Measurement of the speed and bounce of tennis courts

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Abstract

Tennis is played on a wide variety of court surfaces, which can be classified as fast, medium or slow according to the coefficient of sliding friction between the ball and the court. The vertical bounce properties depend on the coefficient of restitution. Both coefficients can be measured by using a video camera to film the bounce of a ball incident at a low angle on the court. It is easier to analyse the data when filming at high film rates, around 200 fps or more, but reliable data can also be obtained when filming at the standard 25 or 30 fps rate used in most video cameras. In this paper we describe results obtained at both 25 fps and 500 fps, and consider some of the physics issues involved in interpreting the data.

1. INTRODUCTION

The International Tennis Federation (ITF) actively encourages the measurement and classification of tennis courts to (a) establish minimum levels of quality and high-quality workmanship, (b) improve standards (c) enable comparisons between courts, and (d) protect contractors against unreasonable demands. A description of the recommended procedures for measuring and classifying court surfaces is available in the technical section of the ITF web site (www.itftennis.com). In essence, courts are classified as fast, medium or slow depending on the coefficient of sliding friction (COF or μ) between the ball and the court, and are also be classified as low, medium or high bounce depending on the coefficient of restitution (COR or e_y). A COF > 0.71 is regarded as high (eg clay courts) while a COF < 0.55 is regarded as low (eg grass courts). Similarly, a COR > 0.85 is regarded as high and a COR < 0.78 is regarded as low. Surfaces with a COR < 0.70 are not recommended by the ITF for use as tennis courts.

A difficulty with implementing court testing on a wide scale is that the ITF–recommended procedure, while quite accurate, is somewhat cumbersome and expensive. The basic equipment costs more than \$60,000. For that reason, the ITF and others have been examining alternative methods of testing surfaces that might prove to be more suitable for general use, even if they are not as accurate. In this paper, I describe my own efforts in this regard, conducted with the assistance of Tennis Australia. One of the interests of Tennis Australia is to ensure that all courts prepared for the Australian Open each year are similar in speed and bounce, and that they are similar to nominally identical courts used for lead–up tournaments in other states. It would not be possible or practicable, using the ITF–recommended procedure, to test all newly resurfaced courts at Melbourne Park each year in the limited time available. Partly to overcome that problem, and partly to provide a method of measuring court speed and bounce that could be used by almost anyone interested in doing so, I developed a simple measurement technique based on filming the bounce of a tennis ball with a standard, 25 fps, video camera. This paper describes the technique, the precautions required to obtain valid results and some of the physics issues involved in interpreting the results.

2. SIMPLE MEASURES OF COF AND COR

The COF for a tennis ball sliding on a tennis court can be measured in an elementary fashion by mounting several balls in a box, adding weights to the box, and then dragging the balls across the court at a constant low speed. The COF is the ratio of the horizontal pull force (equal to the friction force if the speed remains constant) to the normal reaction force (the total weight of the box). While this method does indeed measure the COF, the COF is not identical to that for an impact of a ball on the court since the ball speed and contact area are quite different (1). A commercial version of this apparatus, known as the Tortus floor friction tester, is commonly used to measure slipperiness of floors, but it employs a rubber foot rather than tennis ball cloth and it cannot be used on clay or grass surfaces. Nevertheless, the Tortus device has been used with some success on hardcourt surfaces, particularly in the USA, to provide a relative indication of the speed of different courts and to monitor variations in speed of different sections of the same court.

The COR is easier to measure since it requires only that a tennis ball be dropped vertically from a height of about 150 cm onto the court. The rebound height needs to be measured accurately, preferably by filming the bounce with a video camera. The ball itself should be tested by dropping it in like manner onto a smooth, heavy surface such as concrete or a granite slab. The COR is defined as the ratio of the rebound speed to the incident speed, which is equal to the square root of the rebound height to drop height ratio. The COR for an approved ball dropped onto a smooth, heavy surface is about 0.75, and the COR for a ball dropped onto a tennis court should normally be greater than 0.70, although on some grass courts the COR can be as low as 0.60. The resulting value of the COR is usually lower than that measured when a ball impacts obliquely on a court, but it does provide useful information. For example, the method can be used to locate faulty patches in a court surface.

3. VIDEO FILM TECHNIQUES

Ideally, the COF and COR for a particular court surface should be measured under conditions that are close to those encountered during normal play. In other words, they should be measured in terms of the changes in the horizontal and vertical components of the speed of a ball when it impacts on the court. That is indeed the basis of the procedure recommended by the ITF for its official testing method using apparatus known as the Sestee device. However, the official Sestee apparatus is not available in Australia, is very expensive, is not very portable and requires a highly accurate and reproducible ball launcher capable of launching a tennis ball at 30 ± 2 m/s. Furthermore, the Sestee device is not suitable for use on clay courts since the optical components are mounted in a darkened box and are affected by dust.

In 2004 I devised a relatively simple technique in 2004 to overcome these problems, using a home–made ball launcher and an inexpensive video camera operating at 25 fps (frames per second). The ball launcher contained a spring–loaded mechanical throwing arm that was hand–operated and projected the ball horizontally at a speed of 10 m/s. The camera was mounted on a tripod to film the bounce. In this manner, six bounces could be filmed at each of several different spots on all 22 courts at Melbourne Park in only two days. It took several weeks to analyse the film, using simple motion analysis software, although the task has now been simplified and streamlined using more sophisticated software that automatically tracks the motion of a moving object from one frame to the next. An additional improvement in the software was commissioned by Tennis Australia so that the corrections outlined in the Appendix are automatically included when analysing the data.

