handle. As shown in Fig. 4, a hand-held

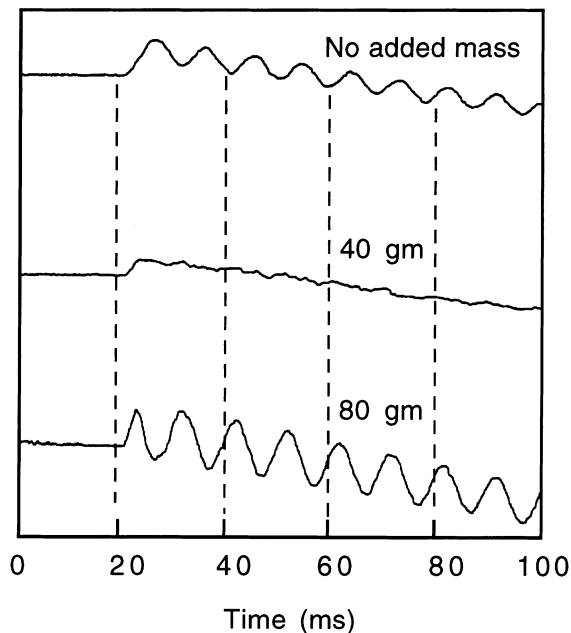


Figure 6 Effects of adding 40 and 80 g masses to the end of the handle for a freely suspended racquet. The traces show the velocity waveforms at a point 12 cm from the end of the handle, for an impact near the tip of the racquet. The phase difference between the top and bottom traces varies as a function of time since the vibration frequencies are slightly different.

racquet deflects downwards during the impact, at a position 12 cm from the end of the handle, but the deflection is upwards in Fig. 6 even with 80 g added to the handle. The large mass required to shift the rotation axis can be attributed to the effect of the hand and arm on the racquet dynamics, provided the dynamics are modelled correctly. As shown by Casolo and Ruggieri (1991), the effective mass of the forearm is less than the actual mass since the racquet applies an impulsive force to the end rather than the centre of the arm, the other end of the arm being pivoted at the elbow. Furthermore, the arm is not rigidly attached to the handle, due to the flexibility of the wrist. Consequently, the effect of the hand and the arm cannot be simulated correctly simply by adding a fixed mass to the end of a freely suspended racquet. The dynamics of the situation can be modelled as shown in the following Section.

Effect of the arm on racquet dynamics

A simple model of the effect of the arm on racquet dynamics, consistent with the above observations, is shown in Fig. 7. The racquet is approximated as a beam of mass M and length L connected by a pivot joint to the forearm, which is represented as a beam of mass M_F and length L_F . It can be assumed that the other end of the forearm is pivoted about the elbow, but it is assumed for simplicity that the elbow does not translate during the impact. The impact of a ball on the racquet can be represented by an impulsive force, F , applied at a distance b from the racquet CM, the CM being located a distance h from the end of the handle. The handle will exert an impulsive force F_R on the forearm, resulting in a reaction force $-F_R$ on the handle. The equations of motion are then

$$F + F_R = M dV/dt \quad (1)$$

$$Fb - F_R h = I_{cm} d\omega/dt \quad (2)$$

and

$$F_R L_F = I_F d\omega_F/dt \quad (3)$$

where V is the velocity of the CM of the racquet, I is the moment of inertia of the racquet about its CM, I_F is the moment of inertia of the forearm about the elbow, ω is the angular velocity of the racquet and ω_F is the angular velocity of the forearm. The velocity of the pivot joint at the wrist is given by

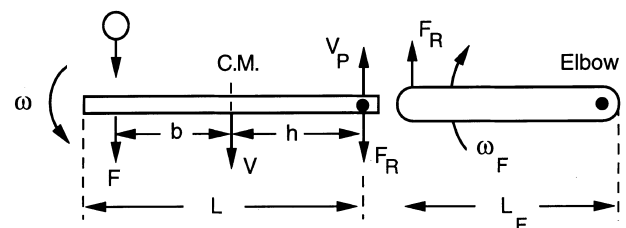


Figure 7 Model used to evaluate the effect of the forearm when a ball impacts at a distance b from the racquet centre of mass. The racquet is pivoted at the wrist and the forearm is pivoted at the elbow.

