RACQUET TECH DECEMBER 2001 p13,21

# HOW STIFF IS A STIFF RACQUET?

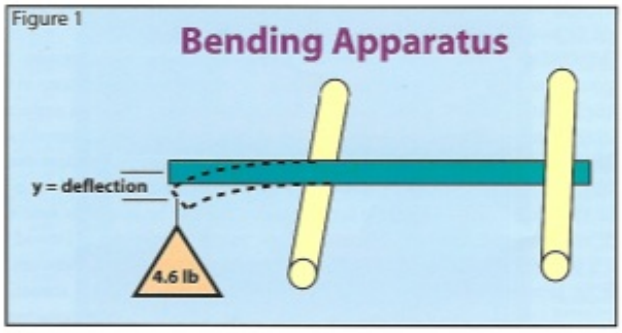
BY ROD CROSS AND CRAWFORD LINDSEY

Modern tennis racquets vibrate a lot less than old wood racquets because they are a lot stiffer. This adds to racquet power since less energy is wasted in vibrations of the frame. The April 2001 issue of Racquet Tech lists the flex values of 409 different racquets. The stiffest was 81 and the most flexible racquet was 50, as measured on a Babolat RDC (Racquet Diagnostic Center) machine. A racquet with a flex of 81 is hard to bend and a racquet with a flex of 50 is easier to bend. These numbers are useful when comparing racquets, but they don't mean much by themselves. An obvious question is whether a racquet with a flex reading of say 80 is twice as stiff as a racquet with a flex reading of 40, or is there some other relation between stiffness and flex? And does a flex reading of 80 mean that the racquet is stiffer than a steel rod?

To sort this out, measurements were made on some aluminum and brass bars of known stiffness. The bars were cut to 70 cm lengths and tested in (a) a Pacific racquet diagnostic machine, calibrated in DA units, (b) a Babolat RDC (Racquet Diagnostic Center) calibrated in RA units and (c) a home-made bending machine made from two rigid steel rods 25 cm apart. For the homemade machine, a weight of 2.09 kg (4.6 lb) was attached to the free end and the deflection of the free end was measured with a dial gauge. On the homemade machine, the distance from the weighted end to the nearest rod was 39.5 cm. In the Pacific machine, the two bending rods are 27.2 cm apart and a weight is attached at a point 31.8 cm from the nearest rod. In the Babolat machine the two rods are 11 inches (27.9 cm) apart and the weight is attached at a point 13 inches (33.0 cm) from the nearest rod.

Normally, a racquet is located so that one rod is on top of the handle and a weight is attached to the tip of the racquet.

Figure 1




If a horizontal bar is located between two rigid rods, as in Fig. 1, and a weight is placed on the free end then the bar will bend by an amount that is proportional to the weight. The deflection y (in mm) at the free end is given by

$$y = kW/S$$

where W is the weight (in lb or kg), S is the stiffness of the bar and k is a constant that depends on the length of the bar and the distance between the two rods. The value of k is not important if we just want to compare the relative stiffness of different bars or racquets. It is necessary only to keep the distance and length constant. The stiffness S depends on the type of bar (steel, aluminum, brass, graphite, etc.) and it depends on the width B, and thickness, A, of the bar. The material property is specified by a number called Young's modulus, E. The stiffness of a solid, rectangular cross-section bar is given by $S = E.B.A^3$. The stiffness of a bar will double if we double its width, but it increases by a factor of 8 if we double its thickness. Young's modulus for brass is 35% larger than aluminum, meaning that a brass bar is 35% stiffer than an aluminum bar of the same cross-sectional dimensions. Steel is three times stiffer than aluminum. The graphite/epoxy material used to make racquets has about the same Young's modulus as brass but it is four or five times lighter.

A square brass bar measuring 3/8 x 3/8 in. (9.53 x 9.53 mm) deflected by 11.01 mm on the homemade machine, about twice as far as a racquet with a flex of 60. A brass bar 3/8 x 1 inch deflected by 4.00 mm across the wide side and by only 0.68 mm across the narrow edge. The stiffest racquet tested was a Prince Viper which deflected by 2.20 mm and had a flex of 81 in both DA and RA units. The bar closest to this was a square aluminum bar 16.0 x 16.0 mm which deflected by 2.08 mm.