The 2004 technique provided useful information on the COF and COR of all the courts used for the Australian Open. It was established using this technique that wear and tear after resurfacing could alter the speed of the court and that the wear was affected by the curing time allowed before the courts were re–used and by subtle effects due to ventilation and climate during the curing process. Attention was also focussed, as a result of this work, on the need to ensure that the texture of the white lines was the same as the texture of the rest of the court, otherwise the white lines could end up being more slippery, especially when wet or covered in perspiration.

In 2006, the ball launcher was replaced with a commercial ball launcher to examine the effect on measured COF and COF of using higher ball speeds, around 20 m/s. The effect was larger than expected, and was traced to the fact that ball spin had not properly been taken into account when analysing the video film. Ball spin alters the trajectory of the ball as a result of the Magnus force. The trajectory is also affected by the drag force and the gravitational force. The trajectory and the change in ball speed with time is significant since it is necessary to extrapolate the data, when filming at 25 or 30 fps, to determine the speed of the ball immediately before and after impact with the court. At low ball speeds, spin is not a significant effect. After making allowance for spin, it was found that the COF and COR do not depend strongly on the incident ball speed, as expected, and that reliable data could be obtained over a relatively wide range of incident ball speeds, from 10 to 30 m/s. The same result was found by Capel–Davies (2).

The reliability of the data depends to a large extent on the court surface being measured. No two bounces are ever the same and different parts of a court will be subject to different amounts of wear. For that reason, it is usually necessary to take averages over 6 to 9 bounces and over 3 or more parts of the court to obtain reliable measurements of the COR and COF for any given court. Because of the variability of the bounce, highly accurate measurements of bounce speed and angle are not warranted. An accuracy of one or two percent is more than adequate in most cases of interest.

A further improvement in experimental technique became possible in 2008 with the release of inexpensive, high frame rate Casio video cameras, capable of recording at rates up to 1200 fps. By capturing several ball positions immediately before and after each bounce, the corrections for spin, gravity and drag are negligible since the ball travels almost in a straight line path at constant speed just before and just after impact. Similar accuracy can be achieved by filming at 25 fps, when proper allowance is made for the ball trajectory, but additional information on ball spin can be obtained when filming at high frame rates since it is then possible to zoom in to see marks on the ball more clearly. In this respect, high speed video has an advantage over the Sestee device since the Sestee is not set up to measure ball spin. High frame rate Casio video cameras are still not available in Australia and may have limited appeal to the general sporting community even when they do become available. For that reason, I have outlined in the Appendix the trajectory calculations that are required to obtain accurate measurements when filming at 25 or 30 fps.

4. BOUNCE THEORY

In order to measure the COF, the ball must be incident on the court surface at a sufficiently low glancing angle that it slides on the court throughout the whole bounce. At higher angles of incidence, the ball will grip the court during the bounce process, in which case sliding friction gives way to static friction. The time–average value of the COF during the bounce will then be lower than that due to sliding friction. The effect is larger than one might expect since the static friction force reverses direction during the bounce (3).

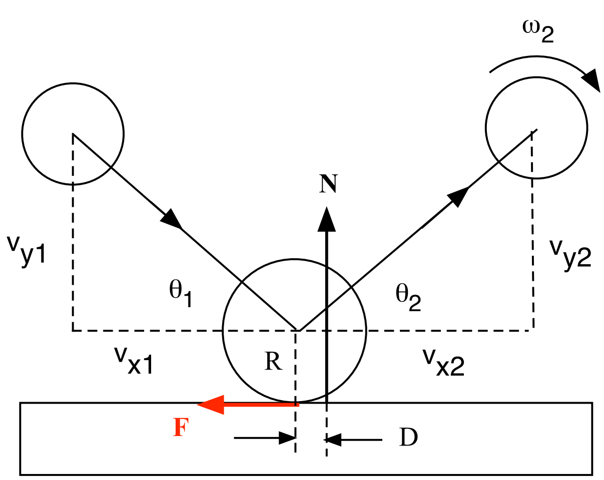


Figure 1: Geometry of the bounce of a tennis ball incident at angle θ_1 on a tennis court. F is the friction force acting at the bottom of the ball and N is the normal reaction force, which acts at a distance D ahead of the centre of mass.

The geometry of the situation is shown in Fig. 1. A ball of mass m, radius R is incident without spin at speed v_1 and at an angle θ_1 to the court surface. The ball bounces at speed v_2, angle θ_2 with angular velocity ω_2. If F is the horizontal friction force (acting in the negative x direction) and N is the normal reaction force (acting in the positive y direction) then $F = -m dv_x/dt$ and $N = m dv_y/dt$ where v_x and v_y are the horizontal and vertical components of the velocity of the centre of mass of the ball. N does not necessarily act through the ball centre of mass. In Fig. 1, it is indicated that N can act along a line at a distance D ahead of the centre of mass. Such an effect can be expected because the front edge of the ball rotates into the court surface while the back edge rotates out of the surface, generating an asymmetry in the distribution of the normal reaction force over the contact region of the ball. A similar effect arises with a vehicle when the brakes are applied, the front end rotating downwards and increasing the normal reaction force on the front wheels.

