# Dynamic properties of tennis balls 

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#### Abstract

Measurements are presented on the dynamic properties of several different tennis balls, during impacts on a force plate. The force on a tennis ball rises rapidly to about half its maximum value in the first $200 \mu \mathrm{~s}$ of the impact due to compression of the cloth cover and the rubber wall near the impact point. The wall then collapses inwards, resulting in a sudden decrease in ball stiffness. Results are presented on the force waveform, the impact duration, ball compression and coefficient of restitution as a function of ball speed.


Keywords: tennis ball, hysteresis, force plate, coefficient of restitution

## Introduction

The properties of tennis balls are rigidly specified by the rules of tennis, more so than most other balls used in major sporting events. Even so, a wide variety of tennis balls with different physical properties is manufactured for the consumer. In Europe, tennis balls tend to be more expensive and more durable, since the consumer expects them to last for several months. In the United States, balls tend to be less expensive and less durable since US players generally prefer to use new balls after a few sets. Some balls are manufactured to near minimum legal size to save on materials costs, while others are made to near maximum legal size to satisfy consumer demand.

There is concern that different balls also play differently, not only because the rules allow some variation in the mass, diameter and coefficient of restitution, but also because the rules are not specific regarding ball properties under actual playing conditions. The compressibility of a tennis ball is specified and tested under static conditions, and the coefficient of restitution, $e$, is specified and

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tested for a low speed collision with a concrete slab. When dropped from a height of 100 inches $(2.54 \mathrm{~m})$ onto a concrete slab, an approved ball must bounce to a height between 53 and 58 inches $(1.35-1.47 \mathrm{~m})$. The advantage of this test is that it is easily implemented by both the manufacturers and the testing authorities. In principle, it should also be relatively easy to measure and specify the coefficient of restitution at higher ball speeds, but the required apparatus has not yet been developed to a point where standards can be simply and reliably specified or enforced.
Furthermore, there is interest in possible alteration of the rules concerning ball properties, in order to make the sport more attractive to spectators and television audiences. For example, the ball speed could be reduced by making the ball larger or by lowering the coefficient of restitution. The latter method is used to reduce the ball speed in top levels of competitive squash. An alternative solution for tennis balls might be to tailor the coefficient of restitution to decrease, as the ball speed increases, by a specified amount. This already happens, but the amount is not specified. One might suspect that a heavier ball would also be slower, but this is not necessarily the case. Even though a heavier ball may come off the strings at a lower speed, the subsequent drag force through the air will result in
a relatively small deceleration compared with the deceleration of a light ball. These effects are currently under examination.

In this paper, results are presented on the properties of several types of tennis ball, the objective being to provide a more scientific basis on which ball properties can be evaluated. For this purpose, the dynamic behaviour of each ball was measured by bouncing it vertically off a force plate containing an array of large area piezo elements. This is obviously not the same as a bounce off the strings of a racquet, but it provides a valid measure of the dynamic properties of the ball and it provides data of direct relevance to the bounce of a ball off the court surface.

## Apparatus

A measurement of the force acting on a ball when it impacts with a rigid surface provides data on the elastic properties of a ball under dynamic conditions. These properties can be quite different from those measured during a static compression. The apparatus used in this experiment to measure ball properties under dynamic conditions is shown in Fig. 1. The upper part of Fig. 1 shows the rotating wheel ball launcher, and the lower part shows the force plate used to measure the force on the ball. An upper force plate was also used, to measure the rebound speed of the ball.

The incident and rebound ball speeds were measured under conditions where the ball was incident vertically downwards on the lower force plate, passing through a vertical tube of internal diameter 75 mm . Two horizontal, circular metal plates, with 80 mm diameter holes, were mounted at the base of the tube so that the ball could pass through the 80 mm holes. Several small piezos were mounted between the plates to form a force plate in order to detect the time of arrival of the rebounding ball. No spin was imparted to the ball. The ball did not rebound straight back through the tube, but always rebounded at a small angle to the vertical, thereby striking the upper force plate. A horizontal laser beam passing through holes near the bottom of the tube was used, in conjunction