$$V_P = L_F \omega_F = b\omega - V \quad (4)$$

In these equations, it is assumed that ω is measured in an anticlockwise sense (as in Fig. 7) and that ω_F is measured in a clockwise sense as appropriate for an impact near the tip of the racquet (as in Fig. 8). Similarly, V is taken as positive when the racquet moves downwards in Fig. 7 and V_P is taken as positive when the pivot joint moves upwards.

Since the racquet exerts a force on the forearm at the pivot joint, an effective mass of the forearm, M_E , can be defined by the relation $F_R = M_E dV_P/dt = M_E L_F d\omega_F/dt$. So from eqn 3, $M_E = I_F/L_F^2$. For example, if the forearm is approximated as a uniform beam, then $I_F = M_F L_F^2/3$ so $M_E = M_F/3$. Equations (1)–(4) can be combined to show that $dV/dt = x d\omega/dt$ where

$$x = \frac{I_{cm} + M_E b(b + b)}{Mb + M_E(b + b)} \quad (5)$$

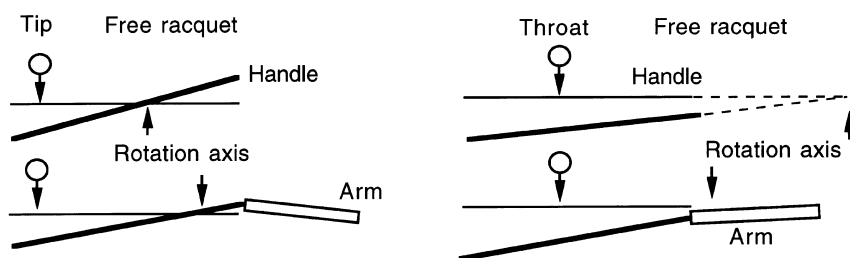
Now consider a point on the racquet located a distance x to the right of the CM where the racquet velocity is $V - x\omega$. This point will coincide with the axis of rotation of the racquet if its velocity remains constant during the impact, or if $dv/dt = x d\omega/dt$. Consequently, the location of the conjugate point, i.e. the axis of rotation, is given by eqn 5. This relation reduces to the well-known expression $x = I_{cm}/(Mb)$ when $M_E = 0$, corresponding to a freely suspended racquet (Brody 1979). It can also be seen from eqn 5 that if $x = b$, then $b = I_{cm}/(Mb)$ which is the same result that one obtains for a freely suspended racquet. This particular value of b defines the centre of percussion, since $F_R = 0$ for this value of b , and the racquet behaves as if it were completely free. For

any other impact point, the forearm constrains the motion of the racquet, and the conjugate point is shifted closer to the end of the handle than for a free racquet, regardless of whether the conjugate point lies within the handle or beyond the end of the handle. This effect is shown schematically in Fig. 8

The results of the previous Section can be modelled with the measured parameters $M = 0.37$ kg, $I_{cm} = 0.017$ kg m², $b = 0.33$ m and $b = 0.27$ m, corresponding to an impact 0.08 m from the tip of the racquet. When $M_E = 0$ then $x = 0.17$ m, meaning that the axis of rotation is located 16 cm from the end of the handle when the racquet is freely suspended. However, when $M_E = 0.6$ kg, $x = 0.29$ m, so the axis of rotation is shifted to a point 4 cm from the end of the handle. Both calculations are consistent with the observed results. This value of M_E is consistent with an approximate estimate of the mass of the forearm, about 1.8 kg, but x does not depend strongly on M_E when M_E is larger than the mass of the racquet. For the same racquet parameters, the centre of percussion (COP) is located at $b = 0.14$ m, assuming that the axis of rotation coincides with the end of the handle. This locates the COP 5 cm from the centre of the strings, as illustrated in Fig. 9.