Integrating over the duration of the impact, we find that

$$\int F\,dt = m(v_{x1} - v_{x2}) \tag{1}$$

and

$$\int N\,dt = m(v_{y2} - v_{y1}) \tag{2}$$

where subscripts 1 and 2 denote values before and after the bounce respectively. If the ball slides throughout the impact then $F = \mu N$, in which case

$$\mu = \frac{(v_{x1} - v_{x2})}{(v_{y2} - v_{y1})} \tag{3}$$

An accurate measurement of the COF therefore requires that all four velocity components in Eq. (3) be measured as accurately as possible, especially the two components in the numerator. Relatively small errors in each v_x component can combine to produce a large error in $(v_{x1} - v_{x2})$. The same problem does not arise with the denominator since v_{y1} is a negative quantity (the ball being incident in the negative y direction). The value of the COR, e_y, is defined by the relation

$$e_y = -\left(\frac{v_{y2}}{v_{y1}}\right) \tag{4}$$

The ball will slide throughout the impact provided that the bottom of the ball does not come to rest. If the ball rotates at angular velocity ω then the bottom of the ball slides along the court at speed $v_x - R\omega$. During the impact, v_x decreases with time and ω increases with time as a result of the torque $FR - ND$. The sliding condition is met provided the ball bounces with $v_{x2} > R\omega_2$. A measurement of ω_2 should ideally be made to ensure that the sliding condition is met. On most court surfaces, the sliding condition is met provided that θ_1 is less than about 17°, although on slow surfaces such as clay it may be necessary to reduce the angle of incidence so that θ_1 is about 10°, as described in Sec. 6.

The angular velocity of the ball can be estimated theoretically if we assume for simplicity that $D = 0$. Then $FR = I d\omega/dt$, where I is the moment of inertia of the ball about its centre of mass. In that case, $\omega_2 = (R/I)\int F\,dt = (mR/I)(v_{x1} - v_{x2})$

From a measurement point of view, an improved technique would be to project the ball with backspin rather than without spin, since the ball could then be incident at a larger angle and still slide throughout the impact. During the sliding process, friction acts to reduce the backspin to zero and then causes the spin direction to reverse. Small errors in the alignment of the video camera can lead to large errors in the estimated angles of incidence and reflection, especially when the angle of incidence is small. For example, if the angle of incidence is overestimated by one degree, then it is overestimated by 10% when the angle of incidence is ten degrees. The bounce angle will then be underestimated by 10%, leading to the COR being underestimated by about 16% and the COF being underestimated by about 4%. Backspin allows the angle of incidence to be increased, thereby reducing this source of error.

5. BALL TRAJECTORY

By filming the bounce of a ball with a video camera, images of the ball are captured at intervals of T seconds, as indicated in Fig. 2. If the ball happens to be incident at say 20 m/s, and the exposure time is set to say 1/1000 s, then the ball will travel 20 mm during the exposure and will appeared blurred in the film. A position error of 5 or 10 mm in the location of the ball CM may therefore be unavoidable, but it can be minimised by filming in bright sunlight using the smallest possible exposure time. Care should be taken, when using a video camera for this purpose, that the camera has available exposure times that can be set manually to 1/1000 s or less and that the camera does not automatically adjust the exposure time according to the available light level (as it does in the "Sports mode" setting of some video cameras).

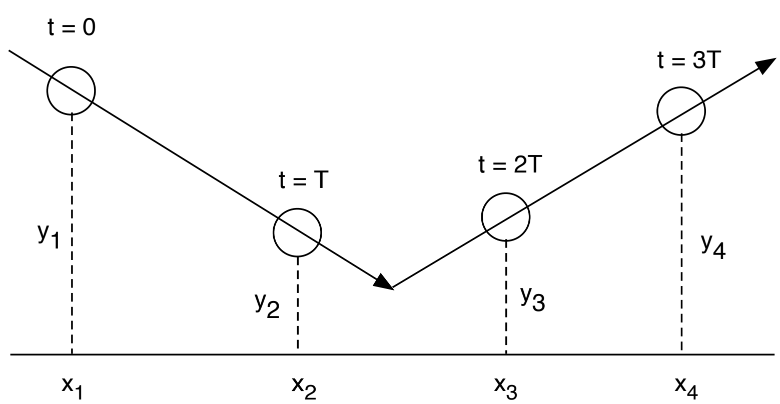


Figure 2: Two positions of the ball before it bounces, and two after the bounce, recorded at equal time intervals T. The (x, y) coordinates can be measured with respect to an arbitrary origin.

Suppose that two positions of the ball are recorded before the bounce (positions 1 and 2) and two positions are recorded after the bounce (positions 3 and 4). It is assumed that the ball is not in contact with the court at positions 2 or 3, and that the camera is aligned with its y axis in the vertical direction and with its central axis perpendicular to plane containing the ball. The average incident and rebound speed components are then

$$v_{x1} = (x_2 - x_1)/T \qquad v_{y1} = (y_1 - y_2)/T \tag{5}$$

and

$$v_{x2} = (x_4 - x_3)/T \qquad v_{y2} = (y_4 - y_3)/T \tag{6}$$

which can be substituted in Eqs. (3) and (4) to estimate the COF and the COR. The estimate can be improved if allowance is made for changes in ball speed with time due to the lift, drag and gravitational forces. Given that the impact duration of a tennis ball with a tennis court is about 5 ms, and that the impact occurs at say time t_0, we would like to estimate the two velocity components at time $t_1 = t_0 - 0.0025$, just before impact, and the two components at time $t_2 = t_0 + 0.0025$, just after impact.

