Measuring the effects of lift and drag on projectile motion

The trajectory of a projectile through the air is affected both by gravity and by aerodynamic forces.[1] The latter forces can conveniently be ignored in many situations, even when they are comparatively large. For example, if a 145 g, 74 mm diameter baseball is pitched at 40 ms^{-1} (89.5 mph) it experiences a drag force of about 1.5 N. The gravitational force on the ball 1.42 N. Nevertheless, the trajectory of a baseball pitched without spin is not strongly affected by the drag force. Because the ball is relatively heavy and the flight distance is relatively small (about 60 ft) the drag force reduces the ball speed by only about 10% by the time it reaches the batter. As a result, the time taken for the ball to reach the batter is only about 5% longer than in a vacuum, and the actual trajectory is also very similar.[2] In situations such as this, it is difficult to obtain accurate measurements of the drag force by measuring the trajectory of a ball.

A more significant change in the trajectory of a baseball results if the ball is launched at a higher angle and travels a long distance toward the fence, or if the ball is spinning. A spinning ball experiences a force that increases with the spin and is known as the Magnus force.[3] The Magnus force on a 90 mph ball pitched at 2000 rpm generates a positive or negative lift force of about 1.0 N, depending on the spin direction. That force acts at right angles to the path of the ball and will cause the ball to deviate by about 24 inches vertically (if the ball has topspin or backspin) or horizontally (if the ball has sidespin) by the time the ball reaches the batter.[2] That is a serious problem for batters, but teachers can conveniently describe the motion of a pitched ball to a first approximation by ignoring the aerodynamics. As a separate issue, is not difficult to calculate the deviation caused by an additional 1.0 N sideways or vertical lift force acting on a 145 g mass while the ball is traveling to the batter. The answer is $s = 0.5at^2$ where $a = F/m$. The deviation caused by ball spin is closely related to the "break" or "movement" of the ball described in TV broadcasts, although the break due to the spin is usually measured from a point starting well in front of the pitcher rather than right from the pitcher's hand.[2]

More dramatic changes in the trajectory of a projectile occur when the projectile is relatively light and when the gravitational force is significantly smaller than the lift and drag forces. A paper airplane is an obvious example, although the aerodynamics is generally too complicated to describe in simple terms. A simpler example is the flight of a light cylinder or a spherical ball. If a light, horizontal cylinder or ball is projected horizontally with backspin, and if the Magnus force is larger than the gravitational force, then the cylinder or ball will rise through the air before falling back to the ground. The lift and drag forces then play a dominant role in determining the trajectory and both forces can be easily measured. In general, aerodynamic forces are difficult to measure accurately by filming the flight of a projectile, although good results for the lift force on a baseball were obtained recently by Nathan.[4] Even so, he needed ten high speed cameras to film the flight of the ball over a reasonable distance and was not able to obtain an accurate measure of the drag force. Better results could have been obtained had Nathan removed the wool from inside the ball and stiffened the cover by other means. The lift and drag forces would have remained the same but the ball trajectory would then have provided a more sensitive measure of those forces.

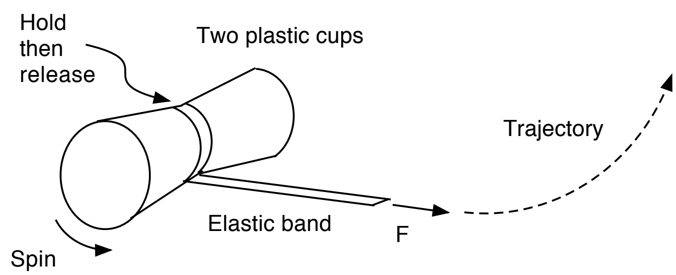

Figure 1: Two plastic cups joined end to end can be launched with backspin at about 5000 rpm and at about 10 ms^{-1} using an elastic band. The cups were held in one hand and the band was stretched by applying a force, F, using the other hand. The trajectory is dominated by aerodynamic forces rather than by gravity.