According to the above theoretical model, the reaction force F_R acting on the end of the forearm should be zero for an impact at the centre of percussion. Previously, it has been assumed that for an impact at the COP, the reaction force on the hand would be zero (Brody 1979, 1981). One might expect that the force on the hand should be essentially the same as the force on the forearm. However, the measurements presented in the

Figure 8 Schematic diagram comparing the motion of a free and a hand-held racquet when a ball is dropped near the tip or throat of the racquet, showing the racquet position before the impact (thin line) and after the impact (thick line). For a hand-held racquet, the axis of rotation shifts to a point closer to the wrist.



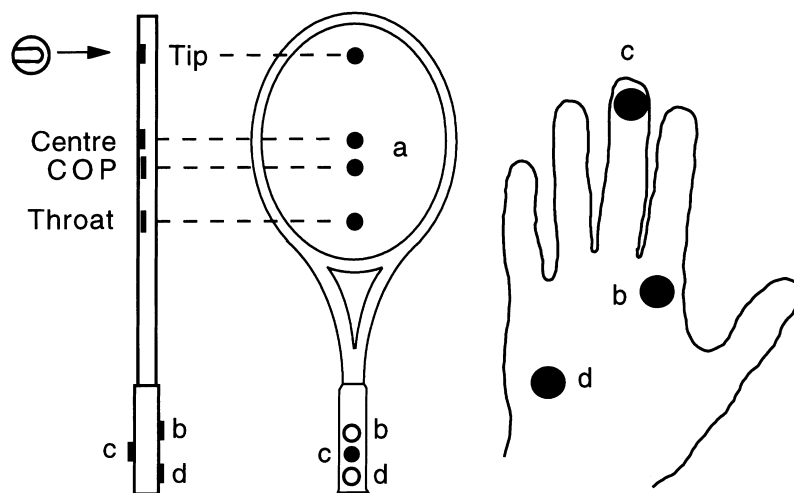


Figure 9 The force acting on the hand was measured at three points b, c and d, for four impact positions on the strings, as labelled. The distance from the Centre to the COP was 5 cm.

following section surprisingly show that the forces acting on different parts of the hand can be quite large, even when the force on the forearm is zero. The result is easily interpreted in terms of the net force on the hand. This will remain zero if the forces on the upper and lower parts of the hand are equal and opposite, even if these forces vary with time. Recent measurements of the hand forces by Hatze (1998) are consistent with the results described below, but Hatze concluded that the COP was of limited significance since the forces on different parts of the hand vary with time.

Forces acting on the hand

In order to measure the forces on the hand, a small piezo was located on the handle, underneath the hand, as described in the section on Experimental Techniques. The impulsive forces acting on the hand were measured at three different points under the hand, and for four different impact points on the strings, as indicated in Fig. 9. The racquet was held firmly by the right hand in a stationary position with the strings in the horizontal plane, and a tennis ball was dropped onto the strings from a height of 20 cm. The results of this experiment are shown in Fig. 10. Absolute values of the force were not calibrated, but the relative magnitudes can be compared with the 150 mV positive signal

recorded when the handle was gripped firmly by the hand (or – 150 mV when the grip was released). The grip waveform decayed to zero with a time constant of 70 ms, representing the discharge time constant of the 7 nF piezo through the 10 M Ω voltage probe. This component of the force waveform therefore decayed to zero prior to each impact measurement. The forces shown in Fig. 10 therefore represent the change in the force at each point as a result of the impact. The largest impulsive force signal was –80 mV, representing the first negative peak recorded at position *d* for an impact at the tip.