Figure 1 Apparatus used to measure the dynamic properties of a tennis ball. The grounded circuit board was used as an electrostatic shield, since the ball charged electrically during compression. The Zn paste provided good electrical contact and minimized mechanical cross-talk between the four piezos.
with a photodiode, to record the time at which the ball passed the beam. The time at which the ball first contacted the lower force plate, and the time at which it rebounded, were also recorded. The lower force plate was located 38.3 cm below the upper force plate. After making small corrections for the gravitional acceleration downwards and the deceleration of the rebounding ball, the incident and rebound ball speeds could be determined to within about $2 \%$. However, the ball did not always rebound with the same speed when the incident speed was kept constant, so all results presented below are averages over two or three separate measurements. These measurements were made after 'conditioning' a new ball by bouncing it several times at high speed on the force plate. The variability in rebound speed and rebound angle,
also commonly observed in the standard 100 inch drop test, can be attributed to (a) the effect of the seam (b) the fact that balls are not required to be perfectly spherical and (c) the wall thickness is not required to be perfectly uniform.

The lower force plate was constructed by the author, since force plates with a suitably high frequency response are not available commercially. Commercial force plates are designed primarily for biomechanical applications and suffer from resonances in the plates for short duration impacts (Cross 1999a). The apparatus shown in Fig. 1 is quite simple, although it was developed after many trial and error iterations to obtain a reliable response. The main component is an array of four square piezoelectric ceramic plates, each of dimensions $50 \times 50 \times 4 \mathrm{~mm}$. Each plate has two silvered electrodes bonded to and covering each of the large surfaces on opposite sides of the plate. The four piezos were arranged in a large square of dimensions $100 \times 100 \times 4 \mathrm{~mm}$, connected electrically in parallel, and were mounted on a steel backing plate of dimensions $140 \times 140 \times 25 \mathrm{~mm}$. The dimensions of the array were chosen so that the contact area of the ball remained less than the area of the array. A single, suitably large area piezo was not available commercially. A similar system was described recently for testing baseballs (Hendee et al. 1998). The latter authors did not give any details of the frequency response of their system. The force plate shown in Fig. 1 provided an accurate response, free of plate resonances, for impacts of duration between $100 \mu \mathrm{~s}$ and 100 ms , as measured by bouncing a steel ball on the plate and by walking on the plate. The low frequency response was limited by the $R C$ time constant of the system, where $C$ represents the capacitance of the piezos and $R=10 \mathrm{M} \Omega$ is the resistance of the voltage probe used to record the output signal.

## Theoretical model of ball impact on a rigid surface

The dynamics of the bounce of a ball can be predicted approximately by assuming that it obeys

Hooke's law $F=-k x$, where $F$ is the force acting on the ball, $x$ is the ball compression and $k$ is the effective spring constant of the ball. This leads to the result that $F$ vs. $t$ is a half-sine waveform of duration

$$
\begin{equation*}
\tau=\pi \sqrt{m / k} \tag{1}
\end{equation*}
$$

where $m$ is the mass of the ball (Cross 1999b). There is no energy loss in the ball in this case. However, it is shown below that the force on a tennis ball is a strongly nonlinear function of the ball compression, and the dynamics cannot be expressed simply in any analytical form. Nevertheless, it is also shown below that Eq. (1) provides a useful estimate of the impact duration, provided that $k$ is interpreted as a time-average value of the ratio $F / y$ where $y$ is the displacement of the centre of mass of the ball.

When a ball of mass $m$ impacts vertically on a rigid surface at speed $v_{1}$, it experiences an impulsive force, $F$, which is typically $100-1000$ times larger than $m g$. The force is given by $F=m \mathrm{~d} v / \mathrm{d} t$ where $v=\mathrm{d} y / \mathrm{d} t$ is the velocity of the centre of mass (CM) of the ball and $y$ is the vertical displacement of the CM of the ball. A measurement of $F$ vs. $t$ can therefore be used to obtain $y$ vs. $t$ by numerical solution of the equation

$$
\begin{equation*}
\mathrm{d}^{2} y / \mathrm{d} t^{2}=F / m \tag{2}
\end{equation*}
$$

assuming that at $t=0, y=0$ and $\mathrm{d} y / \mathrm{d} t=v_{1}$. A plot of $F$ vs. $y$ represents a dynamic hysteresis curve, analogous to the static hysteresis curve obtained when one plots $F$ vs. ball compression under static conditions (Brody 1979). The rules of tennis currently specify bounds for such a static hysteresis curve, but do not refer directly to dynamic hysteresis measurements. Typical $F$ vs. $t$ waveforms observed with the force plate are shown in Fig. 2, together with the corresponding $y$ vs. $t$ and $F$ vs. $y$ curves computed from Eq. (2).

