

Tennis physics, anyone?

Rodney Cross

After a tennis ball lands on the court, it slows down, spins up, and squashes. Friction on the ball can even reverse direction, pushing it forward.

Rod Cross is an associate professor in the school of physics at the University of Sydney, Australia.

Tennis is unique among major sports in that it is played on a wide variety of surfaces. Indeed, some say that tennis played on the red clay of continental Europe is a different game from tennis played on grass at the sport's most prestigious event in Wimbledon, UK. Why is the surface so important? The answer lies largely in the physics of ball bounce, which, of course, has applications to many sports.

Follow the bouncing ball

The manner in which a ball bounces off a surface is not as simple as you might expect, even when the ball is spherical and the surface is flat, rigid, and heavy enough to remain at rest. In that situation a ball incident at an oblique angle and without spin will bounce with reduced speed, with topspin, and with an angle of reflection roughly equal to the angle of incidence. You can easily observe those qualitative features by bouncing a basketball off a heavy surface. A more quantitative analysis might address several questions: At what speed does the ball bounce, at what angle, and with how much spin? At what angle of incidence is the spin greatest? What difference does it make if the incident ball is spinning?

Two broad cases are of interest: Either the ball is in the pure sliding regime and slides through the duration of the bounce or it isn't. When a ball slides on a horizontal surface, the friction force acts to slow the ball horizontally and to impart a torque that causes it to rotate. For a ball of radius R , horizontal velocity v_x , and angular velocity ω , the relative speed between the bottom of the ball and the surface is $V_x \equiv v_x - R\omega$. Suppose that ω is initially zero. As the ball continues to slide, v_x decreases and ω increases. At angles of incidence less than about 20° with respect to the surface, the

ball will rebound while it is still sliding—that is, before V_x has reached zero.

In that pure sliding regime at any given incident speed, both the change in the horizontal speed and the spin imparted to the ball increase as the angle of incidence increases. However, when the angle of incidence is greater than about 20° , a different set of rules applies. In that case the change in the horizontal speed is a fixed fraction of the incident horizontal speed, regardless of the angle of incidence, and the spin imparted to the ball decreases as the angle of incidence increases. In the limit where the ball is incident at right angles, a ball that hits the surface without spin also rebounds without spin, since the sliding-friction force on the ball vanishes.

In many ball sports, spin is just as important as speed, due in part to its aerodynamic effect on the ball and in part to its effect on the way the ball bounces. Tennis is no exception, and players routinely employ both “slice” shots with backspin and topspin shots that go way over the net but still land in play. If the ball is hit with backspin or the surface is slippery, the ball will slide throughout the duration of the bounce for a larger range of incident angles. If the ball is struck with topspin, then the pure sliding regime exists over a smaller range of incident angles.

At large angles of incidence to the surface, the bottom of the ball comes to a stop when $v_x = R\omega$, before the ball has had a chance to rebound. At that instant the rotational speed of the bottom of the ball relative to the center is precisely equal to the translational speed of the center, as is the case when a ball is rolling. However, a rolling ball does not normally squash like a tennis ball. If the bottom of a tennis ball comes to rest on a court surface while the ball is squashed, then the ball will grip the court in the same way that your shoe grips the court when you push your foot down. The friction force on a ball or on a shoe that grips the surface is due to static rather than sliding or rolling friction.

If the bottom of a ball remains at rest while the top of the ball is still rotating forward, what happens then? Experiments show that the friction force on the bottom of the ball drops to zero and reverses direction during the grip phase. As a result, the ball distorts in the tangential direction and in the direction perpendicular to the surface. When the friction force reverses, it actually pushes the ball forward. In the course of the bounce, the ball will eventually lose its grip. The normal reaction force on the ball, after reaching a maximum, drops back toward zero. Then, once the static friction force is insufficient to maintain the tangential elastic distortion of the ball, the grip is released and the bottom of the ball starts to slide backward before it rebounds off the surface.

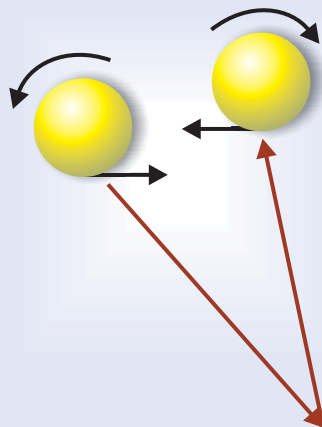


Figure 1. A bouncing elastic SuperBall reverses the horizontal (straight black arrows) and vertical velocity components of its contact point. As a result, the spin (curved arrows) can reverse direction and the angle of reflection can be quite different from the angle of incidence (brown arrows).