The results obtained for an impact at the centre of the strings are easiest to interpret since the racquet frame does not vibrate in that case. The force on the hand is observed to increase at the base of the index finger (waveform b) and decrease at the base of the little finger (waveform d), during and after the impact, indicating clearly that the handle moves towards the base of the index finger and away from the base of the little finger. With respect to the hand, the racquet therefore rotates about an axis that is located between the index and little fingers. Since the force on the middle finger remains small at all times, the axis of rotation within the hand is located almost exactly in the middle of the hand. The actual axis of rotation of the racquet in the laboratory frame may be differ-

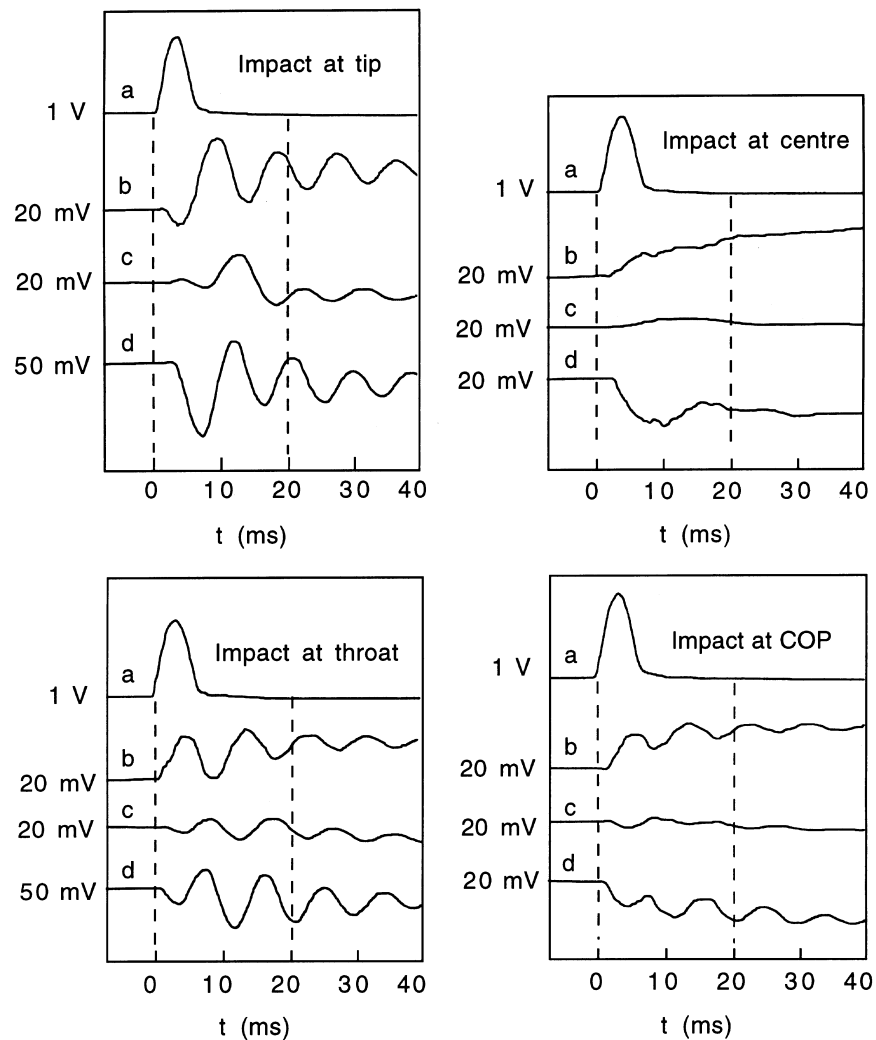


Figure 10 Forces acting on the hand at points *b*, *c* and *d*, as labelled in Fig. 9. Waveform *a* represents the force of the ball on the strings, as measured by a small piezo on the strings. The vertical gain of the oscilloscope used to record waveforms *b*, *c* and *d* was either 20 mV or 50 mV per division, as labelled. A positive force indicates an increase in the force on the hand. Zero impulsive force corresponds to the horizontal part of each trace prior to impact.

ent since the hand itself rotates about an axis through the wrist and it can translate as a result of motion of the forearm.

