Enhancing the Bounce of a Ball

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In sports such as baseball, softball, golf, and tennis, a common objective is to hit the ball as fast or as far as possible. Another common objective is to hit the ball so that it spins as fast as possible, since the trajectory of the ball through the air is strongly affected by ball spin.[1] In an attempt to enhance both the coefficient of restitution (COR) and the spin of a golf ball, I conducted several experiments to see what would happen when a 45-g, 42.8-mm diameter golf ball bounced on: (a) a 58-mm diameter, 103-g Super Ball®; (b) an 8-mm thick, 56-mm diameter circular disk of Super Ball material cut from a large Super Ball and glued to a 3.4-kg lead brick; and (c) a 3-mm thick sheet of rubber glued to a 3.4-kg lead brick. (See Fig. 1.)

For comparison, I also bounced the golf ball off (d) the strings of a tennis racquet and (e) a 14-kg polished block of granite, and I bounced the large Super Ball off the same surfaces. The ball was thrown by hand, without spin, at a speed of about 4 m/s and at an angle of about 25° to the vertical onto the flat, horizontal surfaces. A sufficient number of bounces were recorded so that data could be obtained at an angle of incidence $\theta = 25 \pm 1^{\circ}$ to the vertical in all cases. To measure the bounce off tennis strings, the racquet frame was clamped so that the racquet itself would not bounce.

To measure the COR for a golf ball bouncing off the large Super Ball, the Super Ball was allowed to rest on the granite surface and the golf ball was dropped vertically onto the Super Ball. In each case, the bounce was recorded using a Casio EX-F1 video camera operating at 300 frames/s. Relatively cheap versions of this camera, also made by Casio, are now readily available. A line drawn around the equator of each ball was used to measure the change in rotation angle of the ball from one frame to the next, and the average angular velocity was measured by recording the rotation angle for at least four consecutive frames. The vertical and horizontal components of the ball speed before and after each bounce were obtained by plotting the position of the center of the ball as a function of time over at least four consecutive frames. The ball linear speed and the rotation speed were both measured with an estimated maximum error of about 1%.

Fig. 1. The 8-mm thick Super Ball disk and the golf ball used in this experiment.

Table I. Results of bounce measurements (SB = Super Ball).

Ball	Surface	θ (deg)	COR	e_x	S
Golf	Granite	25	0.87	0.03	14.8
Golf	Rubber	25	0.59	0.51	20.5
Golf	SB disk	25	0.81	0.55	23.6
Golf	Tennis	25	0.91	-0.01	15.0
Golf	58-mm SB	0	0.57	0	
SB	Granite	25	0.78	0.49	14.9
SB	Rubber	25	0.78	0.41	14.5
SB	SB disk	25	0.78	0.57	18.2
SB	Tennis	25	0.91	-0.10	9.0
SB	58-mm SB	0	0.78	0	

Experimental results

The results of the various bounce experiments are summarized in Table I. For a ball incident obliquely on a heavy surface, the normal COR is defined as the negative ratio of the normal component of the velocity of the contact point just after the bounce to the normal component just before the bounce. The tangential COR, e_x, is defined in an analogous manner[2-4] in terms of the horizontal velocity of the contact point (at the bottom of the ball). If the ball translates at horizontal velocity v_x and spins at angular velocity ω, then the horizontal velocity of the contact point is $v_x - R\omega$, where R is the ball radius.

The quantity S in Table I is defined as $S = \omega / v$, where ω is the angular velocity of the ball after it bounces and v is the incident speed of the ball. At any given angle of incidence, ω is proportional to v, so the quantity S is a convenient spin factor that is proportional to the ball spin but is independent of the speed of the incident ball. The values of COR and e_x were measured with an estimated maximum error of $\pm$ 0.01, while S was measured with an estimated error of $\pm$ 0.1.

On the granite surface, the golf ball had a higher COR than the Super Ball. Not all Super Balls bounce equally well. The particular Super Ball tested had a relatively low COR. The highest COR was achieved using the strings of a tennis racquet. The COR was 0.91 for both the golf ball and the large Super Ball.