The primary force in the vertical direction is that due to gravity, so a better estimate of the v_y components can be obtained by assuming that the ball follows a trajectory given by

$$y_{in} = y_B + v_B t + 4.9t^2 \tag{7}$$

before the ball bounces, and

$$y_{out} = y_A - v_A t - 4.9t^2 \tag{8}$$

after the ball bounces. The unknown constants can be determined by fitting these trajectories to the measured y coordinates. The impact time t_0 can be determined as the time at which $y_{in} = y_{out}$, while the speeds at time t_1 and t_2 can be determined by differentiating Eqs. (7) and (8).

Further corrections to the estimated speeds can be obtained by applying corrections for the lift and drag forces acting on the ball (5). For example, the drag force on a ball is given by

$$F_D = \frac{1}{2} C_D \rho A v^2 \tag{9}$$

where C_D is the drag coefficient, about 0.55 for a tennis ball, $\rho = 1.21$ kg/m^3 is the density of air at 20 C, $A = \pi R^2$ is the cross–sectional area of the ball and v is the ball speed. A tennis ball has a mass of 57 g and radius $R = 0.033$ m, so the deceleration, a_d, is given by $a_d = 0.020v^2$ or 8.0 m/s^2 when $v = 20$ m/s. If the ball is filmed at 25 fps, then it will slow down from 20.0 m/s in one frame to 19.68 m/s in the next frame. Even though the correction for drag is small, it can make a relatively large difference when measuring the change in ball speed due to the bounce. Details of the speed correction techniques used by the author are given in the Appendix.

An alternative method of fitting the data would be to plot the positions of the ball in four or five frames before and after the bounce, and to fit a high order polynomial to the data. In theory, that technique could be used to measure the acceleration of the ball and to extrapolate the data to determine the velocity immediately before and after the bounce. However, this technique was found to be inferior since it is very sensitive to small errors in the measured coordinates of the ball. By zooming out to observe more ball positions before and after the bounce, the error in the measured positions of the ball increases. The problem here is that small errors in the ball position result in large errors in the estimated velocity of the ball, which are then magnified to generate even larger errors in the estimated acceleration of the ball. For example, if one films the vertical drop of a ball in an attempt to measure the acceleration due to gravity, then a ball position error of only 1 mm can easily lead to a value of g that is 50% higher or lower than the accepted value. The problem does not arise when filming at high frame rates, since it is then possible to fit a straight line to the data, as described in the following section.

6. HIGH FRAME RATE DATA

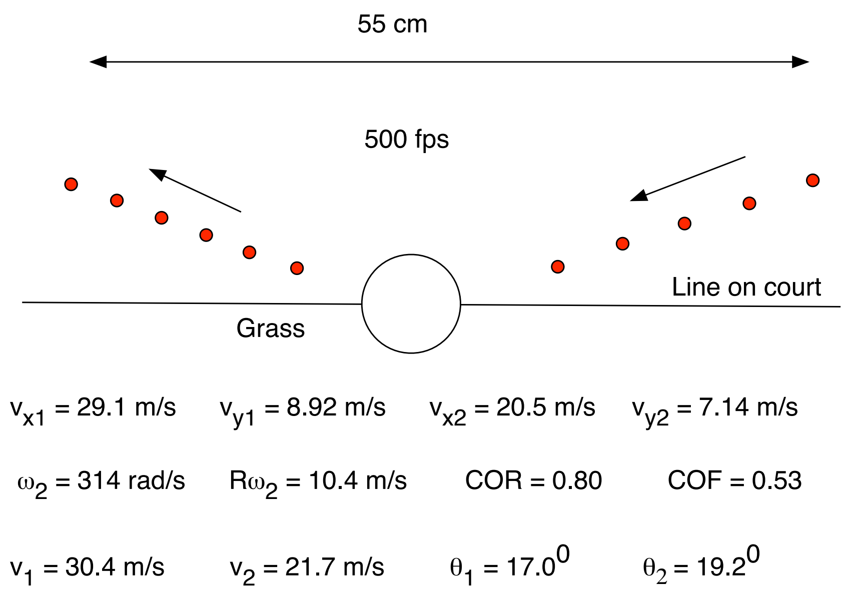
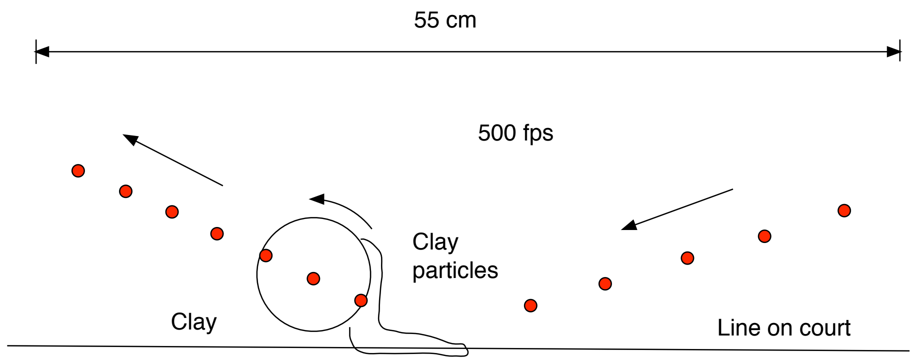


Figure 3: Results obtained for a grass court by filming the bounce at 500 fps. The circle shows the image of the ball during the bounce, and the red dots denote the position of the centre of the ball at 2 ms time intervals.

