The bounce of a ball

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In this paper, the dynamics of a bouncing ball is described for several common ball types having different bounce characteristics. Results are presented for a tennis ball, a baseball, a golf ball, a superball, a steel ball bearing, a plasticene ball, and a silly putty ball. The plasticene ball was studied as an extreme case of a ball with a low coefficient of restitution (in fact zero, since the collision is totally inelastic) and the silly putty ball was studied because it has unusual elastic properties. The first three balls were studied because of their significance in the physics of sports. For each ball, a dynamic hysteresis curve is presented to show how energy is lost during and after the collision. The measurement technique is quite simple, it is suited for undergraduate laboratory experiments, and it may provide a useful method to test and approve balls for major sporting events.

I. INTRODUCTION

The dynamics of a collision between a ball and another object can be determined, in principle, from the initial conditions and the functional form of the force acting on the ball. If the collision is elastic, then the force, F, acting on a ball during the collision is given approximately by Hooke's law, $F=kx$, where x is the ball compression. The collision can then be modeled as one between two springs.[1] The spherical geometry introduces a complication that was first analyzed by Hertz[2] for a force law of the form $F=kx^{3/2}$. If the collision is inelastic, then the relevant force law is generally an unknown function of the properties of the colliding objects. The force law is, in fact, often irrelevant since most problems of this type are cast in the form of conservation equations describing conditions before and after the collision. However, a measurement of the force provides useful information on the behavior of the objects during the collision, on the duration of the collision and on the elastic properties of the objects.

The collision of a ball always involves some loss of energy. For example, if a ball of mass m is dropped from a height h_1 onto a surface and it rebounds to a height h_2, then the loss of energy is $mg(h_1-h_2)$. The energy loss can be expressed in terms of the coefficient of restitution, e, defined in the case of a rigid surface by $e=v_2/v_1=\sqrt{h_2/h_1}$, where v_1 is the incident speed of the ball and v_2 is the rebound speed. The coefficient of restitution (COR) has been measured for many objects and surfaces, but very little information is available on the energy loss process itself or on the force acting on a colliding ball. For example, the energy may be dissipated in the ball during the collision as a result of internal friction, or energy may be lost as a result of a permanent deformation of the ball or the surface. Alternatively, energy may be stored in the ball as a result of its compression and subsequently dissipated after the rebound either in internal modes of oscillation or by a slow recovery of the ball to its original shape. A review of head-on collisions between solid metal spheres was presented 40 years ago in this journal by Barnes.[2,3] Since that time, there have been many other articles on colliding balls,[4,5] but only one[6] included force wave forms.

The energy loss can be predicted approximately from measurements of the static hysteresis curve obtained by plotting the ball compression as a function of applied force for a complete compression and expansion cycle. However, such measurements do not allow for the fact that the dynamic properties of the ball may differ from its static properties. In this paper, dynamic hysteresis curves are presented for several different balls bouncing vertically off a piezo element mounted on a heavy brass rod. The curves were obtained by plotting the displacement of the center of mass, rather than the ball compression, since it is much easier to measure the velocity of a bouncing ball than to measure (or interpret) its dynamic compression.

II. DYNAMICS OF THE BOUNCE

A rigorous analysis of the bounce of a ball is complicated by several factors, one being that in practice, a relatively soft ball can easily squash to half its original diameter and also squash asymmetrically, in which case the relation between the compression of the ball and the displacement of its center of mass is not easily determined. Another complicating factor is that the ball compression versus the applied force relationship is not only nonlinear but may also vary with frequency, in which case a static force versus compression curve is not particularly relevant, and dynamic curves for a spherical object are not readily available, if at all. A simple experiment using a mass on the end of a rubber band is described by Papadakis[7] to illustrate the differences between the dynamic and static properties of rubber. Even a steel ball can be locally compressed beyond its elastic limit in a relatively low-speed collision.[2] Despite these complicating factors, the bounce of a ball can be analyzed at an elementary level using a combination of elementary mechanics and experimental data on the force wave forms.