The area enclosed by a dynamic hysteresis curve, for a complete compression and expansion cycle, represents the loss of energy during the collision.


Figure 2 A sample of results obtained with three different balls, including a Dunlop Airloc ball (previously unused, low pressure or depressurized over time but still very firm), Wilson (brand new, pressurized) and a well-used, old, soft, depressurized Slazenger ball.

The ball compresses during the collision, converting the initial kinetic energy to potential energy, and then expands to convert the potential energy back to kinetic energy. However, energy is dissipated in the ball during this process, with the result that the rebound speed, $v_{2}$, is less than $v_{1}$. The reduction in kinetic energy is given by

$$
\begin{equation*}
\oint F \mathrm{~d} y=m\left(v_{1}^{2}-v_{2}^{2}\right) / 2, \tag{3}
\end{equation*}
$$

which can be equated to the energy dissipated in the ball. Eq. (3) is easily derived from the relation $F=m \mathrm{~d} v / \mathrm{d} t=m(\mathrm{~d} v / \mathrm{d} y)(\mathrm{d} y / \mathrm{d} t)$ where $v=\mathrm{d} y / \mathrm{d} t$, but the circumstances are unusual in that (a) the ball is not a rigid body and (b) the force is applied at a point on the ball that remains stationary during the impact. As a result, no work is done by $F$ in
changing the total energy of the ball, so the total energy after the collision (including the energy dissipated in the ball) is the same as the total energy before the collision. A ball can rebound in either a compressed or elongated state depending on the rate at which the ball recovers from the compression and depending on the amplitude of any oscillations excited by the impact. All balls studied in this paper rebounded in a slightly compressed state, so that $y$ remained finite when $F$ dropped to zero at the end of the impact.

## Effects of cloth and rubber on initial impact force

As shown in Fig. 2, the force on a tennis ball rises rapidly during the first 0.2 ms of the impact, to a value that is typically about half the maximum force
at high ball speeds. This effect can be attributed to compression of the cloth and underlying rubber in a small region surrounding the initial impact point. The impact properties of the cloth cover were investigated separately by covering a 6 -inch $(15.24 \mathrm{~cm})$ diameter spherical aluminium ball, of wall thickness 3 mm , with standard tennis ball cloth of thickness about 3 mm . The mass of the ball plus the cloth was 532 g . Without the cover, the impact duration of the ball on the piezo array was 0.8 ms , as shown in Fig. 3. The addition of the cover increased the impact duration to 5 ms for a low speed collision ( 20 mm drop height) or to 4 ms at a slightly higher ball speed ( 60 mm drop height). High speed impacts were not investigated in order to avoid possible damage to the ceramic piezos. Despite the low speed of these impacts, the resulting force on the cloth was within the range of the impact forces acting on a tennis ball at high speed.

The impact properties of the rubber were investigated by a similar procedure, using several strips of


Figure 3 Force vs. time waveforms for a 6 inch ( 15.24 cm ) diameter aluminium ball dropped onto the force plate from a height of 20 mm or 60 mm , as labelled with a prefix. The ball was either completely covered in $3-\mathrm{mm}$ thick cloth (20C and 60 C waveforms), or partly covered in 3 mm thick rubber (20R and 60 R waveforms), or partly covered with a $6-\mathrm{mm}$ thick composite layer of rubber and cloth (20RC and 60RC waveforms). For the curve labelled Al, the ball was dropped from a height of 10 mm to contact directly on the aluminium surface of the ball.