Data obtained by filming at 25 fps have previously been described in Ref. (4). Results obtained at 500 fps are shown in Figs. 3 and 4 for a grass court and a clay court respectively, for a non–spinning ball incident at about 30 m/s and at an angle of incidence of about 17°. The position of the ball was recorded for 5–7 frames before and after the bounce, at 2 ms intervals. Over a time interval of about 10 ms, there is no discernible curvature in the path of the ball, and the data can be fit by straight line segments. The results are shown in the figures. Each bounce was also filmed at 25 fps, using a second camera. When corrected for ball spin after the bounce and for acceleration of the ball, both sets of results were found to give the same COF and COR values to within 3%.

The angular velocity of the ball after the bounce was easy to determine from markings on the ball. For the grass court, $R\omega_2$ was significantly smaller than v_{x2}, indicating that the sliding condition was maintained throughout the bounce and that a reliable value of the COF was therefore obtained. On the clay court, the horizontal speed of the ball decreased by a larger fraction and the ball bounced with a higher angular velocity, with the result that $R\omega_2$



$v_{x1} = 25.1$ m/s $\quad v_{y1} = 7.54$ m/s $\quad v_{x2} = 14.9$ m/s $\quad v_{y2} = 6.89$ m/s

$\omega_2 = 443$ rad/s $\quad R\omega_2 = 14.6$ m/s $\quad$ COR = 0.91 $\quad$ COF = 0.70

$v_1 = 26.2$ m/s $\quad v_2 = 16.4$ m/s $\quad \theta_1 = 16.7^0 \quad \theta_2 = 24.8^0$

Figure 4: Results obtained on a clay court by filming the bounce at 500 fps.

was almost equal to v_{x2} by the end of the bounce. The resulting value of the COF on clay was consistently found to be about 0.70 from one bounce to the next. However, values of the COF on other clay courts have previously been found by the author to be as high as 0.9 at lower angles of incidence, suggesting that the result in Fig. 4 may not have provided a reliable indication of the COF of the court being tested.

For the bounce shown in Fig. 4, it is possible that the ball stopped sliding during the bounce and then began sliding again near the end of the bounce period, with the result that ball bounced with a value of $R\omega_2$ close to v_{x2}. If that was the case, then the average value of the COF during the bounce would be less than the actual coefficient of sliding friction. The only way to be certain of the result would be to repeat the experiment at lower and higher angles of incidence, as indicated in Fig. 5.

One interesting result, shown in Fig. 4, is that clay particles are ejected from the rear of the ball as it bounces off the court. It had previously been thought that clay is simply pushed ahead of the ball as it slides on the court. This may indeed be the case, but the result observed on film shows that clay also sticks to the bottom of the ball and is then ejected from the rear side of the rapidly spinning ball. A mound of clay ahead of the ball may be responsible for the fact that the COR on clay is commonly found to be higher than on other courts, a result that can be attributed to an upward deflection of the ball by the mound.

Data obtained by the author at 25 fps for a different clay court (the Davis Cup clay court constructed in Sydney in 2003), are shown in Fig. 5. This court was constructed in a manner similar to the courts at the French Open, with the assistance of a French clay court specialist. The ball was incident at speeds from 20 to 30 m/s, at angles of incidence from 9° to 33°. The

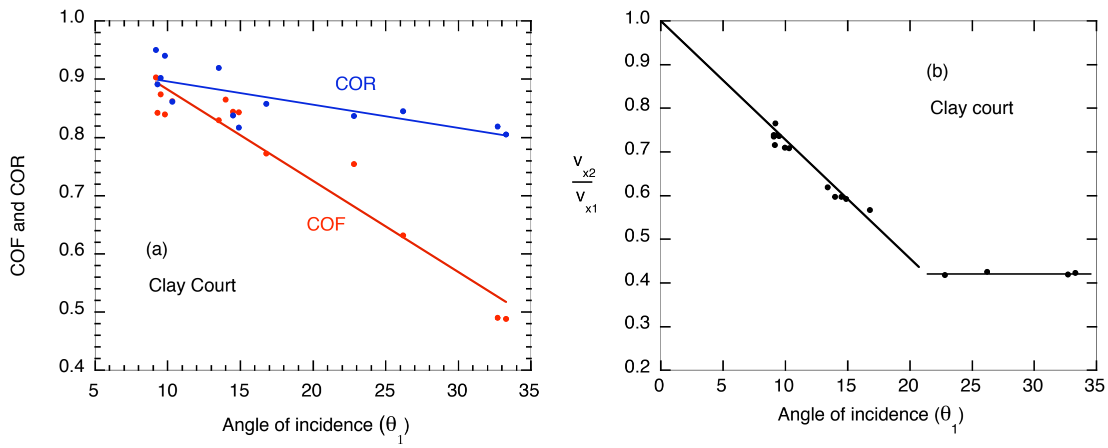


Figure 5: Measurements of (a) the COF and COR and (b) v_{x2}/v_{x1} for a clay court at various angles of incidence. The straight lines are linear fits to the data. In (b) the data are fit by two linear segments. The data were obtained by filming at 25 fps. Each data point represents a single bounce.