A ball dropped vertically onto a surface experiences a vertical impulsive force $F=m\,dv/dt$, where $v=dy/dt$ is the velocity of its center of mass and y is the displacement of the center of mass. F is typically 100–1000 times larger than mg, in which case the gravitational force can be neglected during the impact. For a given or measured force wave form, the ball velocity and the y displacement can be obtained by numerical solution of the equation $d^2y/dt^2=F/m$ with initial conditions $y=0$ and $dy/dt=v_1$ at $t=0$. Regardless of the ball compression and shape of the ball, the work done in changing the kinetic energy of the ball is $\int F\,dy$ and the area enclosed by the F vs y hysteresis loop represents the net energy loss, $0.5m(v_1^2 - v_2^2)$. If the bounce surface is perfectly rigid, the total work done by the force F acting at the bottom of the ball is zero, since the point of application of the force remains at rest. Nevertheless, $\int F\,dy$ represents the change in kinetic energy, which is equal and opposite to the change in potential energy arising from compression of the ball plus any energy dissipated during the collision. The total energy, including the energy dissipated, therefore remains constant.

A simple analysis of the bounce is obtained if one assumes that the bounce surface is not deformed and remains at rest, and that the ball compression, x, is given by $F = -kx$, where k is the spring constant of the ball. If it is also assumed for simplicity that $y = x$ then $d^2y/dt^2 = -ky/m$, so $F = F_0 \sin(\omega t)$, where F_0 is the amplitude of F and $\omega^2 = k/m$. It can be deduced that the ball remains in contact with the surface for a time $\tau = \pi/\omega$, it rebounds with the same speed as the incident speed and the force wave form is a half-sine pulse of amplitude $F_0 = m\omega v_1$. For a tennis ball, $m = 0.057$ kg and $k \sim 2\times 10^4$ Nm^{-1}, giving a contact time $\tau \sim 5.3$ ms, consistent with observations.[8] For a steel ball of the same mass, k is much larger and the contact time is much shorter. The contact time for a small ball bearing colliding with a solid surface is typically only 20–50 μs.

In the case of a Hertzian impact,[2,9] where $F = kx^{3/2}$, or any other impact involving a force law of the form $F = kx^n$, there is also no energy loss so $v_2 = v_1$. In practice, it is found that v_2 is always less than v_1 and that the F vs t wave form is not perfectly sinusoidal or even symmetrical. A measured force wave form can be digitized for a numerical analysis or it can be fitted either by a polynomial or by the first few terms of a Fourier series to obtain analytical solutions. Bounce force wave forms are typically only slightly asymmetrical, so a reasonable first approximation is to consider just the fundamental and second harmonic components. This approximation yields some interesting analytical results, but it does not provide a good fit to experimental data. Consequently, the digitized force wave forms were used to analyze each ball separately, and the results are described below.

III. EXPERIMENTAL TECHNIQUES

The force acting on a ball dropped on a solid surface was measured using a 50 mm diam, 4 mm thick ceramic piezo disk bonded with superglue to one end of a 50 mm diam brass rod of length 100 mm, as shown in Fig. 1. A ball was dropped or thrown at low speed directly onto the piezo disk, and the voltage output was measured, on a digital storage oscilloscope, using a$\times$10 probe connected to light leads soldered to the upper silvered surface of the piezo and to the brass rod. The ball speeds v_1 and v_2, just before and after the impact, were measured by allowing the ball to fall through two horizontal He–Ne laser beams located above the upper surface of the piezo disk and separated vertically by 10 mm, as shown in Fig. 1. The beams were detected with a photodiode and the ball velocity was calculated from the time delays between the photodiode signals and the piezo signal. A small correction was made to the measured velocities to allow for the gravitational acceleration (or deceleration) of the ball after (or before) it crossed the two laser beams.

Using the measured F wave form, and the measured values of v_1 and v_2, it was then possible to calculate the y displacement as a function of time, and to calibrate the sensitivity of the piezo. The piezo was found to generate an output voltage of 1.0 V per 34 N. Other design features and some limitations of this technique are as follows.