A graph of flex reading vs y deflection for the Babolat machine is shown in Fig. 2. Similar results were obtained

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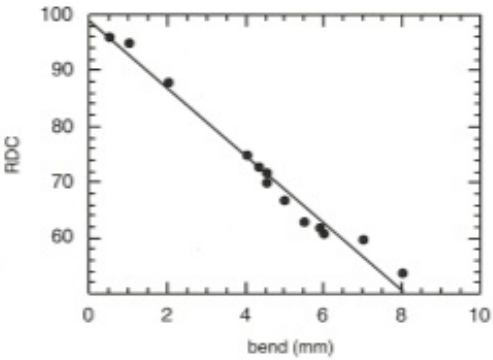
Figure 2



*How Stiff is a Stiff Racquet?*
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on the Pacific machine and on the homemade machine. Zero deflection (no weight applied) gives a reading of 100 which means infinite stiffness. A 1.5 mm deflection on the Babolat gives a reading of 90, a 3 mm deflection on the Babolat gives a reading of 80, and a 6 mm deflection gives a reading of 62. Consequently, a factor of two decrease in stiffness (twice the y deflection) does not correspond to a factor of two decrease in the flex reading.

The Viper stiffness was 81 on both the Babolat and Pacific machines. At the lower end of the stiffness scale, a Volkl Quantum Force racquet registered 62 on the Babolat and 67 on the Pacific machine. ■

# DWELL TIME

BY ROD CROSS

## The Dilemma

Most Saturday mornings I drop off my wife at the supermarket and wander over to the tennis shop to talk tennis with Ron and Steve, both USRSA members, both expert stringers, and both with strong views on all aspects of tennis. Our verbal rallies are just as much fun as an actual game of tennis. Recently, Steve tried to tell me that the ball sits on the strings for a longer time if the racquet is swung toward the ball. In most of my laboratory tests I let the ball impact on a racquet that is initially at rest, either with the head clamped or with the racquet suspended by two long lengths of ordinary string attached to a support above the racquet. That way, the ball doesn't go flying off through the lab window or smash into anything important.

Steve's argument was as follows: If the racquet is initially at rest and suspended by long strings, then a ball hitting the strings will push the racquet away in one direction while the ball bounces off in the opposite direction. That way, the ball loses contact with the strings sooner. But, if the racquet is swung toward a ball, then the racquet and the ball both move in the same direction after the collision. The ball stays on the strings a bit longer because the racquet is following the ball and pushes forward on the ball rather than pulling away from it. The separation between the ball and the strings must be rapid if the ball and strings move off in opposite directions, whereas the separation takes longer if the ball and the strings are both moving in the same direction. That sounds perfectly reasonable and logical, but it is wrong. There is a difference in the distance over which the ball and the strings remain in contact, but there is no difference in the time they remain in contact.

I told Steve that he was wrong because the collision can be observed in a moving frame of reference or in a stationary frame of reference and the collision will be exactly the same, so the dwell times will be the same.

Changing the frame of reference is something that physicists are comfortable with, but it didn't convince Steve. There are several other ways to look at the problem, but they take a bit longer to explain.

## What Determines the Contact Time?

The contact or dwell time of a ball on the strings depends on the stiffness of the ball and the strings, and upon the mass of the ball and the racquet. It does not depend on the speed of the racquet, and it doesn't depend on the speed of the ball. To see how this arises, we can consider a slightly simpler case where a ball is dropped vertically onto the court. If a tennis ball hits the court at a speed of 20 feet/s on the way down, it will bounce upwards at about 15 feet/s. In order to reverse direction, the ball has to come to a complete stop at some stage during the bounce. It comes to a stop because the ball pushes down on the court so the court pushes up on the ball. For every action there is an equal and opposite reaction. That might sound like mumbo jumbo, but it's true. If you are standing at the net and your partner serves a ball into the back of your head, then the ball bounces backward off your head. Something pushed the ball backwards and that something was your head. It's the same with the court.