COF was calculated from Eq. (3) and the COR was calculated from Eq. (4), after making the corrections described in the Appendix.

In Fig. 5(a), the low value of the COF at high angles of incidence is clearly artificial, in the sense that it does not represent the coefficient of sliding friction on clay. Rather, it represents the time–average value of the F/N ratio during the bounce. It is lower than the actual sliding coefficient since the ball grips the court during the bounce. The graph of COF vs θ_1 indicates that the COF may be approximately constant when $\theta_1 < 15°$ and then the COF decreases when θ_1 increases above 15°. However, the data suggests that the COF might even continue to increase as θ_1 decreases below nine degrees. Detailed measurements of the spin of the ball in this region would help to interpret this data, at least in terms of the detailed physics of the process. The fact that the COR continues to increase as θ_1 decreases suggests that the role of the mound of clay ahead of the ball may be important not only in affecting the COR but also in modifying the effective value of the COF.

An alternative measure of court speed can be obtained by plotting the ratio v_{x2}/v_{x1} vs the angle of incidence, as shown in Fig. 5(b). The data can be fit by two straight line segments, one of which passes through the theoretically expected result that $v_{x2} = v_{x1}$ when $\theta_1 = 0$. It might appear from the latter graph that the ball continues to slide at all angles up to about $\theta_1 = 21°$ and grips the court when $\theta_1 > 21°$, but the result for the COF indicates that the ball grips the court when $\theta_1 > 15°$.

Given the uncertainty in the real value of the COF, and the clear experimental behaviour of the v_{x2}/v_{x1} data, the latter ratio could be regarded as a more reliable and more practical indicator of court speed than the former. It may also be more closely related to the perception of players to court speed, given that the v_{x2}/v_{x1} ratio is directly related to the change in horizontal ball speed as observed by players. In addition, there is less scatter in the v_{x2}/v_{x1}

data since the ratio is not as sensitive as the COF to small differences between v_{x1} and v_{x2}.

7. PLAYER PERCEPTION OF COURT SPEED

Experienced tennis players develop a qualitative feel for the speed of various tennis courts and adjust their game to accommodate variations in the speed and bounce height of the ball as it bounces off the court. The speed and bounce height or angle depends not only on the coefficients of sliding friction and restitution but also on the angle of incidence and the incoming spin of the ball. Players combine all these factors together when assessing a particular court, with the result that their perception of the speed of a court does not necessarily correlate in a linear fashion with the measured value of the coefficient of sliding friction. In particular, players perceive that grass courts are faster than the COF would suggest and that clay courts are slower than the COF would suggest (6). The exact reason has not been determined, but it is presumably related to the time taken, after the ball bounces, for the ball to reach the player. That time, in turn, depends on where the player is standing, the speed, spin and angle of the incoming ball, and the rebound height of the ball.

The ITF has developed a formula for classifying the speed of a court in terms of a quantity known as the court pace rating (CPR). The CPR is based on the older SPR (Surface Pace Rating), defined by the relation SPR = 100(1 - μ). For example, if $\mu = 0.7$ then SPR = 30. A fast court therefore has a high SPR rating and a low COF. The CPR is defined by the relation CPR = SPR + k where k is a player perception factor given by k = 150(0.81 - COR). k is therefore zero for a court with COR = 0.81 (a typical, average value), is negative on clay courts (where COR is about 0.9) and is positive on grass courts (where COR is about 0.75). The CPR therefore represents a combined property of a court, based on both its COF and COR, that correlates with the perception of players regarding the speed of the court.

8. CONCLUSIONS

A relatively simple and accurate method of measuring the speed and bounce of tennis courts has been outlined in this paper. The method involves filming the bounce of a tennis ball with a video camera. A camera operating at 25 or 30 fps is adequate, although care needs to be taken to correct for changes in ball speed before and after the bounce as a result of the force of gravity and the aerodynamic lift and drag forces acting on the ball. Alternatively, inexpensive video cameras can now be purchased (at least from overseas sources) that can be used to film the bounce at frame rates around 200 or 300 fps, in which case the velocity of the ball can be measured more accurately, without the need to correct for changes in velocity prior to or after the bounce. The same technique could be used to measure friction and bounce properties of other sporting surfaces, although the results presented in this paper pertain only to tennis courts. It was shown in this paper that, at least for clay courts, the v_{x2}/v_{x1} ratio provides a more direct and more reliable indication of court speed than the measured COF.

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Appendix: Ball speed corrections

Suppose that three positions of the ball are recorded before the bounce and three after the bounce, as indicated in Fig. 6.

Figure 6 shows 3 images of the ball before the bounce, at times $t = 0$, T and $2T$. Three images of the ball after the bounce are shown at times t_4, t_5 and t_6. The coordinates of the ball at the six times are respectively (x_1, y_1), (x_2, y_2) ... (x_6, y_6), as indicated in Fig. 6. It is assumed that the time interval T between the three images after the bounce is the same as the time interval T between the first three images, so $t_5 = t_4 + T$ and $t_6 = t_5 + T$. At high frame rates, the ball positions can be recorded every second or third frame, while frames where the ball remains in contact with the court can be disregarded. The number of frames skipped during the bounce itself is arbitrary but should be kept to a minimum (0 or 1 when recording at 25 fps, and 2 or 3 when recording at 300 fps).