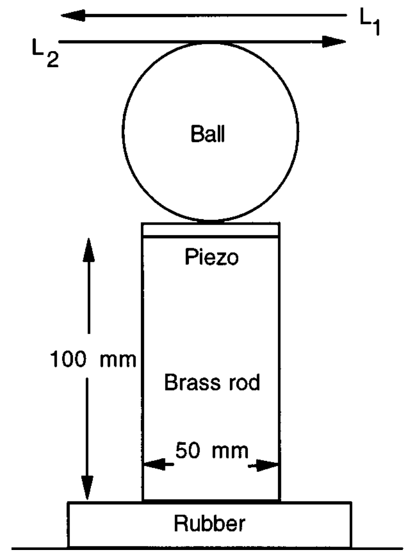

Fig. 1. The arrangement used to measure the ball speed and force wave forms. Here L_1 and L_2 are horizontal laser beams separated vertically by 10 mm.

(1) The capacitance of the piezo disk was 3 nF, but it was artificially increased to 5 nF by connecting a 2 nF capacitor in parallel with the disk in order to increase the RC time constant (of the disk and the 10 MΩ probe) to 50 ms. The force wave forms are reproduced reliably only if the discharge time constant is much longer than the duration of the impulse.

(2) The length of the brass rod was not sufficient to avoid reflections off the far end of the rod. The transit time of a pulse from the upper surface of the piezo to the lower end of the rod was 30 μs, resulting in a standing wave of period 60 μs or frequency 16.7 kHz. This mode was not excited with any significant amplitude by any of the balls tested since the ball contact time was longer than 120 μs in all cases. As a result, the frequency spectrum of the impulse did not extend significantly beyond 10 kHz. To avoid reflections off the table and floor, the rod was isolated from the table with a soft rubber support, as shown in Fig. 1. Simply holding the rod in one hand also provided excellent isolation, but the distance to the laser beams was then not known accurately. In principle, a much longer rod could have been used to delay the reflected pulse, but a rod of length at least 10 m would have been required to avoid the reflected pulse from a tennis ball. A rod of length about 1.5 m is ideal for studying the impact of small steel balls, and it also generates textbook examples of compressional (nondispersive) and transverse (strongly dispersive) wave modes that can be detected with a small piezo at one or both ends.

(3) A large diameter disk was chosen to avoid saturation of the force wave form that would occur if the contact area of the ball exceeded the area of the disk. Even so, measurements for a tennis ball were restricted to velocities less than $8\,\mathrm{ms}^{-1}$ since the contact diameter of the ball exceeded 50 mm at ball speeds greater than $8\,\mathrm{ms}^{-1}$. In the case of a high-speed tennis ball, or a large diameter ball such as a basketball, a piezo larger in diameter than 50 mm would be re-

Table I. Ball parameters.

Ball	Mass	Diameter	v_1	v_2	τ	Δy
Tennis	56.0 gm	64.0 mm	2.95 ms^{-1}	2.38 ms^{-1}	5.75 ms	0.48 mm
Superball	37.4 gm	43.1 mm	3.12 ms^{-1}	2.33 ms^{-1}	3.00 ms	0.17 mm
Golf	45.6 gm	41.5 mm	1.47 ms^{-1}	1.24 ms^{-1}	0.94 ms	0.03 mm
Baseball	143.6 gm	70.5 mm	1.25 ms^{-1}	0.61 ms^{-1}	2.20 ms	0.16 mm
Steel ball	66.6 gm	25.4 mm	0.77 ms^{-1}	0.65 ms^{-1}	0.13 ms	3.5 μm
Plasticene	48.7 gm	36.0 mm	1.47 ms^{-1}	0	3.8 ms	0.07 mm

quired. Such piezos are difficult to obtain, but it is relatively easy to connect any number of small piezos in parallel between two metal plates, with the same polarity, and bonded by a very thin layer of epoxy. Piezos extracted from inexpensive piezo buzzers would be suitable for this purpose. The lower plate should be quite thick (40 mm or more) to avoid transverse oscillations of the structure in the kHz range, and the upper plate should be relatively thin and light to minimize the force on the piezos induced by low-frequency vibrations transmitted from the soft rubber support to the upper plate. A suitable plate can be made from double-sided circuit board, using the upper side as a grounded shield. Such a system has been constructed by the author to measure high-speed tennis ball impacts, and the results will be presented elsewhere.