The force on the ball slowing it down from 20 feet/s to zero depends on the ball stiffness. If you squeeze a ball in your hands with a force $F = 18$ lb, it will squash a distance $x = 1/4$ of an inch. The ball stiffness is $k = F/x = 18/0.25 = 72$ lb/in. When a ball is dropped onto the court it squashes. If it squashes by 1/4 of an inch, the force on the ball is 18 lb. If a stiffer ball, like a baseball, is squashed by 1/4 of an inch, the force on the ball would be around 100 lb.

Let's suppose the average force on the tennis ball is 18 lb. How long does it take to stop? That depends on the mass of the ball. If the ball weighed only 5.7 gm, it would take 0.2 milliseconds. If it weighed 570 gm it would take 20 ms. But it weighs 57 gm and it takes 2 ms. And it takes about 2 ms to accelerate up to 15 feet/s as it leaves the court. So the total dwell time is about 4 ms. The details of this calculation are given in our book *The Physics and Technology of Tennis*.

If you throw a ball onto the court at 40 feet/s, it will bounce up at about 30 feet/s, but it still spends about 4 ms on the court. If the ball hits the court at twice the original speed, it will squash twice as far, so the force on the ball is twice as big. If the force is twice as big and the incident speed is twice as big, then it takes the same 2 ms to come to a stop. Actually, there is a slight reduction in contact time as the ball speed increases, but that's because the ball gets stiffer when it is squashed farther.

## Contact Time on Strings

When a ball bounces off the strings of a hand-held racquet, the contact time is around 5 ms rather than 4 ms (for reasons described later). The ball and string system work the same way as any other mass on a spring. Get a long rubber band, cut it to make a long stretchy string, tape a mass on one end and tie the other end to a support. Pull the mass down 1/4 inch and let go. The mass will bob up and down about once or twice every second. Then pull the mass down 1/2 inch and repeat. This time the mass will move faster, but it bobs up and down at the same rate. It travels faster, but it travels farther (up and down 1/2 inch instead of 1/4 inch), so it takes the same time to complete

each up and down cycle. A tennis ball is not tied onto the strings of a racquet, so it bounces off. But, if you glued a ball onto the strings, it would take 5 ms to complete each half cycle, just like a mass tied onto a rubber band. The contact time is very short compared with the mass on a rubber band because the strings are a lot stiffer than a rubber band.

The contact time of a steel ball on a steel plate is about 0.00005 seconds because a steel ball and a steel plate are both very stiff. And the contact time is the same regardless of whether the ball hits the plate or the plate hits the ball.

## Contact Distance

To consider Steve's argument in more detail, let's go back to the case of a ball hitting the racquet strings. Suppose that the collision time is 5 ms (0.005 s), the racquet is initially at rest and the ball is traveling at 30 feet/s toward the right just before impact (Figure, Case 1). Suppose that the ball bounces off the racquet at 12 feet/s toward the left and the racquet heads off toward the right at 16 feet/s. These numbers are realistic and typical of how most racquets would behave. The average recoil speed of the racquet is 8 feet/s (starting at zero and finishing at 16 feet/s), so the racquet head travels a distance 8 x 0.005 = 0.04 feet = 0.48 inches during the collision. The ball also travels 0.48 inches to the right before it loses contact with the strings, but at the instant it leaves the strings it is traveling at 12 feet/s to the left.

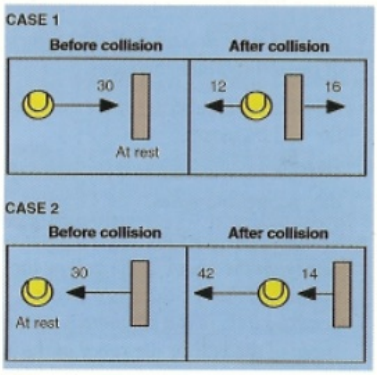

*Figure 1*

Now suppose the ball is at rest and the racquet approaches the ball at 30 feet/s traveling from right to left (Figure, Case 2). That is what happens in a low speed serve. In that case, the ball will end up traveling from right to left at 42 feet/s and the racquet will be traveling at 14 feet/s from right to left just after the ball loses contact with the strings. (I got those numbers by subtracting 30 feet/s from all the previous numbers.) That is what a bug would see if it were flying at 30 feet/s beside a ball traveling at 30 feet/s. The bug would think the ball is at rest and it would see the racquet approaching at 30 feet/s.