An off-centre impact results in vibration of the frame, as well as translation and rotation of the frame. The forces on the hand resulting from vibration increase as the impact point moves further from the centre of the strings. The 'DC' component of the force waveforms is qualitatively similar to that observed for an impact at the centre of the strings, regardless of the impact point. The racquet therefore rotates within the hand about an axis that is near the centre of the hand, regardless of the impact point. If the axis of rotation in the

laboratory frame was located further up the handle towards the racquet head, the handle would move away from the base of the index finger, not towards it. Consequently, for all of the results shown in Fig. 10, the axis of rotation in the laboratory frame is either in the middle of the hand or shifted to a point close to or beyond the end of the handle. For an impact at the throat, the DC component of waveform (*d*) is significantly smaller than at other impact locations, and is close to zero for the first 10 ms, indicating that the axis of rotation of the racquet was close to the end of the handle.

An interesting feature of the results in Fig. 10 is that there is a phase shift of about 90° between the

vibrational components of waveforms b and d. This effect is presumably associated with the fact that these waveforms are recorded on the same side of the handle but on opposite sides of the vibration node under the hand. The node itself is not clearly apparent but is close to the centre of the hand, regardless of impact point, judging by the reduced vibration amplitude of waveform c. For an undamped standing wave, there is a phase shift of 180° between any two points on opposite sides of a node. For a damped standing wave, the phase shift is generally less than 180° since the phase angle varies continuously from $+90^\circ$ to -90° from one antinode to the next. The phase jumps discontinuously by $+180^\circ$ at a node only when the damping is zero.

The results in Fig. 10 are consistent with those obtained by Knudson and White (1989) and by Hatze (1998) who also found that the force increases at the base of the index finger and decreases at the hypothenar eminence, during the impact, corresponding to an impulsive rotation of the racquet in the same (expected) sense as observed in this paper.

There is no impact point on the strings where the forces on the hand are zero everywhere. The vibrational component is zero at all points under the hand for an impact at the vibration node, and this node clearly qualifies as a sweet spot in terms of the qualitative 'feel' of the racquet. The smallest DC forces acting on the hand occur for impacts between the centre of the strings and the COP, and the largest forces occur for an impact at the tip of the racquet. The COP has no special significance with respect to the forces acting on different parts of the hand. However, it is of major significance in determining the force on the forearm, as described in the following Section.

Impulsive motion of the forearm

Since the forces acting on the hand vary from one point to another, a measurement of the force acting at a single point under the hand does not provide a valid indication of the total force of the handle on the hand (or of the hand on the handle). In principle, one could sum the forces on the hand at

many different points under the hand to determine the total force, but this is not a practical proposition. Alternatively, a reasonable assumption is that waveform b represents the net force on the upper part of the hand and waveform d represents the net force on the lower part of the hand. The net force on the hand is then given approximately by the sum of waveforms b and d. Measurements of the impulsive motion of the forearm indicate that this is indeed a good approximation.

In order to measure the impulsive motion of the forearm, a 19-mm diameter piezo was strapped, in wrist-watch fashion, to the forearm around the wrist as shown in Fig. 11. The piezo itself was attached with adhesive tape to a 2-mm thick, 25-mm diameter fibreglass disk in order to avoid bending of the piezo and in order to provide anchor points for the band around the wrist. The output of the piezo was integrated with a 100-ms time constant integrator to measure the velocity waveform. The acceleration waveform is more difficult to interpret since the largest component is due to vibration of the racquet and arm. The polarity of the acceleration signal is therefore dominated by the polarity of the vibration component. The velocity waveform provides a less ambiguous indication of the response of the arm due to rotation and translation. The vibration component is not filtered out, but integration acts to attenuate the amplitude of the high frequency components of the waveform. The velocity measurement was tested for reliability in a number of ways, including simple motion of the arm up or down without the racquet

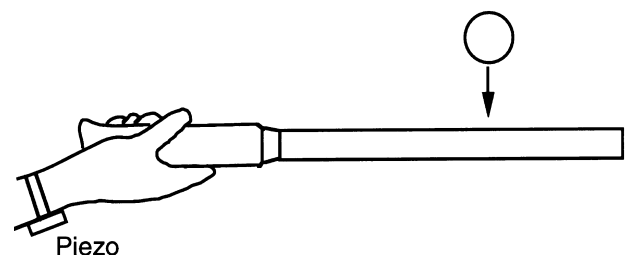


Figure 11 Arrangement used to measure the velocity of the forearm using a piezo strapped to the wrist. The impact points on the strings were the same as those shown in Fig. 9.

and attaching the piezo to a vibrating, cantilevered mechanical arm.