(4) The combined mass of the piezo and brass rod, 1.8 kg, was much larger than that of any of the balls tested, so the energy transferred to the rod was much smaller than the incident energy of the ball. The momentum transfer was not entirely negligible, with the result that part of the *y* displacement observed at the end of the impact could be attributed to motion of the brass rod during the impact. The velocity of the rod after the collision is given by $V=m(v_1+v_2)/M$, where *m* is the ball mass and $M=1.8$ kg is the rod mass. Since the average speed of the rod during the collision is approximately $V/2$, the displacement in time τ is approximately $\Delta y=m(v_1+v_2)\tau/(2M)$. This displacement is shown in the last column of Table I. For the baseball, tennis ball, and steel ball, motion of the rod accounted for about half of the final *y* displacement, and it also accounted for about 5% of the energy lost by these balls. The results presented below were not corrected for this effect, in part because of the unknown effect of the rubber support in restricting motion of the rod. The displacement of the rod is significant only toward the end of the impact, and the area of the hysteresis loop is increased by only a few % as a result. More precise measurements could be obtained either by using a heavier rod, or by suspending the rod horizontally to allow for free motion of the rod during the collision. In the latter case, an appropriate correction based on the measured force wave form could then be made for displacement of the rod.

IV. EXPERIMENTAL RESULTS

Results for the seven balls tested are given in Figs. 2–4, and further details are given in Table I. The force wave forms are all of a similar general form, being an approximate half-sine wave form, but asymmetrical in time. For most of the balls, the maximum force is recorded at a time close to 0.5τ, where τ is the duration of the impact, indicating that the experimental compression and expansion phases are of approximately equal duration. However, the impulse during the compression is larger than the impulse during the expansion, with the result that the ball rebounds at a speed less than the incident speed. The plasticene ball did not bounce and remained permanently deformed after the collision. All of the hysteresis curves have a finite area, indicating that all collisions were inelastic. The golf and superballs have an approximately linear compression phase, with $F\propto y$, and a nonlinear expansion phase.

The *y* displacement wave forms are more closely sinusoidal than the force wave forms, at least during the compression phase. In all cases it was found that the ball rebounds in a compressed state since *y* remains finite at the end of the impact. This was confirmed for the tennis and superballs by aligning the beam L_2, as shown in Fig. 1, so that it grazed the top of the ball when the ball was at rest on the

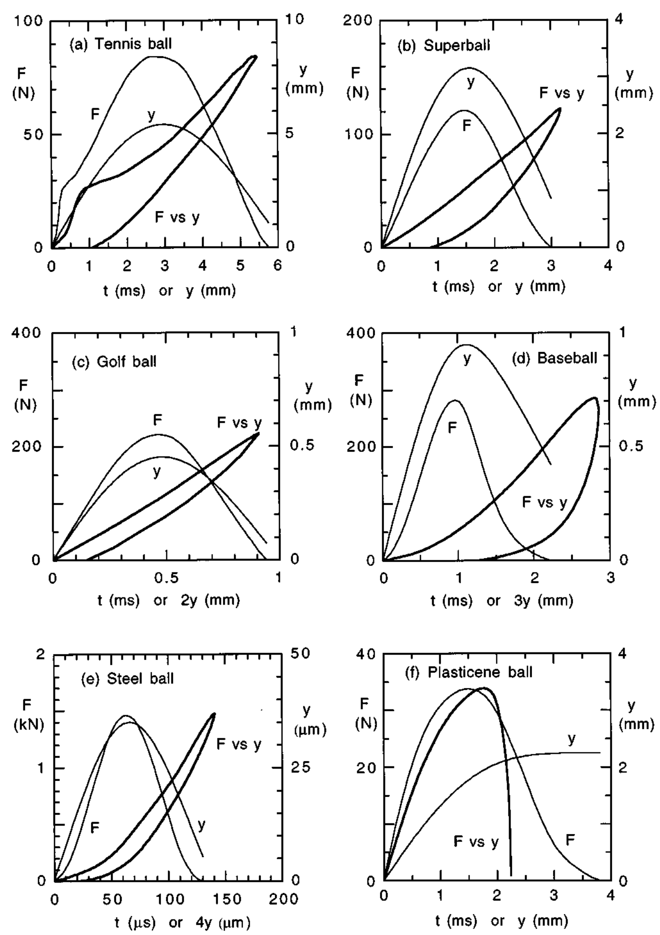

Fig. 2. *F* and *y* vs time for six different balls, together with the corresponding *F* vs *y* dynamic hysteresis curves, where *y* is the displacement of the center of mass. The ball speeds are listed in Table I.