When a moving racquet strikes a stationary ball, the ball speeds up and the racquet slows down. The average speed of the ball during the collision is 21 feet/s (starting at zero and ending at 42 feet/s). Assuming that the collision still lasts 5 ms (which it does), the ball trav-

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els to the left a distance 21 x 0.005 = 0.105 feet = 1.26 inches before it loses contact with the strings. The racquet slows down from 30 feet/s to 14 feet/s, at an average speed of 22 feet/s, and travels a distance 22 x 0.005 = 0.11 feet = 1.32 inches before it loses contact with the ball.

If the ball travels 1.26 inches and the racquet travels 1.32 inches before they lose contact with each other, then something strange has happened. Surely they must travel the same distance before they lose contact.

The reason is that the average speed is not exactly as we have calculated it. It is true that the average of 30 and 14 is 44/2 = 22, but if the racquet spends most of its time traveling at around 30 feet/s and only a short time traveling at around 14 feet/s then the average speed will be greater than 22 feet/s. We only need a slight imbalance here to come up with the correct result that the ball and the strings travel the same distance before they part company. The imbalance arises because the ball is not perfectly elastic. There is a certain force on the ball while it is squashing and a slightly smaller force while it expands back to its original shape. That's why a ball dropped onto the court will not bounce back to its original height. The ball loses some energy during the bounce. The lack of symmetry in the force on the ball and the racquet (the forces are equal and opposite) means that the ball speed and the racquet speed are not quite symmetrical either. In other words, the average speed during a bounce is not exactly the same as adding the initial and final speeds and then dividing by two.

In our first calculation in Case 1, the ball and the strings both travel about 0.5 inches before they part company. I cheated a bit when I got 0.48 inches, since I ignored the lack of symmetry in the force, but if I do the calculation properly, the answer is about 0.50 inches. In both Case 1 and Case 2, the average speeds cannot be calculated exactly from simple averaging, but simple averaging is nearly correct and is close enough to show that Cases 1 and 2 are quite different in terms of the contact distance.

In Case 2, the ball and the racquet both travel about 1.3 inches before they part company. What we see here are two situations where the dwell times are the same, but the distances over which the racquet and ball remain in contact are different. If the racquet and ball fly off in opposite directions, they remain in contact over a relatively short distance. If the ball and the strings move in the same direction after the collision, then they remain in contact over a larger distance.

## Effect of Racquet Mass on Contact Time

There is one situation where Steve's argument can be used to get the correct answer, even though the argument is not correct when comparing Cases 1 and 2. The contact time of a ball on the court is about 4 ms. The contact time on the strings of a hand-held racquet is about 5 ms. If I clamp the head of a racquet to something much heavier, such as a heavy table, then the contact time on the strings is about 7 ms. In Steve's terms, the contact time is lengthened because the racquet head is not pushed away from the ball so the ball sits on the strings for a longer time.

The general rules about contact time are:

- *Contact time decreases if the stiffness of the ball increases.*
- *Contact time decreases if the stiffness of the string plane increases.*
- *Contact time increases if the mass of the ball increases.*
- *Contact time increases if the mass of the racquet increases.*
- *Contact time does not depend significantly on the speed of the ball or the racquet, although there is a slight decrease if there is a big increase in the speed of the ball or the speed of the racquet.*
- *Contact time for a bounce on a hand-held racquet is about 4.5 ms near the racquet tip, about 5 ms in the middle of the strings and about 5.5 ms near the throat area.*

The last rule is another case where Steve's argument is OK. If the ball strikes near the tip of a stationary racquet, the tip recoils rapidly and the contact time is short. If the ball strikes near the throat, the racquet recoils more slowly and the contact time is longer.