The author conducted an experiment of this type, not with a hollow baseball but with two plastic cups joined end–to–end with adhesive tape, as shown in Fig. 1. The overall length of the two cups was 132 mm, the end diameters were 60 mm and the diameter in the middle was 45 mm. A 70 mm diameter, 10 g plastic Christmas ball (the type used to decorate a Christmas tree) was also tried but the lift effect due to ball spin was less dramatic. Plastic or paper cups provide much clearer demonstrations of lift and drag effects. The mass of the two cups plus the tape was 13.2 g. They were projected in an approximately horizontal direction with backspin using an elastic band wrapped around the cups three times. One end of the band was held onto the middle section of the cups with the left hand and the free end of the band was stretched horizontally in the launch direction with the right hand. The trajectory of the cups was filmed at 30 frames/s with a Casio EX-F1 video camera and the spin was subsequently determined by filming at 300 frames/s, zoomed in close near the launch point to observe the rotation of marks on the cups.

Typical results are shown in Fig. 2. The trajectory is dramatically different from the usual parabolic trajectory observed with heavy balls. Instead, the lift force caused the cups to rise vertically in the air while the drag force reduced the speed substantially. The cups then fell at low speed to the ground, curving away from the launch point as a result of the lift force. The lift force acts in a direction perpendicular to the velocity vector and therefore acts approximately horizontally when the cups fall approximately vertically. The spin was not determined for the particular trajectory shown in Fig. 2(a) but was determined for a similar shot as shown in Fig. 2(b). No significant decrease in spin was observed during the first six revolutions captured on film, although there may have been a slight decrease in spin by the end of the trajectory. On striking the ground the cups bounced and rolled about half way back to the original launch point, in a manner similar to that often seen in high trajectory, backspin golf shots, indicating that the cups were still spinning rapidly when they struck the ground.

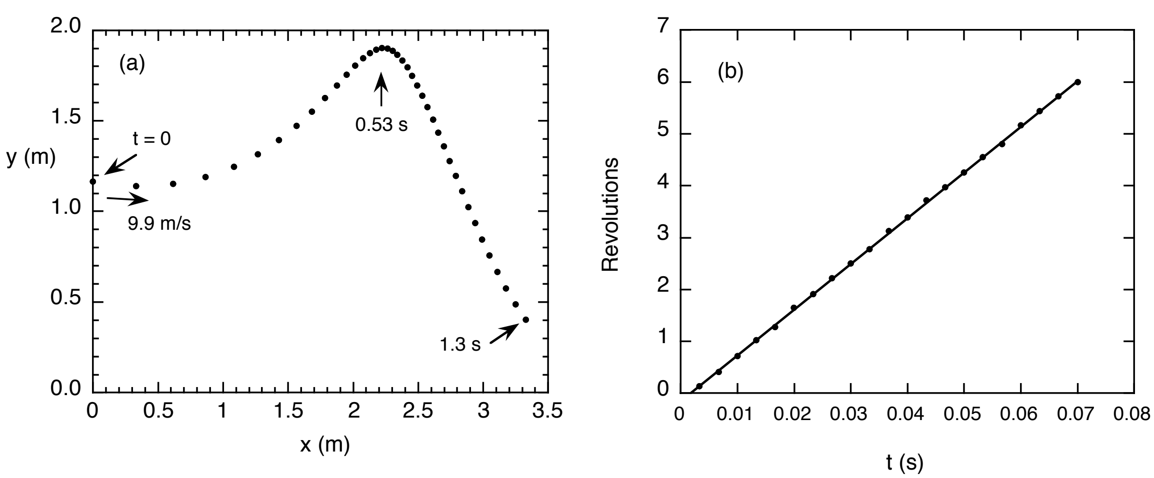

Figure 2: (a) Trajectory of cups launched at 9.9 ms^{-1} showing y vs x at intervals of 1/30 s. (b) Measured spin of cups, showing the number of revolutions vs time at intervals of 1/300 s for a launch similar to that in (a). The straight line fit corresponds to a launch spin of 5200 $\pm$100 rpm.

Trajectory analysis

Consider a ball or cylinder of mass m that is traveling through the air with backspin at speed v and at an angle θ to the horizontal, as shown in Fig. 3. The forces on the object consist of the gravitational force, mg, a drag force F_D acting in a direction opposite the velocity vector, and a lift force F_L acting in a direction perpendicular to the velocity vector. The equations of motion describing the trajectory are

$$ma_x = -F_D \cos\theta - F_L \sin\theta \tag{1}$$

and

$$ma_y = F_L \cos\theta - F_D \sin\theta - mg \tag{2}$$

where a_x is the horizontal acceleration and a_y is the vertical acceleration. From Eqs. (1) and (2) we find that

