Launch of a Vehicle from a Ramp

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A vehicle proceeding up an inclined ramp will become airborne if the ramp comes to a sudden end and if the vehicle fails to stop before it reaches the end of the ramp. A vehicle may also become airborne if it passes over the top of a hill at sufficient speed. In both cases, the vehicle becomes airborne if the point of support underneath the vehicle falls below the trajectory that would be followed by the vehicle in the presence of gravity alone. When the vehicle becomes airborne, the normal reaction force exerted by the ramp or the hill drops to zero, first on the front wheels and then on the rear wheels. Just prior to the vehicle's becoming airborne, the normal reaction force on the rear wheels acts to exert a torque on the vehicle, causing the vehicle to rotate. After the rear wheels become airborne, the vehicle will continue to rotate until it lands some distance from the launch point.

An airborne vehicle is commonly driven by a stunt driver, either as a part of a police chase in a movie or as an attempt by a "daredevil" to break some distance record. Drivers in dune buggies like to become airborne just for the thrill of it. Dune buggies normally have a roll bar in case the vehicle rolls or lands on its front end. Four-wheel drive vehicles are also prone to rolling accidents, partly because of the high center of gravity of the vehicle and partly because the vehicles are often used in rugged terrain. In 2010 the author was contacted by the police to investigate an accident where a four-wheel drive vehicle became airborne at the top of a sand dune, landed on its front end, and rolled onto its soft roof. Two passengers in the rear seat were seriously injured, one becoming a paraplegic. The police asked me to estimate the launch speed and to describe the physics of the situation. As part of that investigation, I filmed a toy vehicle on a ramp in order to ensure that I understood the physics correctly. The results are described in this paper as an interesting practical problem combining awareness of driver safety, projectile motion, and rotation of a vehicle.

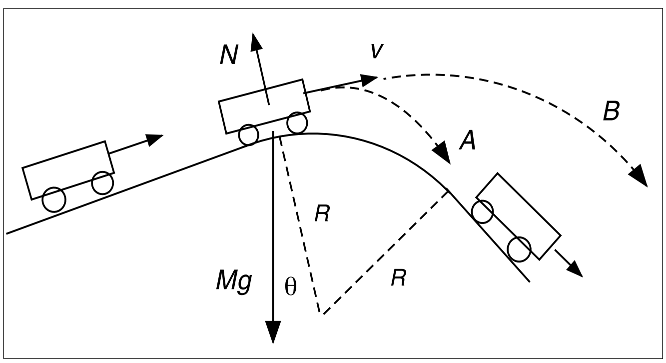

Fig. 1. A vehicle passing over a hill at speed *v* will become airborne and follow trajectory *B* if *v* is large enough. The normal reaction force *N* drops to 0 if the vehicle becomes airborne. Trajectory *A* is an assumed airborne low speed trajectory that intersects the hill, in which case *N* will remain positive and the vehicle will follow the trajectory of the hill rather than following trajectory *A*.

Theoretical considerations

Suppose that a vehicle is traveling up a hill with a gradual slope and that the hill has a steep downward slope on the other side, as shown in Fig. 1. For simplicity, we can assume that the slope remains constant near the bottom of both sides of the hill and that the two constant slope sections are connected by a circular path of radius R.

If the vehicle travels along the curved path at speed v, the normal reaction force N on the vehicle will be given by

$$N = Mg\cos\theta - Mv^2/R\,, \tag{1}$$

where M is the mass of the vehicle and θ is the angle between the vertical and the line of action of N. The normal reaction force therefore decreases as v increases, and will decrease to zero if $v = (gR\cos\theta)^{1/2}$. The vehicle will not become airborne near the bottom of the hill where the slope remains constant (since then $R = \infty$), but it will become airborne before it reaches the top of the hill if v is large enough or if R is small enough. For example, if $R = 10$ m and $\theta = 10°$, then the vehicle will become airborne if v exceeds 9.8 m/s. Conversely, if the vehicle was upside down underneath the road surface in a loop-the-loop ride, v would need to be greater than 9.8 m/s to prevent the vehicle becoming airborne. However, it would subsequently become airborne on the straight downhill slope.

If the vehicle remains in contact with the hill, then it will rotate as it passes over the hill, pointing upward as it ascends and downward as it descends. The origin of the torque required to rotate the vehicle is not immediately obvious, but can be attributed to a larger normal reaction force acting on the rear wheels than on the front wheels. When the vehicle is near the top of the hill, as in Fig. 1, it tends to follow a path tangent to the hill so the load on the front wheels decreases, resulting in a net clockwise torque on the vehicle. The physics

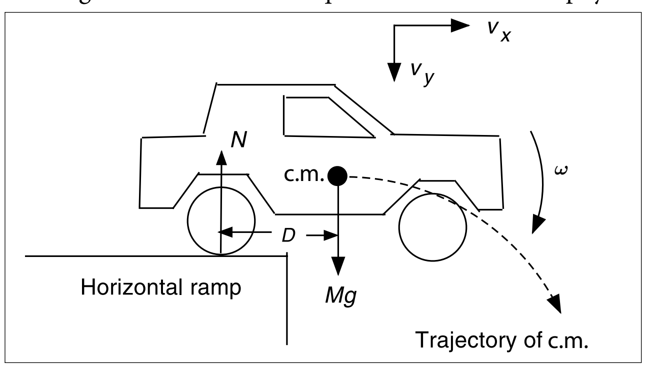

Fig. 2. A vehicle launched off a horizontal ramp will commence to rotate when the front wheels leave the ramp. *N* is the normal reaction force on the rear wheels, c.m. denotes the center of mass, and ω is the angular speed of the vehicle.