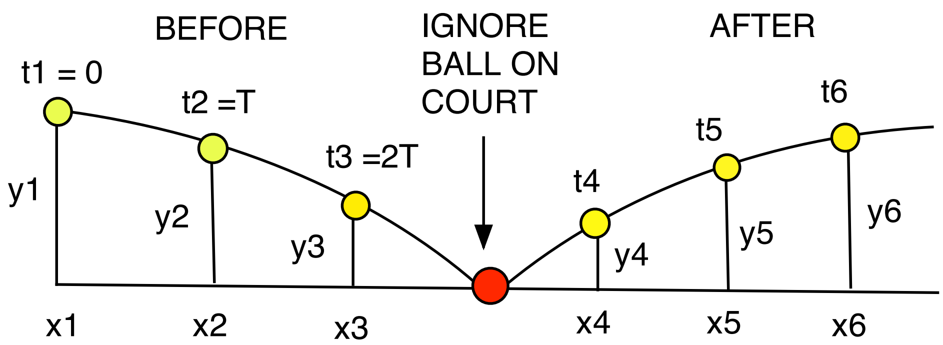

Figure 6: Three positions of the ball before and three after the bounce.

The ball follows a parabolic path before the bounce, and a different parabolic path after the bounce. Over small time intervals, the path is almost a straight line. The velocity of the ball in the vertical (y) direction, at time $t = T$, is given to a very close approximation by $v_{y1} = (y_3 - y_1)/(2T)$. The vertical velocity after the bounce, at time t_5, is $v_{y2} = (y_6 - y_4)/(2T)$. Similarly, the horizontal velocity at $t = T$ is $v_{x1} = (x_3 - x_1)/(2T)$, and the horizontal velocity after the bounce, at time t_5, is $v_{x2} = (x_6 - x_4)/(2T)$.

The coordinates of the ball at time $t = T$ and at time $t = t_5$ are not needed in the following calculations. The positions of the ball at these times can be recorded to provide a useful visual (on–screen) check that the ball position was recorded at equally spaced time intervals, that the ball is following a sensible trajectory, that the ball is not being recorded while it is in contact with the court and that there are no missing frames in the video clip transferred from the camera to the computer.

The velocities here need to be corrected for the fact that the ball speed changes over time due to the force of gravity and the effects of air resistance. As described previously, we are interested in the velocities when the ball first contacts the court, at time t_1 and when it just bounces clear of the court, at time $t_2 = t_1 + 0.005$ s. Time t_o in Fig. 6 denotes the mid point of the contact period, so $t_1 = t_o - 0.0025$ and $t_2 = t_o + 0.0025$. We can use the velocities measured at times $t = T$ and $t = t_5$ to calculate the acceleration of the ball and hence calculate the velocities at time t_1 and time t_2.

Good fits to the ball trajectory, both before and after the bounce, can be obtained with simple quadratic equations of the form $y = at^2 + bt + c$ and $x = dt^2 + et + f$ where a, b,...f are constant coefficients and t is the time. Quadratic fits work at any ball speed and over almost any distance travelled by the ball, but the coefficients depend on ball speed and spin and they are different before and after the bounce. Unfortunately, small errors in the measured ball positions result in large errors in the answers for the COR and COF, even when the ball position is in error by only one pixel.

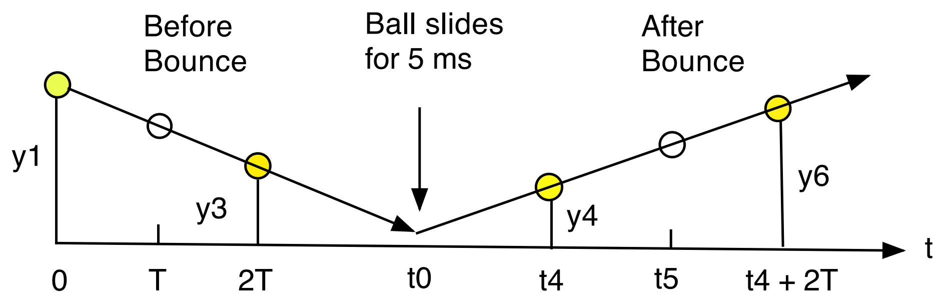

Figure 7: Straight lines can be fitted to the ball coordinates before and after the bounce, to estimate the ball speeds at times $t = T$ and $t = t_5$

From a practical point of view, a better procedure is to fit straight lines of the form $y = at + b$ and $x = ct + d$ to the measured ball coordinates. At time T, the components of the ball velocity are

$$v_{x1} = (x_3 - x_1)/(2T) \qquad\qquad v_{y1} = (y_3 - y_1)/(2T) \tag{10}$$

At time t_5, the components of the ball velocity are

$$v_{x2} = (x_6 - x_4)/(2T) \qquad\qquad v_{y2} = (y_6 - y_4)/(2T) \tag{11}$$

The straight line fits are $y = a_1 t + b_1$ and $x = c_1 t + d_1$ before the bounce, where

$$a_1 = v_{y1} \qquad\qquad b_1 = y_1 \qquad\qquad c_1 = v_{x1} \qquad\qquad d_1 = x_1 \tag{12}$$

and $y = a_2 t + b_2$ and $x = c_2 t + d_2$ after the bounce, where

$$a_2 = v_{y2} \qquad\qquad b_2 = y_4 - a_2 t_4 \qquad\qquad c_2 = v_{x2} \qquad\qquad d_2 = x_4 - c_2 t_4 \tag{13}$$

Small errors in the measured coordinates then result in relatively small errors in the COR and COF, provided a correction is made for the fact that the ball decelerates as a result of the drag force. Additional corrections must also be made for the acceleration due to gravity and the acceleration arising from the spin of the ball after it bounces. When filming at 300 fps or more, these corrections are negligible, but are included below so that the formulas remain valid regardless of the frame rate.