The spin imparted to each ball was largest when bouncing off the 8-mm thick disk cut from a large Super Ball. The spin imparted to the golf ball was larger than that imparted to

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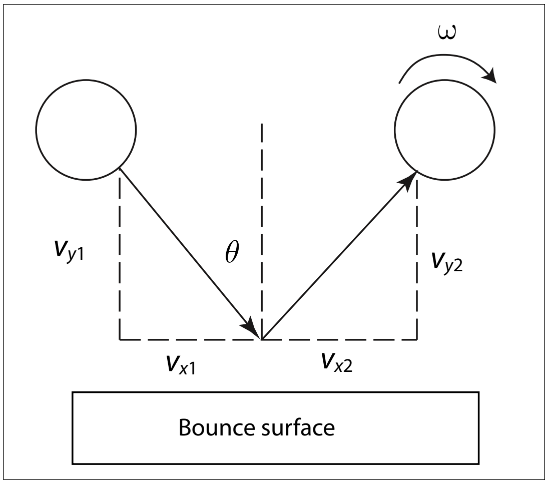

Fig. 2. Geometry of the oblique bounce of a ball off a horizontal surface.

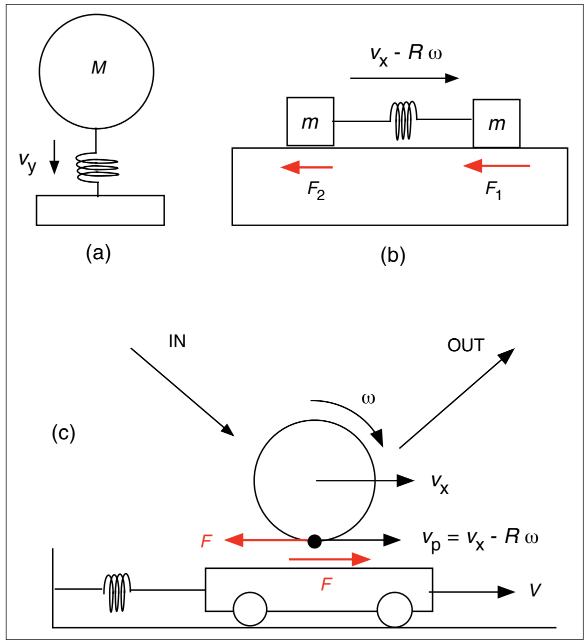

Fig. 3. Simplified models of a ball used to explain the manner in which elastic energy is stored due to (a) motion of the ball in the vertical direction, (b) horizontal motion of a flexible ball on a rigid surface, and (c) the oblique impact of a rigid ball on a flexible surface.

the large Super Ball on all the bounce surfaces, except for the granite surface, where the spin was essentially the same for both balls.

The worst bounce surface for the golf ball, having the lowest COR, was the large Super Ball. The Super Ball bounced up off the granite surface when it was struck by the golf ball, thereby reducing the energy transferred to the golf ball. When the Super Ball was held down onto the granite surface by hand, the bounce was even worse, having a COR of only 0.39. By eye, it appeared that the golf ball didn't bounce at all since the ball bounced to a height of only 20 mm in only 0.064 s before falling back onto the Super Ball.

The rubber surface also resulted in a low COR using the golf ball, although it had no effect on the COR of the Super Ball. The rubber surface enhanced the spin of the golf ball, compared with the spin off the granite surface, but it resulted in a slight decrease in the spin of the Super Ball (compared with the spin off the granite surface).

Discussion

(a) COR:

When a ball bounces on a surface, the normal forces on the ball and the surface are equal and opposite. The compression of the ball and the surface, and the elastic energy stored in each, depend on the relative stiffness of the ball and the surface. Since granite is much stiffer than either of the balls tested, the COR on granite is a measure of the energy lost in the ball. The golf ball was stiffer than all of the impact surfaces other than granite, so most of the impact energy was stored in the surface rather than the ball, and the resulting COR provided a measure of the energy loss in each surface.

The bounce of the golf ball off the Super Ball can also be regarded as a case where most of the elastic energy of the collision was stored in the Super Ball rather than in the golf ball. Part of the elastic energy was returned to the golf ball, and part was retained by the Super Ball since it bounced vertically off the granite surface. The golf ball was dropped from rest through a height of 232 mm, collided with the Super Ball, and then bounced to a height of 75 mm above the initial impact point. The Super Ball bounced to a height of 35 mm. In that respect, 67% of the initial impact energy was retained after the collision, a result that would also be obtained if a ball bounced with a COR of 0.82 off a fixed surface. When the Super Ball was held onto the granite surface by hand, only a small fraction of the elastic energy stored in the Super Ball was returned to the golf ball, the rest being dissipated in the Super Ball and the hand.