The velocity of the forearm was measured under conditions where the racquet handle was held by the right hand, using an eastern forehand grip with the strings in the horizontal plane, and a tennis ball was dropped from a height of 20 cm onto a small piezo attached to the strings. The same waveforms, proportionally larger in amplitude, were observed for impacts at ball speeds up to 15 m s^{-1} . The observed effects were therefore independent of ball speed up to this limit. The racquet and arm were initially stationary, so the observed velocity of the forearm corresponds purely to the impulsive motion generated by the impact of the ball on the racquet. The velocity of the forearm was measured for five different impact points, relocating the piezo for each drop so the ball landed directly on the piezo to generate a reference signal for timing purposes. The results of this experiment are shown in Fig. 12. The absolute values of the forearm velocity were not calibrated, but the results are displayed at the same sensitivity for each drop to provide a comparison of the relative amplitude and polarity of the velocity in each case. The polarity was chosen so that a positive velocity corresponds to motion of the forearm vertically upwards, opposite the direction of the incident ball. One of the waveforms in Fig. 12 corresponds to a drop onto the handle, at a point midway between the hand and the strings.

The most significant effect indicated by the waveforms in Fig. 12 is that there is almost no initial motion of the forearm for an impact at the COP. For an impact at the tip or centre of the strings, the forearm moves initially in the opposite direction to the incident ball. For an impact at the throat of the racquet or on the handle, the initial motion of the forearm is in the same direction as the incident ball. Several other effects are also obvious from these waveforms:

- 1 Within about 20 ms of the impact, the velocity of the forearm drops to zero and reverses sign at most impact locations. After the ball leaves the racquet, one might expect the racquet and forearm velocity to remain constant. Such a

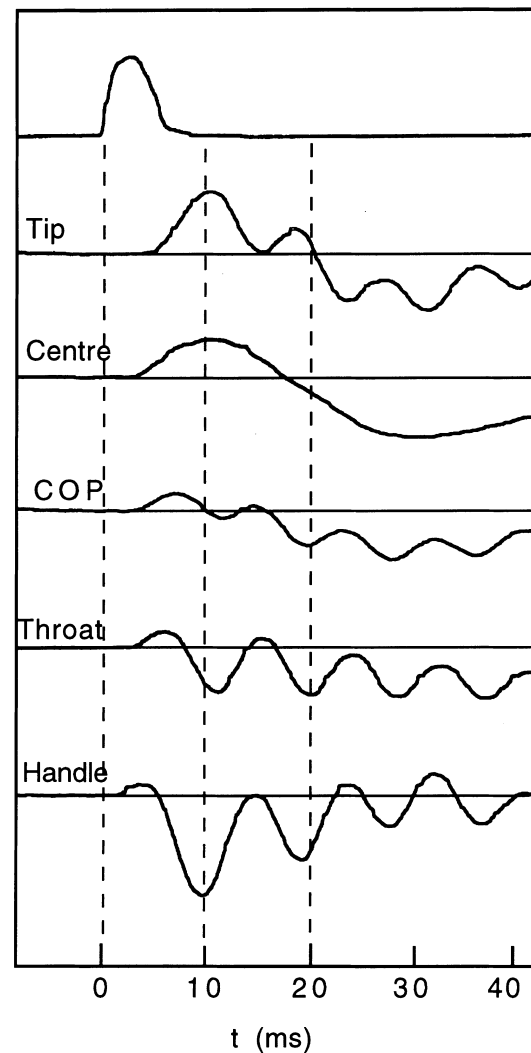


Figure 12 Velocity of the forearm at several different impact points, when the racquet is initially stationary and a ball is dropped onto the strings or the handle at the points indicated. A positive velocity corresponds to motion of the forearm vertically upwards. The horizontal line through each waveform is the zero velocity baseline. The drop height (20 cm) and vertical sensitivity were the same for each impact.