3-mm thick rubber cut from a pressurized tennis ball and glued side-by-side to the bottom section of another 6 -inch diameter aluminium sphere of wall thickness 3 mm . As shown in Fig. 3, the impact duration in this case was 3 ms for a $20-\mathrm{mm}$ drop, and 2 ms for a $60-\mathrm{mm}$ drop, indicating that the rubber is stiffer than the cloth by a factor of about four. Also shown in Fig. 3 are the corresponding impacts when several strips of rubber, with cloth attached to the rubber, are glued to the aluminium ball to form a $6-\mathrm{mm}$ thick composite covering. The combination of the cloth and the rubber is softer than the rubber or the cloth alone, and the impact durations are correspondingly longer.

The impact force waveform for the uncovered aluminium ball is a half-sine wave, indicating that the compression of the ball is linearly proportional to the applied force. The force waveform for the cloth-covered ball is an exponentially rising and falling bell-shaped curve, indicating that the force is exponentially proportional to the compression of the cloth. An analysis of the hysteresis curves for the cloth-covered ball shows that, during the compression phase,

$$
\begin{equation*}
\ln _{\mathrm{e}}(F)=2.98 y+0.84 \tag{4}
\end{equation*}
$$

where $F$ is the force in Newton, and $y$ is the displacement in mm of the centre of mass of the ball. This relation was found to describe the cloth compression accurately in the range $3<F<1000 \mathrm{~N}$. For example, a force of 202 N will compress the cloth to half its original thickness (i.e. $y=1.5 \mathrm{~mm}$ ). As a good approximation, one can assume that the displacement $y$ is equal to the compression of the cloth since the compression of the aluminium was negligible compared with that of the cloth. Similarly for the rubber, the force is exponentially proportional to the compression of the rubber, at least in the range $3<F<1000 \mathrm{~N}$, where

$$
\begin{equation*}
\ln _{\mathrm{e}}(F)=6.06 y+1.94 \tag{5}
\end{equation*}
$$

The combination of the rubber and the cloth was found to satisy the relation

$$
\begin{equation*}
\ln _{\mathrm{e}}(F)=2.0 y+1.20 \tag{6}
\end{equation*}
$$

which is consistent with the fact that, for any given $F$, the total compression is equal to the compression of the rubber plus the compression of the cloth.

During the first 0.2 ms of the collision of a tennis ball with a rigid surface, the ball speed remains essentially constant since the change in momentum, $\int F \mathrm{~d} t=m \Delta v$ is negligible compared with the initial momentum. At an initial speed of $15 \mathrm{~ms}^{-1}$, the centre of mass moves a distance of only 3 mm during the first 0.2 ms . A logarithmic plot of the $F$ vs. $t$ curves in Fig. 2 shows that $F$ increases exponentially with $t$ during this time. Similarly, a logarithmic plot of the $F$ vs. $y$ curves in Fig. 2 shows that

$$
\begin{equation*}
\ln _{\mathrm{e}}(F)=2.12 y+0.28 \tag{7}
\end{equation*}
$$

for compressions up to $y=2.5 \mathrm{~mm}$ (or $F$ up to 265 N ), the constants varying only slightly for different balls. For a given compression, this force is a factor of approximately two lower than that given by Eq. (6). This is consistent with the fact that, for any given $y=v_{1} t$, the contact area of the tennis ball is a factor of about two smaller than that of the larger diameter aluminium ball.

## Subsequent force on ball

For compressions larger than about 1 or 2 mm , there is a sudden transition from a high to a low stiffness state commencing about $0.1-0.2 \mathrm{~ms}$ after the initial contact, depending on the ball speed (see Fig. 2). This suggests that the wall starts to bend at this time since it is much easier to bend rubber than to compress it. The transition point does not occur at a fixed compression or at a fixed value of the impact force. At low ball speeds, the transition occurs at a force $\mathrm{F} \sim 50 \mathrm{~N}$. As the ball speed increases, the transition occurs at a higher force and at earlier times.