The y vs t curve before the bounce intersects the y vs t curve after the bounce at a time t_o when

$$y = a_1 t_o + b_1 = a_2 t_o + b_2 \tag{14}$$

so

$$t_o = \frac{(b_2 - b_1)}{(a_1 - a_2)} \tag{15}$$

The ball speeds in the x and y directions at times t_1 and t_2 can be calculated from the standard relation $v = u + at$ where a is the acceleration and u is the initial velocity. The velocity components at times $t = T$ and $t = t_5$ are given by the straight line fits to the ball positions, as described by Eqs. (10) and (11).

Acceleration of the ball

The drag force of the air on the ball causes it to slow down through the air. The effect is quite important at high ball speeds. For example, at $v_1 = 10$ m/s and a frame rate of 25 frames/s, the COF is overestimated by about 5% if the drag force is neglected. At 20 m/s, the COF is overestimated by about 10%. At 30 m/s, the COF is overestimated by about 15%.

The drag force $F = ma$ on a non–spinning ball is given by Eq. (9). For a tennis ball,

$$a = \frac{dv}{dt} = -0.020v^2 \tag{16}$$

If the ball is spinning, there is a slight increase in the drag force and an additional force arises called the lift force. Furthermore, the ball accelerates in a vertical direction due to the gravitational force. The acceleration of the ball in the x and y directions, resulting from all three forces, is described in Ref. (5) and is given by

$$a_x = -0.0352v(C_D v_x + C_L v_y) \tag{17}$$

$$a_y = -9.8 - 0.0352v(C_D v_y - C_L|v_x|) \tag{18}$$

where $|v_x|$ is the absolute value of v_x in Eq. (18), the -9.8 in Eq. (18) is the acceleration due to gravity, and v is the ball speed given by

$$v = (v_x^2 + v_y^2)^{1/2} \tag{19}$$

The drag coefficient for a spinning tennis ball is given by

$$C_D = 0.55 + 1/(22.5 + 4.2S^{2.5})^{0.4} \tag{20}$$

where $S = [v/(R\omega)]$ is a spin factor. $R = 0.033$ m is the radius of the ball and ω is the angular velocity of the spinning ball, in radians/sec. The lift coefficient is given by

$$C_L = -\frac{1}{(2.02 + 0.98S)} \tag{21}$$

In practice, $R\omega = 1.36(v_{x1} - v_{x2})$ after the bounce and $R\omega = 0$ before the bounce (to a good approximation). If the ball is incident without spin, as it is when testing court surfaces, $C_D = 0.55$ and $C_L = 0$. After the ball bounces, the ball has topspin and then C_D is typically about $0.55 + 1/4$ while C_L is typically about $-1/3.5$.

Speed corrections

The required values of the ball acceleration are

$$a_{x1} = -0.01936 v_1 v_{x1} \qquad\qquad a_{y1} = -9.8 - 0.01936 v_1 v_{y1} \tag{22}$$

where $v_1 = (v_{x1}^2 + v_{y1}^2)^{1/2}$

$$a_{x2} = -0.0352 v_2 (C_D v_{x2} + C_L v_{y2}) \tag{23}$$

$$a_{y2} = -9.8 - 0.0352 v_2 (C_D v_{y2} - C_L |v_{x2}|) \tag{24}$$

where $v_2 = (v_{x2}^2 + v_{y2}^2)^{1/2}$. The quantity $S = [v/(R\omega)]$ after the bounce is calculated with $v = v_2$ and $R\omega = 1.36(v_{x1} - v_{x2})$.

The corrected values of ball velocity at the impact and bounce times are then

$$v_{x1c} = v_{x1} + a_{x1}(t_1 - T) \qquad\qquad v_{y1c} = v_{y1} + a_{y1}(t_1 - T) \tag{25}$$

$$v_{x2c} = v_{x2} - a_{x2}(t_5 - t_2) \qquad\qquad v_{y2c} = v_{y2} - a_{y2}(t_5 - t_2) \tag{26}$$

Equations (25) and (26) are the speeds required to calculate both the COR and COF, as follows:

$$COR = -\frac{v_{y2c}}{v_{y1c}} \qquad\qquad COF = \frac{(v_{x1c} - v_{x2c})}{(v_{y2c} - v_{y1c})} \tag{27}$$

To avoid negative COF, it is best to calculate the absolute value of COF, otherwise COF will be negative if v_{x1} and v_{x2} are negative.

The angle of incidence is given by $\tan(\theta_1) = -v_{y1c}/v_{x1c}$ and the bounce angle is given by $\tan(\theta_2) = v_{y2c}/v_{x2c}$. The correct incident speed v_{1c} and the bounce speed v_{2c} are given by

$$v_{1c} = (v_{x1c}^2 + v_{y1c}^2)^{1/2} \qquad\qquad v_{2c} = (v_{x2c}^2 + v_{y2c}^2)^{1/2} \tag{28}$$