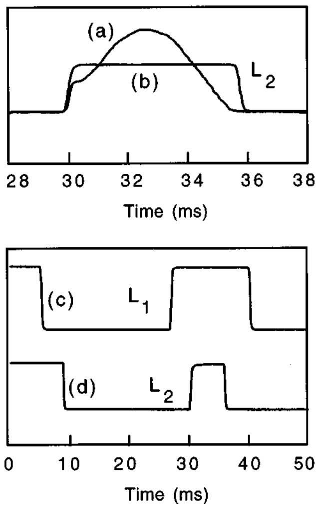

Fig. 3. The force wave form (a), and the laser beam signals (b), (c), (d) for a tennis ball when the lower beam L_2 just grazes the top of the ball, as shown in Fig. 1. L_2 is unblocked during the impact and for 0.4 ms after the impact. Trace (b) is an expanded version of trace (d).

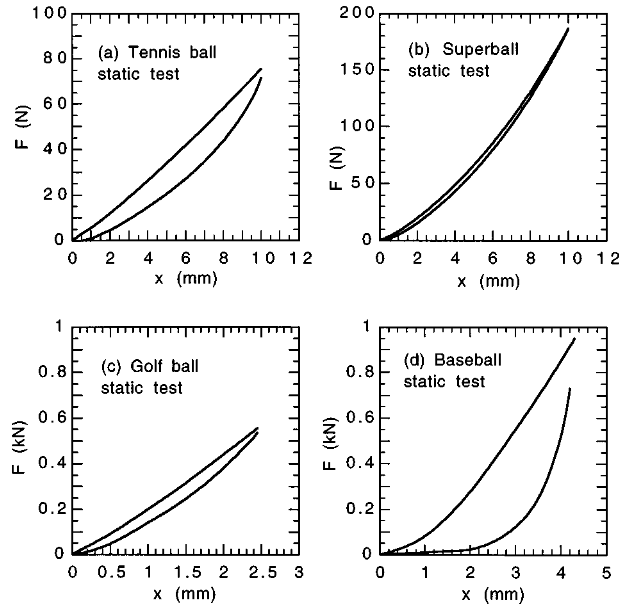

Fig. 5. Static hysteresis curves for four different balls showing the applied force F versus the compression, x, when the ball is compressed and then decompressed between parallel metal plates.

piezo. The results for the tennis ball are shown in Fig. 3, where it can be seen that the beam is blocked just prior to the impact, it is unblocked during the impact, and remains unblocked for about 0.5 ms after the impact. The spatial resolution was not sufficient to observe this effect with the other balls, since the ball compression was too small.

A totally unexpected result was obtained with the silly putty ball. Silly putty has the property that it stretches easily when stretched slowly, it breaks when stretched quickly and bounces when dropped on a surface. When the silly putty ball was dropped on the piezo, it was discovered, with some initial astonishment, that the piezo generated a negative output signal commencing about 30 ms before the ball made contact with the piezo, as shown in Fig. 4. No other ball had this effect, and the effect was observed only with a freshly prepared silly putty ball, created by stretching the putty and rolling it into a ball. The effect was traced to electrostatic charging of the ball to about 1 kV when it was stretched. The effect was simulated by charging a plastic rod and moving it toward or away from the piezo. The capacitance between the ball and the piezo was only a few pF, but this was sufficient to generate a voltage of about 0.2 V across the piezo. The ball was able to hold its charge for about 20 min, despite repeated handling and dropping of the ball.