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of the situation is more obvious if we consider the launch of a vehicle at speed v_x from a horizontal ramp, as shown in Fig. 2. In that case, the center of mass continues in a horizontal direction at the instant the front wheels leave the ramp, even though the normal reaction force on the front wheels drops to zero. If there is no horizontal braking or accelerating force on the vehicle, then the horizontal component of the velocity will remain equal to v_x, even after the rear wheels lose contact with the ramp.

The equations of motion for the vehicle while the front wheels are airborne and the rear wheels are in contact with the ramp are

$$Mg - N = M\frac{dv_y}{dt} \tag{2}$$

and

$$ND = I_{\text{c.m.}}\frac{d\omega}{dt}\ , \tag{3}$$

where v_y is the vertical speed of the center of mass (c.m.) of the vehicle, ND is the torque acting about the c.m., $I_{\text{c.m.}}$ is the moment of inertia about the c.m. and ω is the angular velocity of the vehicle. Since the vehicle pivots about the rear axle, $v_y = D\omega$ so $dv_y/dt = D d\omega/dt$ and we find from Eqs. (2) and (3) that

$$N = \frac{Mg}{(1 + MD^2/I_{\text{c.m.}})}\ . \tag{4}$$

For a uniform object of mass M and length L, $I_{\text{c.m.}}$ is about $ML^2/12$. If we take $L = 3D$, then $I_{\text{c.m.}}$ is about $3\,MD^2/4$ and hence N is about 0.43 Mg. While all four wheels are on the ramp, $N = 0.5\,Mg$ if the weight is distributed equally, in which case there is a slight drop in N on the rear wheels when the front wheels become airborne.

Since N remains constant while the front wheels are airborne, the angular velocity of the vehicle will increase to a value $NDT/I_{\text{c.m.}}$ by the time the rear wheels become airborne, where $T = W/v_x$ is the time interval between the front and rear wheels becoming airborne and W is the distance between the front and rear wheels. A vehicle proceeding slowly over the ramp rotates faster than a vehicle launched at high speed since the torque acts for a longer time. Since N and $I_{\text{c.m.}}$ are both proportional to M, the rotation rate is independent of the mass of the vehicle.

Toy vehicle experiment

The situation shown in Fig. 2 was examined using a toy vehicle of mass 43.9 g, length 78 mm, and with $D = 26$ mm (see Fig. 4). Its moment of inertia was measured by mounting the vehicle as a physical pendulum with an axis at one end, giving $I_{\text{c.m.}} = 2.43 \pm 0.05 \times 10^{-5}\ \text{kg}\cdot\text{m}^2$. The vehicle was projected at three different speeds off the edge of a table and filmed with a video camera to determine its launch speed, the trajectory through the air, and its angular velocity after the launch. The rotation speed results, shown in Fig. 3, compare well with the theoretical prediction. At very low speeds the vehicle slid to a stop without being launched, since the front wheels dropped almost vertically off the table and the undercarriage slid on the corner of the table.

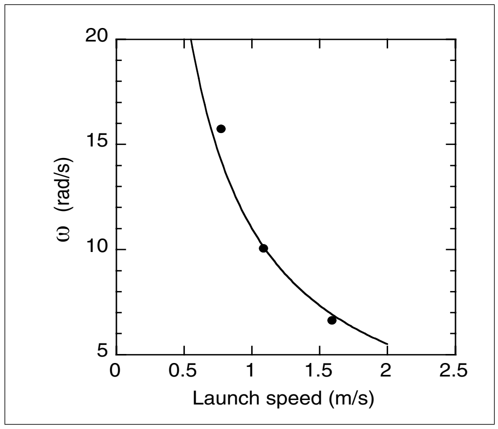

Fig. 3. Measured angular velocity of toy vehicle after launch vs launch speed for three different launch speeds (black dots). The solid curve is the theoretical prediction.

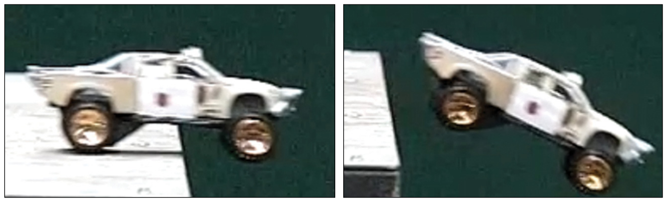

Fig. 4. Video images of toy vehicle launched off the edge of a table.

Case study

The case I investigated for the police involved a vehicle that crashed when it became airborne on a sand dune. The vehicle approached the top of the dune on an incline of 7°. The dune fell sharply on the other side at an angle of 35° to the horizontal. There was no gradual change in slope at the top of the dune and no significant disturbance at the top of the dune created by the vehicle. The vehicle therefore became airborne in a direction tangential to the approach side and was observed by witnesses to land in an approximately vertical position before rotating forward onto its roof. The center of mass of the vehicle fell through a vertical height of 2.8 m and traveled a horizontal distance of 5.1 m before the vehicle nose-dived into sand at the bottom of the dune. A simple trajectory calculation showed that the vehicle was launched at a speed of 24 km/h. The estimated rotation angle of the vehicle was about 80°, consistent with witness statements.

Rod Cross *has lectured and researched physics at Sydney University since 1968 and now works mainly on the physics of sport. Sports physics movies can be viewed at www.physics.usyd.edu.au/~cross. He has authored two books on the physics of tennis, a forensic physics book,* Evidence for Murder, *and* Physics of Baseball and Softball.
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