If a ball strikes a heavy concrete slab, the contact time is only about 4 ms because the slab is very stiff. If the ball strikes the strings of a racquet clamped to the slab, the contact time increases to about 7 ms because the strings are much softer than the slab. If the racquet is lifted off the slab and held by hand, the contact time is about 5 ms because the ball pushes the racquet away and quickly loses contact with the strings. But, if the ball is at rest and is struck by a racquet, the contact time is still 5 ms because there is no difference in mass or stiffness of either the ball or the racquet.

## Contact Distance When Serving

If you serve a ball at 100 mph, the ball remains in contact with the strings for about 5 ms and is pushed forward a distance of about 9 inches before it leaves the strings. At the instant the strings first contacted the ball, the racquet was traveling at about 71 mph at the point of contact (and around 100 mph at the tip). At the instant that the ball leaves the strings, the ball is traveling at 100 mph and the racquet head is traveling at about 36 mph at the point of contact. They are not traveling at the same speed when they lose contact because the strings stretch and eject the ball out of the frame at a higher speed than the frame itself. That's what the strings are there for. In addition, the ball gets squashed and it pushes itself off the strings as it returns to its normal shape.

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# FRAME VIBRATIONS

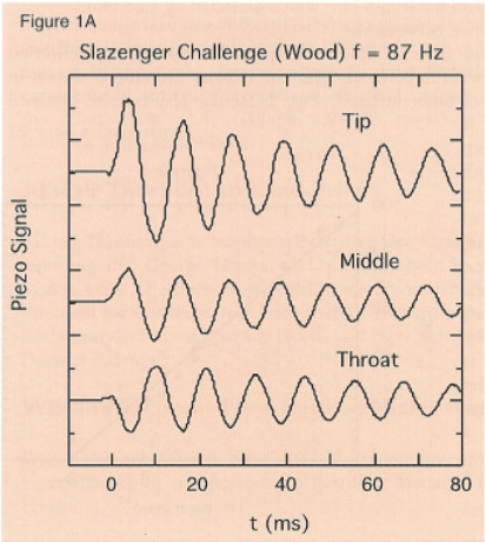
BY ROD CROSS AND CRAWFORD LINDSEY

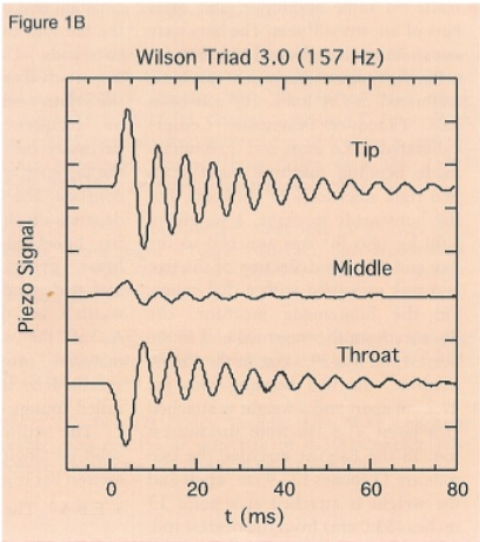
In September this year, we collected a bunch of racquets to measure and compare the frame vibrations. There was no intention of finding the best or worst racquet, since that is next to impossible. Some players like light racquets and others like heavy racquets. Some like stiff strings or frames, others like loose strings or flexible frames. And some like to feel a racquet vibrate and others don't. Still, it would be useful to know if the vibration level in a given racquet is high, medium, or small, in the same way that it is useful to know if a racquet is light or heavy, midsize, or oversize etc.