$$F_D = -m(g \sin\theta + a_x \cos\theta + a_y \sin\theta) \tag{3}$$

and

$$F_L = m(g \cos\theta - a_x \sin\theta + a_y \cos\theta) \tag{4}$$

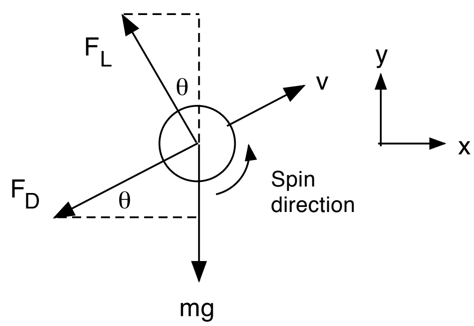

Figure 3: The forces acting on a ball with backspin include the gravitational force, mg, the drag force, F_D, and the lift or Magnus force, F_L.

The measurements of $x(t)$ and $y(t)$ shown in Fig. 2(a) were fitted with sixth order polynomials to smooth out small errors in the position measurements. The resulting fits were differentiated to obtain the velocity components v_x and v_y and differentiated again to obtain a_x and a_y. The angle θ was obtained from the slope dy/dx, and g was taken as 9.8 ms^{-2}. The resulting values of F_D and F_L are shown as functions of ball speed in Fig. 4 and can be compared with the gravitational force, 0.13 N, acting on the cups.

As expected, both F_D and F_L increase with velocity. Conventionally, drag and lift forces are expressed in the form

$$F_D = 0.5 C_D \rho A v^2 \qquad \text{and} \qquad F_L = 0.5 C_L \rho A v^2 \tag{5}$$

where ρ is the density of air (1.2 kg.m^{-3}), A is the cross–sectional area of the projectile, v is the velocity, C_D is the drag coeficient and C_L is the lift coefficient. At low speeds or at low Reynold's numbers, C_D is about 0.5 for a sphere and about 1.2 for a cylinder.[5] The results in Fig. 4 indicate that C_D and C_L are both equal to 1.2 ± 0.1 at $v = 10$ m/s. In Fig. 4, F_D is proportional to v^2 when $v > 4$ ms^{-1}, while F_L is approximately proportional to v. The latter result can be explained by the fact that the lift force is proportional to $C_L v^2$ but C_L depends on the ratio of the spin to the velocity. If there is no spin then $C_L = 0$ and there is no lift. If C_L is approximately proportional to spin/v (as observed by Nathan for a sphere[4]) then F_L is approximately proportional to v, as observed.

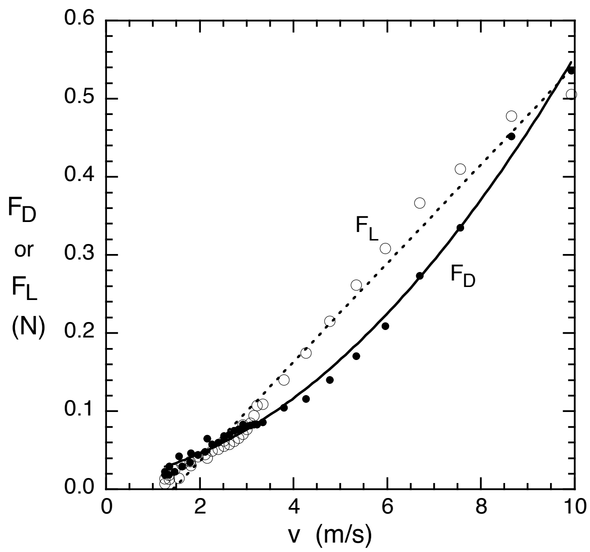

Figure 4: Measured values of the drag force, F_D, shown by black dots, and the Magnus force, F_L, shown by open circles. The solid and dashed lines are best fits to the data, giving $F_D = 0.007v + 0.0047v^2$ and $F_L = 0.063v - 0.089$.

References

[1] V. Pagonis and D. Guerra, "Effects of air resistance", *Phys Teach.* **35**, 364–368 (1997).

[2] R. Cross, *Physics of Baseball and Softball*, Springer NY (2011).

[3] R. Cross, "Aerodynamics of a party balloon", *Physics Teach.* **45**, 334-336 (2007).

[4] A. Nathan, "The effect of spin on the flight of a baseball", *Am. J. Phys.* **76**, 119–124 (2008).

[5] http://en.wikipedia.org/wiki/Drag_coefficient