(b) Spin:

Spin is imparted to a ball mainly as a result of the horizontal friction force F acting on the bottom of the ball. The normal reaction force contributes nothing to the spin if the normal reaction force acts vertically through the center of the ball. The resulting spin depends on the moment of inertia I of the ball about an axis through its center of mass. For a solid sphere of radius R, $I = 0.4\,MR^2$, where M is the mass of the sphere. Since the golf ball had a smaller value of I than the large Super Ball, by a factor of 4.2, one might expect that the golf ball would spin faster than the Super Ball after bouncing. However, the spin also depends on the tangential COR.

Consider the bounce geometry shown in Fig. 2, where a ball of mass M and radius R is incident without spin at angle θ and with velocity components v_{x1} and v_{y1}. The ball bounces with velocity components v_{x2} and v_{y2}, and with angular velocity ω. The horizontal impulse is given by

$$\int F dt = M(v_{x1} - v_{x2}) \quad (1)$$

and the torque on the ball is given by $FR = Id\omega/dt$, so $R\int Fdt = I\,\omega$ and hence

$$\omega = (v_{x1} - v_{x2}) \,/\, 0.4R. \qquad (2)$$

Given that

$$e_x = -(v_{x2} - R\omega)/v_{x1}, \qquad (3)$$

we find that

$$\omega = 5(1 + e_x)v_{x1}\,/7R, \quad \text{so} \quad S = \omega/v = 5(1 + e_x)\sin\theta\,/7R. \quad (4)$$

When $\theta = 25^{\circ}$, we find that

$$S\,(\text{golf ball}) = 14.1(1 + e_x) \text{ and } S\,(\text{Super Ball}) = 10.3\,(1 + e_x). \quad (5)$$

There is no simple method available to calculate e_x for any given ball and surface since it depends on the elastic properties of both surfaces, but e_x is typically close to zero for slippery surfaces (such as a golf ball bouncing on granite or tennis strings) and cannot exceed one, in the same way that the normal COR cannot exceed one. Since e_x can vary from about zero to one depending on the elastic properties of the ball and the surface, the spin imparted to a ball can vary by a factor of up to about two, depending on whether none or all of the tangential elastic energy is recovered. The spin imparted to the golf ball and the Super Ball on the granite surface was coincidentally about the same, despite the large difference in their moment of inertia, since e_x for the Super Ball is larger than e_x for the golf ball. Note that e_x is undefined when $\theta = 0$ since v_{x1} and v_{x2} are then both zero.

(c) Simplified bounce models:

It is easy to account for the elastic energy stored in a bouncing ball, due to its motion perpendicular to a rigid surface, in terms of the simplified model shown in Fig. 3(a). If the ball is represented by a mass in series with a spring, then the impact duration and impact force can be easily calculated in terms of appropriate mass and stiffness values.[5] Figure 3(b) shows a simple model designed to account for the elastic energy stored in a ball due to its motion parallel to a rigid surface. The bottom part of the ball is represented by two equal masses connected by a spring. The bottom of the ball slides along the impact surface at speed $v_x - R\omega$. Friction acting on the bottom of the ball causes the ball to start rotating as soon as it impacts the surface. The front edge of the ball rotates into the surface at angular speed ω, while the back edge of the ball rotates out of the surface at angular speed ω. Consequently, the normal reaction force at the front of the ball will be larger than the normal reaction force at the rear of the ball, and the friction force F_1 will be larger than F_2. The front edge of the ball will decelerate more rapidly in the horizontal direction than the back edge, with the result that the spring will compress.

At angles of incidence less than about 50°, the bottom of the ball comes to a complete stop during the bounce, with $v_x - R\omega = 0$, causing the ball to grip the surface.[3] As the ball rises up off the surface, the ball loses its grip and starts sliding again. It is during this stage that elastic energy stored in the spring is released, resulting in a finite positive value of e_x. Since the back of the ball loses its grip before the front of the ball, the result is an increase in the tangential speed (backwards), and hence in the angular velocity of the ball at large values of e_x. The ball bounces with $R\omega > v_x$ when e_x is positive. Note that a rolling ball rolls with $v_x = R\omega$.

If the ball is stiffer than the bounce surface, then most of the tangential elastic energy resides in the surface rather than in the ball. Figure 3(c) shows a simple model that allows elastic energy to be stored in an external spring during an oblique impact. If the ball grips the surface during the impact, then the two will engage like gears. Initially, the ball slides on the surface and the resulting friction force acts to stretch the spring. The spring can subsequently pull the surface backward, thereby enhancing the spin of the ball. Calculations based on the model in Fig. 3(c) have been published elsewhere.[6]

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