result is predicted from eqns 1,2,3,4, since if $F = 0$ then $F_R = 0$. The experimental results indicate the presence of other forces acting on the forearm. In order to hold the racquet in a steady horizontal position prior to the impact, the upper arm exerts a force on the forearm and the forearm exerts a force on the wrist to keep it

locked in position. The results indicate that these forces may act as restoring forces to return the forearm to the horizontal position with a response time of order 10 ms. If this is the case, then the rapid response of the muscles in the arm is perhaps faster than one might expect intuitively. However, the tendons connecting muscle to bone act as passive springs, with a spring constant of order 10^5 N m^{-1} (Alexander 1992). Acting on a mass of order 1 kg, the half period of oscillation would be of order 10 ms, as observed.

- 2 There is a significantly longer propagation delay in the response of the forearm, compared with the delays shown in Figs 1, 2 and 10. This can be attributed partly to the fact that the velocity increases slowly from zero even if the applied force increases rapidly. For an impact on the strings lasting 7 ms, the velocity of the forearm should reach a maximum about 9 ms after the ball first contacts the strings, assuming that the impulsive force on the forearm is delayed by about 2 ms. An additional delay might be introduced by the response time of the hand to rotate about an axis through the wrist. The long delay observed for an impact at the tip of the racquet appears to be due to the fact that the rotational, translational and vibrational components of the forearm velocity sum to zero for the first half cycle of oscillation. The first half cycle is positive

for an impact at the throat or the COP, so the first half cycle should be negative for an impact at the tip.

- 3 Vibrations in the racquet frame result in a significant vibration of the forearm. This does not alter significantly the vibration frequency of the racquet, since the frequency is determined primarily by wave reflection at the end of the handle rather than the end of the arm. This situation can be compared with the more obvious example of a piano wire where the frequency is determined by the mass and length of the wire, not the whole piano. In the present case, the hand acts to shift the vibration node in the handle closer to the end of the handle, thereby increasing the wavelength of the fundamental mode and decreasing the vibration frequency slightly. However, the situation is probably complicated by the fact that some wave reflection occurs at the hand, as well as the end of the handle, and this will act to decrease the effective length of the racquet.

Summary

A summary of the effects observed in this paper, during and immediately following an impact, is presented in Fig. 13. These drawings are based on the observations that (i) the axis of rotation of a

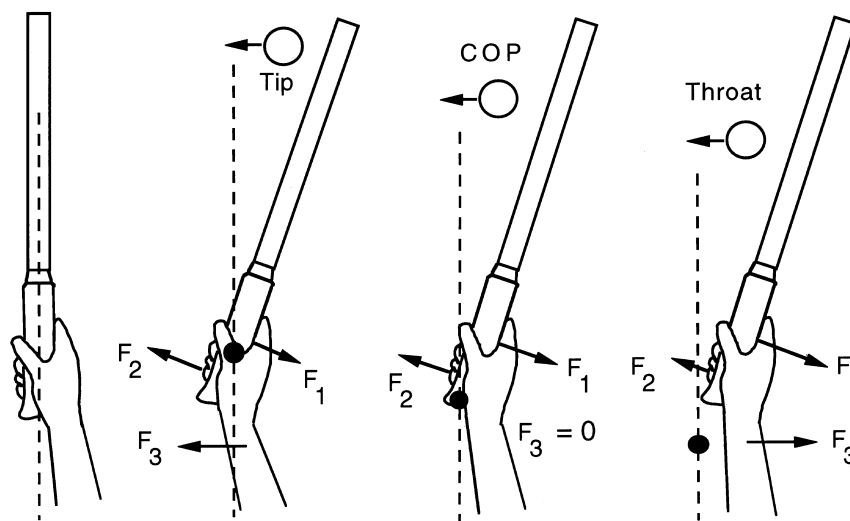


Figure 13 A summary of the observed rotation and translation of a hand-held racquet. The drawing at the left shows the positions of the racquet, hand and forearm prior to impact. F_1 denotes the force acting on the upper part of the hand. F_2 denotes the force acting on the lower part of the hand. F_3 denotes the force acting on the forearm near the wrist and is approximately equal to $F_1 + F_2$. The black circle indicates the axis of rotation of the racquet in the laboratory frame. The dashed vertical line represents the position of the long axis through the handle, prior to impact.