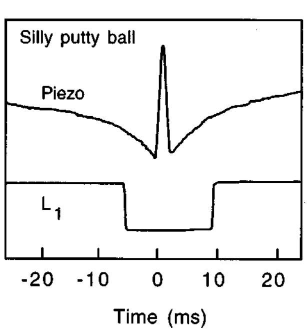

Fig. 4. The force wave form observed with a silly putty ball, and the L_1 laser beam signal observed when L_1 is located 11 mm above the piezo surface. The ball had a mass of 12.9 gm and a diameter of 29 mm. The ball blocks the beam 5 ms before it hits the piezo and unblocks the beam 7 ms after it leaves the surface.

V. COMPARISON OF STATIC AND DYNAMIC HYSTERESIS CURVES

Four of the balls were measured under static load conditions using commercial test equipment in the Faculty of Engineering at Sydney University. The results are shown in Fig. 5. Each ball was compressed between parallel steel plates at a uniform rate over a period of one minute, held at this compression for one minute and then allowed to expand at a uniform rate, over a period of one minute, back to its original shape. The break in the curve at maximum compression is due to relaxation of internal stress in the ball during the one minute pause between the compression and expansion cycles. The static and dynamic hysteresis curves cannot be compared directly since (a) the dynamic curve is plotted as a function of the y displacement of the center of mass, and the static curve is plotted as a function of the ball compression, x; and (b) both sides of the ball were compressed equally in the static test, whereas only the contact side of the ball is compressed in a dynamic bounce. If it is assumed that $y = x/2$ for a static compression and that $y = x$ for a dynamic compression then the dynamic and static curves yield similar values for the effective spring constant $k = F/y$ at maximum compression. Alternatively, the dynamic value of F/x at maximum compression is about twice that of the static value.

The area enclosed by a static hysteresis curve is less than that of the corresponding dynamic curve for the same compression. This is particularly evident for the superball, where the energy loss is almost negligible during a static compression and expansion. The effect is less pronounced for a baseball since the static and dynamic hysteresis losses are both relatively large. The superball tested did not bounce particularly well, a result that could possibly be attributed to microscopic cracks in the ball. Old superballs, with visible cracks in their surface, bounce even worse. The dynamic tennis ball results are unusual in that the ball is much stiffer during the initial impact than at later times, resulting in a pronounced kink in the force wave form and in the dynamic hysteresis curve. The kink was also observed with other tennis balls, old and new, pressurized and unpressurized.

Brody[8] has also measured the static hysteresis curve for a tennis ball, using a hemispherical cap to avoid static compression of the upper surface. His results are qualitatively similar to those shown in Fig. 5(a) and indicate that the ball tested by Brody was slightly stiffer and probably newer. The tennis ball used throughout this experiment was an old, relatively soft ball. The static hysteresis curve shown by Brody, as well as the static curve shown in Fig. 5(a), both enclose an area that is only about 50% of the actual energy loss when a tennis ball bounces off a rigid surface. The dynamic curves in Fig. 3 account for 100% of the energy loss since the y displacement has been calibrated from measurements of v_1 and v_2. Part of the discrepancy between the actual loss and the loss estimated from the static curves can be attributed to the increased initial stiffness of the ball during an actual bounce. Part of the discrepancy is also due to losses in the cloth cover. It is known that the cloth cover on a tennis ball contributes significantly to the energy loss, since a rubber ball without a cloth cover bounces better than one with a cloth cover. The effect of the cloth would not be apparent in a static compression test if the cloth recovers elastically from a compression during the test, but not during the short period of the impact. A similar relaxation effect is commonly observed with paper, since paper unfolds very slowly after bending or folding.

The increased stiffness of rubber for a high-speed compression can be modeled approximately by the relation $m\, d^2y/dt^2 = -ky - \gamma\, dy/dt$, where k is the effective spring constant and $\gamma\, dy/dt$ is a velocity-dependent force term related to the viscosity of the rubber.[7,10] Such a model results in a hysteresis curve of finite area since the model equation describes damped harmonic motion. The hysteresis curve in this case commences with $y=0$ and $F=-\gamma v_1$ at $t=0$. The model hysteresis curve bears a resemblance to the tennis ball data, for an appropriate choice of γ, but it does not give a good fit and is not relevant to any of the other balls. There is no evidence of any velocity-dependent force acting on any of the other balls, since $F=0$ at $t=0$ for all of the balls. Consequently, the energy loss in all cases appears to be due to a time-dependent relaxation of the internal stresses in the ball. Such an effect is referred to simply as an ''elastic aftereffect'' in the rheology literature.[10] The effect is complicated by the fact that a spectrum of different time constants is usually required to describe the relaxation. In the case of the steel ball, losses in the ceramic piezo and the brass rod may account for almost all of the energy loss.