Experimental results indicate that the wall deforms approximately as shown in Fig. 4. The most direct result is obtained by visual inspection of the


Figure 4 Deformation of a ball about 2 ms after initial contact, showing the formation of an internal bubble.
interior of a ball that is cut in half and compressed by hand on a flat surface. The force waveform of a ball half, when dropped or thrown on the piezo array, is very similar to that of a complete ball, indicating that deformation of the bottom half of the ball is responsible for the general features observed in the force waveform. A small vertical force applied around the perimeter of a ball half results in the contact area deforming into a relatively flat, circular disk. As the force is increased, the wall buckles as shown in Fig. 4. When the force is sufficiently large, a hemispherical ball turns inside-out. The formation of an interior bubble in the ball can be attributed to a horizontal component of the applied force, transmitted through the wall to the contact area, resulting in an unstable buckling of the contact region under compression. This effect is probably assisted by the fact that the initial contact area compresses in a vertical direction and will therefore tend to bounce off the surface while the rest of the ball is still moving towards the surface. Such an effect would help to explain why the transition to a low stiffness state occurs earlier in time and at a larger force as the incident ball speed increases.

A dynamic test of the buckling effect was performed by mounting a $2-\mathrm{mm}$ thick, 13 mm diameter piezo disk on top of the lower force plate, as shown in Fig. 5. The small disk was surrounded by a $2-\mathrm{mm}$ thick, $80-\mathrm{mm}$ square piece of circuit board containing a $15-\mathrm{mm}$ diameter hole for the small piezo, so that the ball impacted on a flat surface and so that the small piezo was not subject to bending. The large piezo array responded to the


Figure 5 Arrangement used to measure the force at selected positions under the ball.
total impact force, and the small piezo sampled the force acting on a $13-\mathrm{mm}$ diameter section at the bottom of the ball. The sampled location under the ball could be varied, depending on where the ball landed. Typical results are shown in Fig. 6, indi-


Figure 6 Evidence for the deformation shown in Fig. 4. Waveform (a) shows the total force acting on the ball at an impact speed of about $7 \mathrm{~ms}^{-1}$. Waveform (b) shows the force on a $13-\mathrm{mm}$ diameter piezo located on the force plate directly below the ball centre, and waveform (c) shows the force on the small piezo when the initial impact point is 20 mm away from the centre of the small piezo.
cating that the vertical force on the initial contact area is a minimum when the total force on the ball is a maximum.

As shown in Fig. 2, a force peak is sometimes observed at the end of the contact period, especially at high ball speeds and with old or depressurized balls. This peak is due to the internal bubble 'popping' back out of the ball at the end of the impact, and was observed on both the large piezo array and the small piezo at the centre of the array. However, the waveform from the small piezo was found to be noisy at high ball speeds due to the fact that the small piezo bounced off the force plate and lost electrical contact with the lower surface when the force on the small piezo dropped to zero.

## Pressurized vs. unpressurized balls

Tennis balls can be classified as either pressurized or unpressurized (or 'pressureless') depending on whether the air pressure inside the ball is higher than or equal to atmospheric. Pressurized balls are usually packaged in a pressurized can to prevent gradual leaking of the air through the ball. However, some pressurized balls, such as the Dunlop Airloc ball, do not need to be stored in a pressurized can, since they are basically a thick-walled unpressurized ball filled slightly above atmospheric pressure to increase the coefficient of restitution slightly. There is no internal metallised coating in a tennis ball to prevent the air leaking out. As the air gradually leaks out of an Airloc ball, its coefficient of restitution decreases, but it remains within the specified limits even when the internal pressure drops to atmospheric. The internal pressure of a pressurized ball is typically 6-12 psi above atmospheric pressure, $P_{\mathrm{A}}$, where $P_{\mathrm{A}}=14.7 \mathrm{psi}=$ 101 kPa . Upressurized balls are required to have the same mass, external diameter, coefficient of restitution, and static stiffness as a pressurized ball, within permitted small variations, and are manufactured using a thicker wall to increase the wall stiffness. The density of the rubber compound is therefore lower than the rubber used in pressurized balls. A pressurized ball has a rubber wall thickness of about 3 mm , whereas unpressurized
balls have a rubber thickness of about $4-4.5 \mathrm{~mm}$. The rubber in a pressurized ball is a high density material with a high filling content, low polymer content and has a relatively low gas permeability in order to maintain the excess pressure for as long as possible. Both types of ball are covered with a yellow cloth material, about 3 mm thick, glued onto the rubber. The difference between pressurized and unpressurized balls can be demonstrated by drilling a small hole in the ball or by inserting a pin through the wall of a pressurized ball to release the compressed air. This has no observable effect on an unpressurized ball, but a large effect on a pressurized ball. A depressurized ball is noticeably softer and has a lower coefficient of restitution, immediately on release of the air.