It is easy to compare the frame vibrations for impacts at the tip, middle, and throat of any given racquet, but it is a bit harder to compare the vibrations of one racquet with another. In order to measure the vibrations of the handle, it is necessary to attach a vibration sensor to the handle. The vibrations that are sensed depend on how well the sensor is attached to the handle, and that depends on the shape and flatness of the grip. To get around this problem, we attached the sensor to the flat side of a block of wood shaped to fit a handle and then bolted the wood block firmly to the butt end of the handle. A thin rubber pad between the grip and the block was used to help make good contact and to avoid damaging the grip. The sensor itself was a thin, flat ceramic piezo disk that generates a voltage when the handle moves or vibrates. Such a disk acts as a simple accelerometer and can be found in piezo buzzers or musical greeting cards. The voltage signal was recorded with a data acquisition system fed to a computer.

Typical voltage signals from the piezo are shown in Fig. 1, for impacts at the tip, middle, and throat of (a) an old Slazenger wood racquet, (b) a Wilson Triad 3.0 racquet, and (c) a Prince Viper racquet. Each of the racquets was suspended freely from a horizontal beam by two lengths of string so that it was free to rotate and vibrate. A tennis ball was also suspended from the beam, as a pendulum, so that it could swing towards the stringbed and impact at the same speed on every racquet and at the exact spot we wanted.

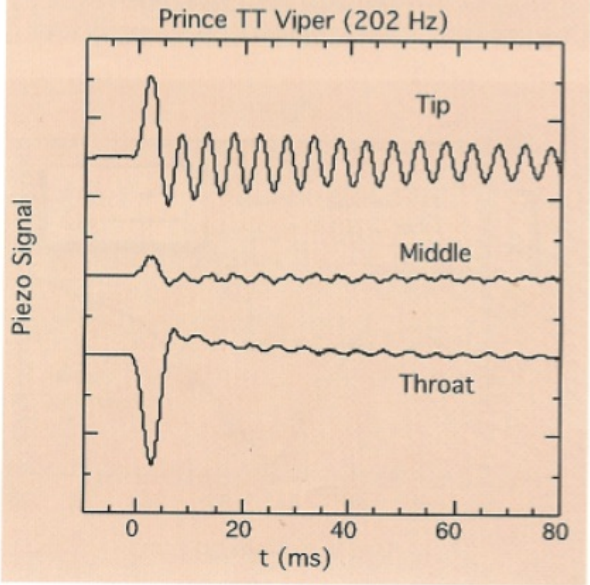
The results in Fig. 1 are atypical. The vibrations in the handle are always largest for an impact near the tip of the racquet, and smallest for an impact at the vibration sweet spot about 15 cm from the tip. In a modern racquet, the sweet spot is near the middle of the strings. In an old wood racquet, 15 cm is about half way between the middle of the head and the throat. Consequently, handle vibrations remain relatively large for an impact in the middle of the strings of an old wood racquet (Fig. 1). The initial motion of the handle is either towards the incoming ball for an impact near the tip, or away from the incoming ball for an impact near the throat. This can be seen from the piezo traces where the first part of the signal is positive for an impact near the tip and negative for an impact near the throat. There is another sweetspot near the first, called the center of percussion (COP). If a ball impacts at the



COP then the racquet rotates about an axis through the butt end of the handle. In that case, the butt end does not move towards or away from the incoming ball. The first sweetspot results in minimum vibration of the handle. The second results in minimum sudden motion of the handle and hence minimum shock.

Figure 1C




The three racquets shown in Fig. 1 were chosen because they each illustrate some interesting differences between racquets. Old wood racquets vibrate a lot more than modern graphite racquets and they vibrate at a lower frequency. Both of these effects are due to the fact that old wood racquets are heavier and more flexible than graphite racquets. The Triad is interesting because the vibrations are strongly damped by a rubber Iso-Zorb strip glued between the head and the handle. Additional strong damping occurs in all racquets when the handle is held by hand. Damping doesn't mean that the first vibration cycle is significantly reduced in amplitude. Damping refers to the fact that the subsequent vibrations get smaller quicker, and it may also act to reduce the amplitude of the first vibration to some extent.