VI. BALL VIBRATIONS

An estimate of the losses due to vibrations induced in the tennis ball was obtained by gluing a small (4mm×4mm) piezoelectric ceramic element, of thickness 0.3 mm, onto a tennis ball and measuring the induced voltage by means of light wires soldered onto the element. Results are shown in Fig. 6 for a case where the ball was dropped from a height of 10 cm onto the 50 mm diam piezo. When the small piezo element is located near the bottom of the ball, the force wave form observed is similar to that observed with the large piezo element, but there is a delay of about 0.4 ms between the two wave forms. The pulse decreases in amplitude and changes shape as the location of the element is rotated away from the bottom of the ball toward the top of the ball. The top of the ball is only slightly effected by the compression and expansion of the bottom of the ball, but there is a small-amplitude oscillation at ~700 Hz. The oscillations are global in extent and persist for about 2 ms after the ball rebounds. The 0.4 ms delay observed between the large and small piezo signals is roughly consistent with the fact that the initial impulse propagates around the ball to give a period of oscillation of ~1.5 ms. The delay also coincides with the transition from a high to a low stiffness state, indicating that the ball surface may deform into a bending mode when the impulse propagates to a point about 30° from the bottom of the ball. Since the ball is hollow, it bends more easily than a solid ball, and it is much easier to bend rubber than to compress it.

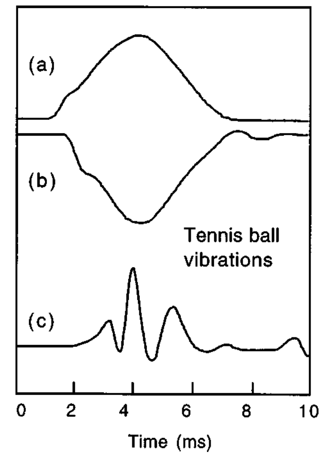

Fig. 6. Results for a tennis ball showing (a) the output of the 50 mm diam piezo, (b) the output of a small piezo (at 50 mV/div) mounted on the ball at a point 30° away from the contact point, and (c) the output of the small piezo (at 10 mV/div) mounted on the top of the ball, i.e., at a point 180° away from the contact point.

The amplitude of the oscillation shown in Fig. 6(c) is relatively small when measured in terms of the displacement of the ball surface. The induced voltage in the piezo is proportional to the displacement of the surface, but it is also proportional to the square of the frequency. Given that the stored energy in the ball is proportional to the compression squared and that the piezo output is proportional to the applied force and hence to the second derivative of its displacement, it is clear that the 700 Hz signal represents a relatively small-amplitude, low-energy oscillation. An absolute value for the energy stored in the oscillation was not obtained, since the piezo was not calibrated and since it responds to bending as well as to a force perpendicular to the surface. Even at high impact speeds, ball vibrations do not store a large amount of energy after the rebound. High-speed video film of a ball impacting with concrete at 100 mph has recently been obtained by the International Tennis Federation. The film was recorded at 18 000 frames/s and shows the ball oscillations clearly. Several frames from this video are shown schematically in Fig. 7. The video image is consistent with the results in Fig. 6 and shows that when the ball compresses to about half its original diameter, the surface opposite the contact surface oscillates with an amplitude of about 1 cm during the impact and at lower amplitude for several ms after the ball rebounds.

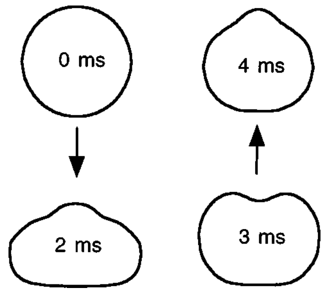

Fig. 7. Cross section of a tennis ball during a 100 mph collision with a concrete slab. The ball impacts at $t = 0$ and rebounds at $t \sim 4$ ms.