The air inside a pressurized ball cannot exert any net force on the ball. The pressure has the effect of increasing the stiffness of the rubber, in the same way that a rubber band or a string or membrane is stiffer for transverse displacements when it is stretched and under tension. For an internal pressure of 80 kPa , and an internal diameter of 3 cm , it is easy to show that the wall tension is $1200 \mathrm{Nm}^{-1}$ and that the wall stiffness is increased by about $30 \%$ as a result of the internal pressure. In practice, unpressurized balls tend to be slightly stiffer than pressurized balls, and the impact duration is correspondingly shorter.

## Typical results

Measurements of the peak force, contact duration, ball compression and the coefficient of restitution, $e$, were made for a variety of pressurized and unpressurized tennis balls, as a function of ball speed, $v_{1}$. For this purpose, a ball launcher was constructed using two counter-spinning wheels of variable speed. The maximum ball speed was limited to $17 \mathrm{~ms}^{-1}$ with this apparatus. Within this range of ball speeds, all new balls tested were very similar in performance. Consequently, results for only two balls are presented in this paper. One set of results is given for a Dunlop Airloc ball, partly because this ball (like unpressurized balls) can be tested over a period of several months without any


Figure 7 The maximum force, $F_{\text {max }}$, and the impact duration, $\tau$, as a function of the incident ball speed, $v_{1}$, for the Dunlop Airloc ball.


Figure 8 The maximum displacement of the CM, $y_{\text {max }}$, and $e$ as a function of $v_{1}$ for the Dunlop Airloc ball.
observable change in properties. The ball had a mass of 58.1 g and a diameter of 67.0 mm . The other ball was a Wilson 'US Open' pressurized ball, of mass 56.5 g and diameter 65.0 mm , tested immediately after opening a can of new balls.

Results for the Dunlop ball are shown in Figs 7 and 8 , and results for the Wilson ball are shown in Figs 9 and 10. Figures 7 and 9 show the peak force, $F_{\max }$, and the contact duration, $\tau$, as a function of ball speed. Figures 8 and 10 show $e$ and the parameter $y_{\text {max }}$ as a function of ball speed. $y_{\text {max }}$ represents the maximum displacement of the CM, calculated from the measured force waveform and ball speed, as described above.


Figure 9 The maximum force, $F_{\text {max }}$, and the impact duration, $\tau$, as a function of $v_{1}$ for the Wilson ball.


Figure 10 The maximum displacement of the CM, $y_{\max }$, and $e$ as a function of $v_{1}$ for the Wilson ball.

The standard 100 inch drop test corresponds to an incident ball speed of $7.05 \mathrm{~ms}^{-1}$, and the range of $e$ at this speed, specified in the rules of tennis, is $0.728<e<0.762$. Both of the balls tested have very similar values of $e$, but the Wilson ball was slightly softer (both in terms of the qualitative feel of the ball and the measured $F / y$ ratio) with the result that, for any given ball speed, the peak force was smaller, $y_{\text {max }}$ was larger and $\tau$ was longer, each by about $10 \%$.