The Viper had the smallest vibrations of all the racquets tested. It was also one of the stiffest and it vibrated at the highest frequency (202 Hz with the wood block attached or 227 Hz without the wood block). Frame vibrations are almost completely absent in this racquet for an impact near the throat. The single large spike in the piezo waveform is due to the sudden acceleration of the handle when the ball strikes the strings. That cannot be avoided and it therefore occurs in all racquets. It results in shock but no vibration in the Viper (or in any other racquet that vibrates at 200 Hz or above). With other racquets, the effects of rotation and vibration merge together right from the beginnining of the impact, increasing the overall acceleration of the handle and the shock felt by the hand.

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**Comparison of 19 racquets**

If one ignores the first half-cycle in the piezo waveforms, then the rest of the signal is due to vibration of the handle. The piezo generates a signal that is proportional to the handle acceleration. The actual vibration distance or amplitude is given by the acceleration divided by the vibration frequency squared. This was measured for each racquet at several positions from the tip to the throat, and the results are shown in Figs. 1D to 8, using the same scale on each graph so that the vibrations of all racquets can be compared directly. The scale itself was not calibrated, but the same settings and ball speeds were used to measure every racquet at every impact point. The vibration amplitude was measured at the beginning of the vibration, immediately after the first half-cycle. The amplitude is zero at the first sweetspot near the middle of the strings, but this was not recorded on every racquet since we just measured impacts at fixed distances from the tip of each racquet. The curves in Figs. 2–10 are smoothed-out curves fitted to those points by the computer. ■

Figure 7

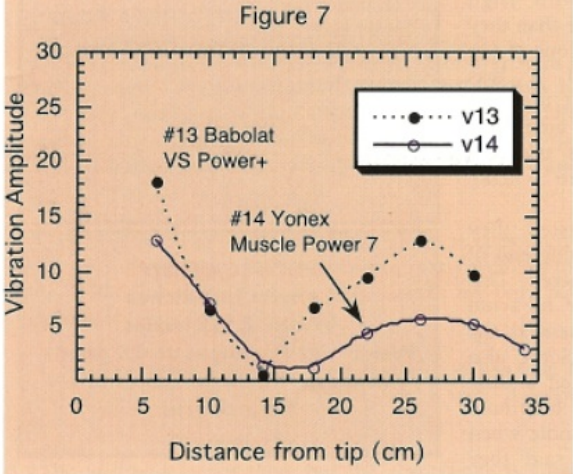
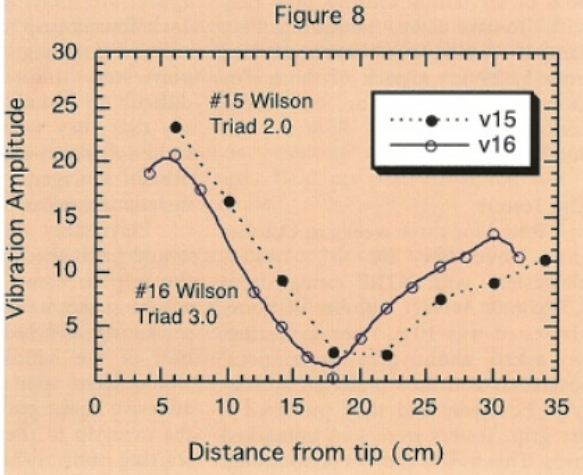