VII. EFFECTS OF BALL SPEED

It is well known that the coefficient of restitution decreases, the impulsive force increases, and the ball contact time decreases as the ball speed increases. The Hertz model[2,9] for colliding solid spheres indicates that $F_0 \propto (v_1)^{1.2}$ and $\tau \propto (v_1)^{-0.2}$, where F_0 is the force amplitude and τ is the duration of the impact. These relations were checked for the superball and the tennis ball, colliding with the 50 mm piezo disk/brass rod structure, for incident ball speeds in the range $1-8\ \mathrm{ms}^{-1}$. For the superball, it was found that $F_0 \propto v_1^n$ and $\tau \propto v_1^m$, where $n = 1.15 \pm 0.05$ and $m = -0.22 \pm 0.01$. For the tennis ball, it was found that $n = 1.10 \pm 0.05$ and $m = -0.07 \pm 0.01$. The superball therefore behaves in a manner that is close to Hertzian, but the tennis ball behaved more like a simple spring where $F_0 \propto v_1$ and τ is independent of v_1. The force law for a golf ball has been measured by Jones,[9] who found that a golf ball is close to Hertzian over a wide range of ball speeds up to $80\ \mathrm{ms}^{-1}$.

The static force law for a superball was checked by plotting the static compression curve in Fig. 5(b) on a log–log graph, as shown in Fig. 8, indicating that $F \propto x^{1.32}$. The fact that the dynamic compression phase of the golf and superballs is almost linear is therefore surprising. The dynamic ball compression, x, was not measured in this experiment. The dynamic results imply that y is approximately proportional to $x^{3/2}$ for the compression of a golf or superball. Such a result might be obtained, for example, if the ball compresses symmetrically for small x, so that $y \sim x/2$, and asymmetrically for large x, with $y \sim x$. Energy dissipation during the compression phase might also help to linearize the F vs y relation.

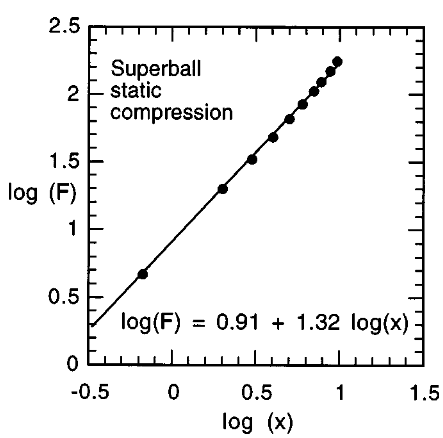

Fig. 8. The static compression curve for a superball [Fig. 5(b)] plotted on a log–log scale.

VIII. DISCUSSION

In this paper, dynamic hysteresis curves have been presented for a number of common ball types bouncing off a heavy brass rod. The results indicate that all balls studied (apart from the plasticene ball) rebound in a slightly compressed state, but the major energy loss occurs during the bounce rather than after the bounce. The study was limited to impacts at low ball speeds off a flat surface. The technique could easily be extended to study impacts at higher speeds or to study other balls. Such a study would be particularly useful in regard to the testing and approval of balls used in ball sports.[11]

The current rules regarding tennis balls are quite specific regarding static compression tests, although the specified equipment to be used is relatively ancient and somewhat operator dependent. There are no rules at all regarding the static compression of a golf ball or a baseball. In regard to dynamic tests, a tennis ball must have a COR of 0.745 $\pm 2.3\%$ when dropped from a height of 100 in. onto a concrete slab. There are no rules regarding the COR of a tennis ball in a high-speed collision. Surprisingly, there are no official rules at all concerning the COR of a baseball. The dynamic rule for a golf ball is that it must not travel faster than 250 ft (76.2 m) per second when hit by apparatus specified in the rules. Particularly in the case of tennis balls, where a wide range of pressurized and unpressurized balls are manufactured to meet current specifications, it is observed that different balls can behave quite differently under actual playing conditions. The techniques described in this paper would provide a useful method of distinguishing and understanding these differences.

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