## Discussion

The actual change in ball diameter, or the dynamic compression was not measured in this experiment,
but this is likely to be about $70 \%$ larger than the displacement of the CM. Qualitatively, it was found that balls with a low static stiffness also have a low dynamic stiffness. However, even if the actual dynamic compression, $x$, could be measured, it would be difficult to make a quantitative comparison between static and dynamic measurements of ball stiffness or hysteresis. During the initial stages of the ball compression, the dynamic stiffness, $F / y$, is almost an order of magnitude larger than the static stiffness, $F / x$. The rules of tennis specify that for a static load of $18 \mathrm{lb}(8.165 \mathrm{~kg})$, the ball shall have a forward deformation, $x$, of more than 0.220 of an inch $(5.59 \mathrm{~mm})$ and less than 0.290 of an inch ( 7.37 mm ). Consequently, the static stiffness, $k$, must be in the range $10.9<k<14.3 \mathrm{kNm}^{-1}$ during the compression phase. If we take the Wilson ball in Fig. 2 as an example, then the dynamic stiffness $F / y=87.5 \mathrm{kNm}^{-1}$ at $t=0.2 \mathrm{~ms}$ and $F / y=$ $34.1 \mathrm{kNm}^{-1}$ at maximum compression (i.e. at $t=1.7 \mathrm{~ms})$. The latter figure is a measure of the average slope of the $F$ vs. $y$ hysteresis curve. It is therefore a reasonable measure of the time average dynamic stiffness and is also consistent with the observed duration of the impact, as indicated by Eq. (1) ( $\tau=4.1 \mathrm{~ms}$ ). Similarly, the impact durations shown in Figs 7 and 9 are also consistent with Eq. (1) if $k$ is interpreted as $F_{\text {max }} / y_{\text {max }}$.

Differences in the coefficient of restitution between different balls may translate to differences in rebound speed when a ball impacts on a racquet. Differences in ball stiffness or contact duration may translate to differences in the rebound angle off a racquet. For example, suppose that a racquet is swung towards an incoming ball with an angular velocity of $30 \mathrm{rad} \mathrm{s}^{-1}$ and the ball contacts the strings at normal incidence. If the contact duration is 4 ms , and the average angular velocity of the racquet during the collision is $25 \mathrm{rad} \mathrm{s}^{-1}$, then the ball will leave the racquet after the racquet has rotated through an angle of 0.1 radian or $5.7^{\circ}$. Neglecting the effects of ball spin, the ball will leave the racket at an angle approximately normal to the strings. If the contact duration was 5 ms , the racket would rotate by $7.2^{\circ}$ during this time. A difference in rebound angle of
$1.5^{\circ}$, extended over a travel distance of 20 m , will change the ball displacement by 0.5 m . Such an effect should be apparent to good players as a difference in the ball, rather than an error of judgement on their part. It is also likely that players could pick the difference between the two balls by the feel (i.e. the hardness or softness as indicated by squeezing the ball by hand) and by the sound of the balls.

One of the interesting differences between pressurized balls and unpressurized balls is that they sound different. This effect does not show up in the force waveforms, but is obvious when the sound of the impact is recorded by means of a microphone located near the impact point. Figure 11 shows the impact force waveform and a simultaneous measurement of the microphone output for a pressurized ball (Wilson) and an unpressurized ball (Tretorn Plus). The microphone was located 15 cm from the impact point. Both balls generate a sound waveform that corresponds roughly to the compression and expansion of the ball, and the waveform for both balls also contains a $1.2-\mathrm{kHz}$ frequency component (period 0.8 ms ). The high frequency component is more strongly damped in the pressurized ball, which therefore sounds duller. The denser rubber of the Wilson ball appears to damp the vibrations more rapidly. This was not studied in detail, but examination of force waveforms, such as those in Fig. 3, would provide a useful estimate of the dynamic hysteresis losses in the different rubber compounds.

## Conclusions

With the aid of a simple force plate, it is possible to obtain a large amount of information on the dynamic properties of tennis balls. The standard 100 inch bounce test, together with static compression tests, have provided sufficient information to date to regulate the game of tennis, and will continue to do so for some time into the future. Measurements of the type described in this paper are perhaps more of purely scientific interest, but also provide data that may help to shape future


Figure 11 Output of a microphone located 15 cm from a ball impacting on the force plate, and the corresponding force waveform, for an unpressurized (Tretorn) and a pressurized (Wilson) ball. In both cases, the ball was dropped onto the force plate from a height of 50 cm .
developments in the specification and testing of ball properties. For example, the standard static compression test dating from the 1930s is somewhat operator dependent and could possibly be replaced with a simpler, more relevant and more reliable impact duration test or a dynamic compression test as described above.

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