Figure 8



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# Vibration Amplitudes

Figure 1D

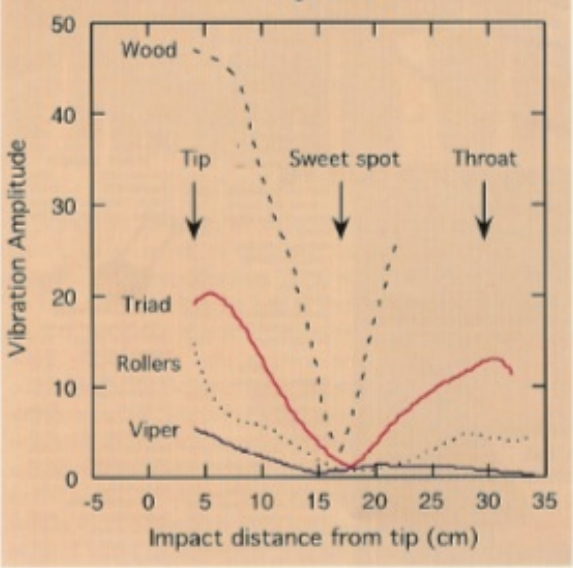


Figure 2

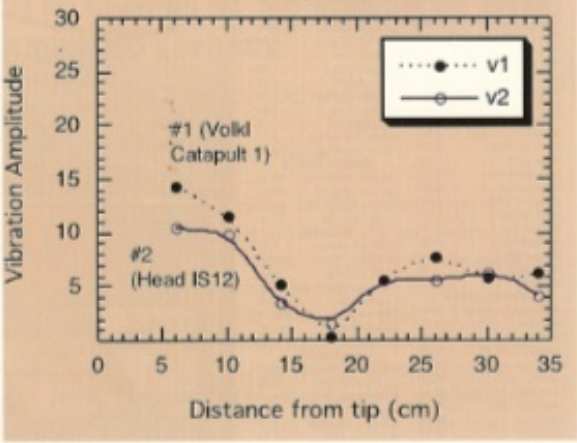


Figure 3

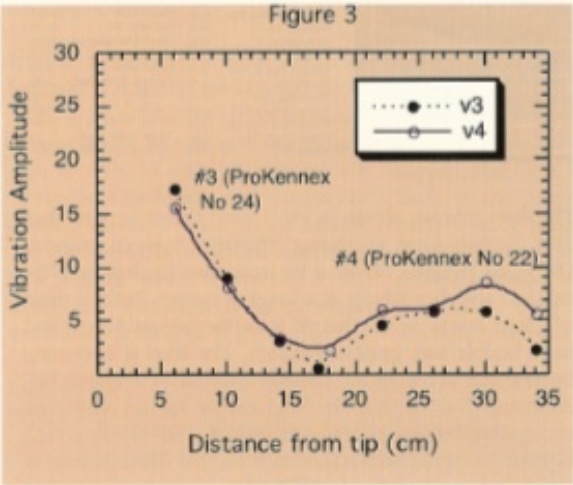


Figure 5

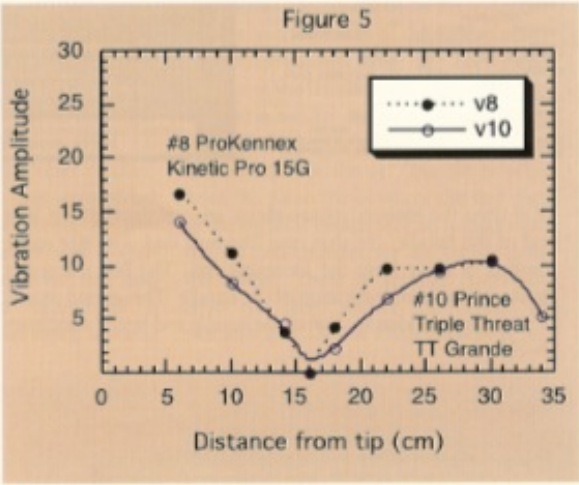


Figure 4

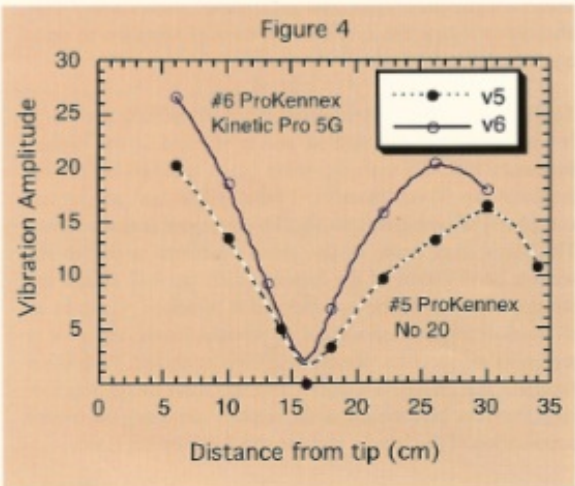


Figure 6