hand-held racquet, in the laboratory frame, lies within the hand for an impact at the tip of the racquet; (ii) measurements of the forces on the hand show that the axis of rotation, in the laboratory frame, lies within the hand or beyond the end of the handle and (c) the forearm is not deflected for an impact at the COP. On a longer time scale, starting 10–20 ms after the impact, recoil of the racquet and internal forces in the arm modify the initial response of the forearm. These effects are not included in Fig. 13.

The total force on the hand was not measured, but one would expect that it is at least qualitatively similar to the force on the forearm since the force on the forearm is transmitted from the handle via the hand and wrist. The relation between the forces on the hand and forearm clearly depend on the biomechanical linkages, and could be determined in principle by an independent experiment and modelled by connecting springs. However, a reasonable interpretation of the above observations is that the racquet exerts a torque on the hand, as represented by the forces F_1 and F_2 in Fig. 13. The force F_1 is represented by waveform b in Fig. 10 and the force F_2 is approximately equal to and opposite waveform d, since if the racquet handle moves away from the base of the little finger it moves towards the tip of the little finger on the opposite side of the handle.

The net force on the hand, $F_1 + F_2$, is transmitted to the forearm as the force F_3 shown in Fig. 13. This interpretation is qualitatively consistent with the results in Fig. 10. For example, for an impact at the COP, waveforms b and d are approximately equal and opposite, indicating that there is essentially no net force on the hand. In fact, if the forearm remains at rest and if the hand rotates about an axis through the wrist, then the centre of mass of the hand will translate slightly as a result of its rotation about the wrist. As a result, the condition for the forearm to remain at rest is that F_1 must be slightly larger than F_2 . For an impact at the tip or centre of the racquet, the DC component of waveform d is significantly larger in magnitude than waveform b, indicating that there is a net force on the hand acting in a direction opposite the

direction of the incident ball. For an impact at the throat of the racquet, the DC component of waveform d is significantly smaller than b, at least for the first 10 ms following the initial impact, indicating that there is a net force on the hand acting in the same direction as the incident ball.

Conclusions

The primary purpose of this work was to determine experimentally whether the hand has a significant or a negligible effect on the dynamics of the collision between a tennis ball and racquet. The results show that the hand plays a more significant role than previously suspected since an impulse is transmitted from the strings to the hand well before the ball leaves the strings. The effect of the hand on the outgoing ball speed was not investigated; however, it was found, by comparing hand-held and freely suspended racquets, that (a) the vibration node in the handle is shifted to a point under the hand; (b) the axis of rotation of the racquet is shifted to a point under the hand or close to the end of the handle; (c) the vibrational forces on the hand and forearm are zero for an impact at the vibration node in the centre of the strings and (d) the force on the forearm is minimized for an impact at the centre of percussion (the force is not zero since a small vibrational component is present). These impact points are the well-known sweet spots of a tennis racquet, but their significance in relation to the forces acting on the hand and the forearm has not previously been studied in any detail.

For a freely suspended racquet, the location of the COP is not uniquely defined since there is no unique axis of rotation in the handle. The COP can therefore be located anywhere on the strings, depending on which axis one chooses in the handle. In the case of a hand-held racquet, the location of the COP can be defined uniquely in terms of the impulsive motion of the forearm, at least for a short period during and immediately following the initial impact. The relevant axis of rotation in the handle then passes through the end of the handle, at least when the racquet is held by one hand. This may not be the case for a two-handed stroke. On a

longer time scale, motion of the forearm is determined by restoring forces within the arm as well as by the effects of the impact of the racquet with the ball.

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