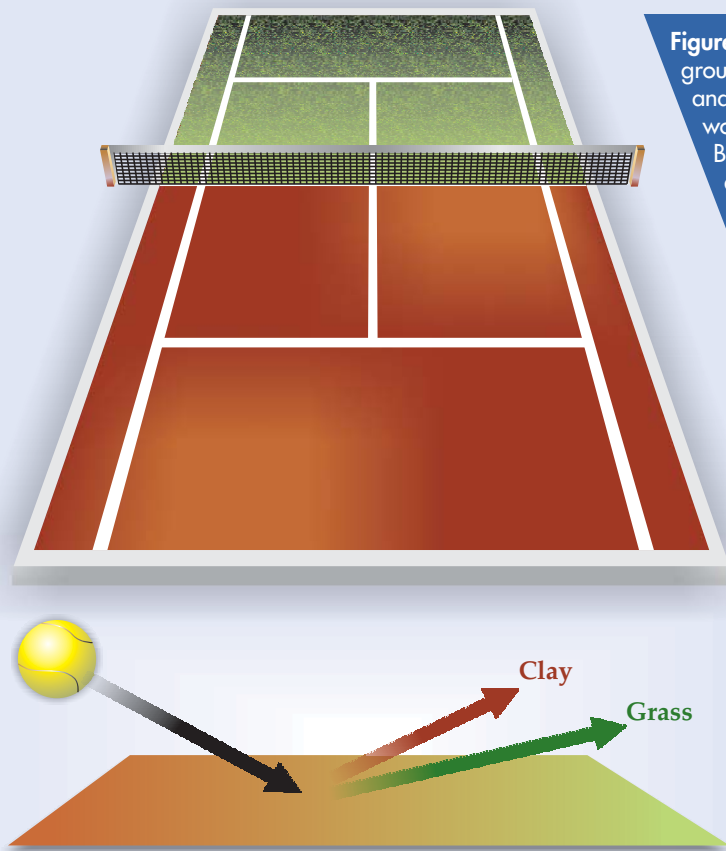


Figure 2. A half-grass, half-clay tennis court was the battleground for a 2007 exhibition match between Roger Federer and Rafael Nadal. Federer, then ranked number one in the world, was considered to be the stronger player on grass. But the clay court game is significantly different—the higher and slower bounce, as illustrated below the court, is one reason why—and Nadal is the better clay court player. Nadal won the exhibition in a third-set tiebreaker.

If a ball grips the surface during the bounce, it will rebound with $R\omega > v_x$, since the ball leaves the surface while it is sliding backward. If the ball slides forward throughout the bounce, then it rebounds with $R\omega < v_x$. Measurements of v_x and ω after the bounce indicate not only whether the ball gripped the surface but also the extent to which the ball recovers the elastic energy stored due to its tangential deformation.

The court demands restitution

The change in a ball's vertical velocity component during the course of its bounce is measured by the coefficient of restitution e_y , the ratio of the up component of the rebound speed to the down component of the incident speed. For most sports balls, e_y lies in the 0.5–0.8 range, and it drops slightly as incident speed increases. One can also consider a tangential coefficient of restitution, e_x , defined in terms of the x component of the velocity of the ball's contact point, V_x . One takes e_x to be the negative ratio of V_x after the bounce to that before the bounce. For most balls that grip, e_x ranges from +0.1 to +0.3. Otherwise, e_x is a negative quantity, since the ball slides throughout the bounce and V_x is positive both before and after.

A perfectly elastic ball, if it existed, would have $e_x = e_y = 1$, and it would spin twice as fast after bouncing as a ball with $e_y = 1$ and $e_x = 0$. A SuperBall is impressively, if not perfectly, elastic. Because it recovers most of its stored elastic energy, it bounces higher and spins faster than most other balls. Moreover, as illustrated in figure 1, both V_x and spin can reverse during the bounce.

Tennis racquet strings are also highly elastic, and they feature in one of the more bizarre stories in tennis history. Normally, the strings in a racquet are woven. But in the 1980s, some journeyman players introduced spaghetti stringing to professional tennis. The technique involved strings that were not woven at all but that could move freely in a direction par-

allel to the string plane. The tangential energy stored in the strings was almost completely recovered as the ball bounced off them. The result was a doubling in the amount of spin imparted to the ball, a peculiar flight path through the air, an equally strange bounce off the court, and some remarkable upsets. Spaghetti stringing was banned by the International Tennis Federation soon after its introduction.

The spaghetti-stringing story shows that coefficients of restitution depend not only on the elastic properties of the ball but also on the elastic and frictional properties of the surface off which it bounces. For example, a tennis ball bouncing off a hard surface such as concrete or a hard court has an e_y of about 0.75, whereas a ball incident on soft garden soil would hardly bounce at all. On a grass tennis court, e_y varies from about 0.6 to 0.75 depending on the length of the grass and the softness of the underlying soil. Clay courts, made from crushed brick or

tile, are unusual in that a ball digs a hole in the surface. As a result, the ball gets deflected upward, e_y is about 0.85, and the ball slows down considerably in the horizontal direction. Figure 2 illustrates the differing bounces of a ball on grass and clay along with a hybrid clay–grass court on which the world's two best players faced off last year.

The coefficient of sliding friction on clay can be as high as 0.9 at low angles of incidence, making clay the slowest of all tennis court surfaces. It is the surface of the French Open, one of tennis's four Grand Slam jewels. The grass at Wimbledon, with a coefficient of sliding friction of 0.5–0.6, is the fastest surface used in a Grand Slam event. Clay is doubly unusual in that the small particles on the surface behave like ball bearings under a player's feet, allowing the player to slide more easily than on a hard surface. The coefficient of sliding friction is therefore quite low underfoot and quite high under the ball.

Do the differences between clay and grass really matter to a professional player? Indeed they do. This past June, Rafael Nadal defeated Roger Federer to win the French Open. When he repeated the feat a month later to end Federer's five-year streak as Wimbledon champion, it was the first time in 28 years that the same man won the year's premier tournaments on clay and on grass.

Additional resources

- ▶ H. Brody, "How Would a Physicist Design a Tennis Racket?" *PHYSICS TODAY*, March 1995, page 26.
- ▶ R. Cross, "Grip-slip Behavior of a Bouncing Ball," *Am. J. Phys* **70**, 1093 (2002).
- ▶ R. Cross, C. Lindsey, *Technical Tennis: Racquets, Strings, Balls, Courts, Spin, and Bounce*, Racquet Tech Publishing, Vista, CA (2005).
- ▶ R. Cross's webpage, www.physics.usyd.edu.au/~cross. ■