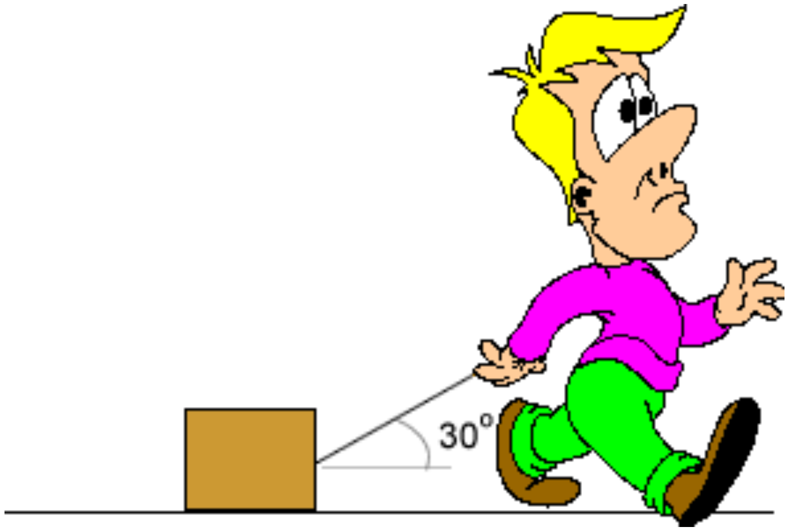
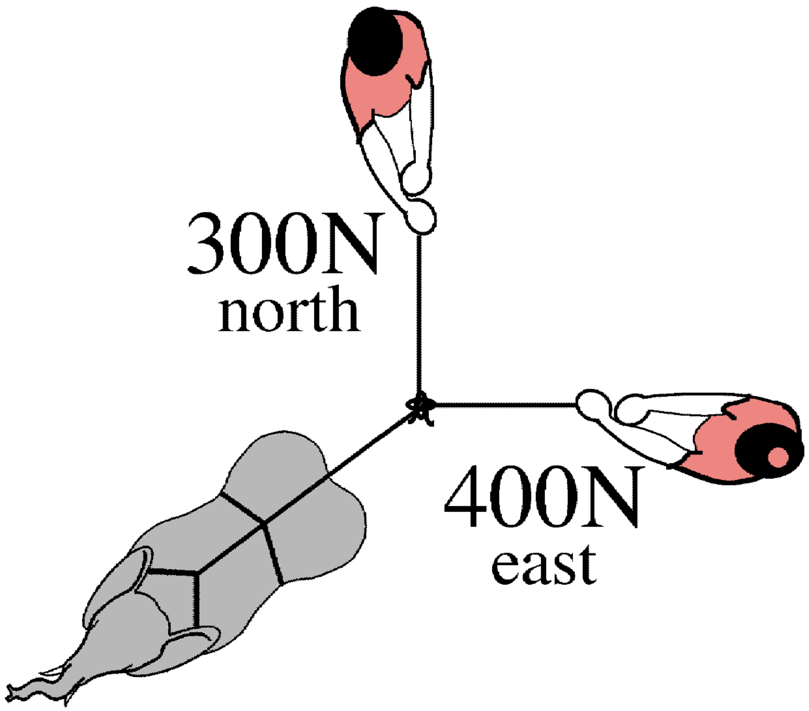
Lecture 7

Centre of mass
and
Torque

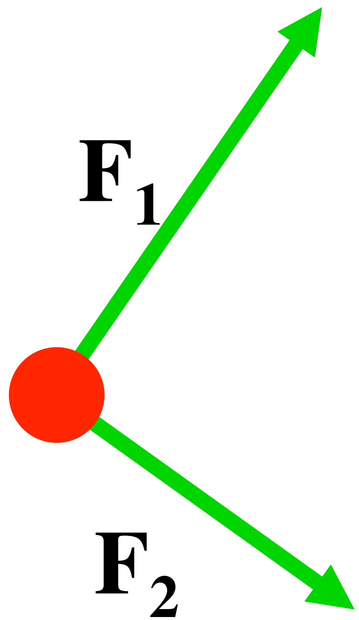
Pre-reading: KJF §7.2 and 7.3


30°


300N
north
400N
east

What is the dot?

- In a free-body diagram, we treat an object as though it is a single point.
- Can we choose any point within the object to represent the whole body?



KJF §7.3

CENTRE OF MASS

KJF §7.3

Centre of Mass (CM)

The centre of mass (CM) of an object or system of particles is the point that moves in agreement with Newton's 3 laws of motion as though all of the mass were concentrated there

At equilibrium, a body free to rotate will hang so its CM is vertically under the suspension point

KJF §7.3

CM for Many Particles

- Suppose there are n particles. If the coordinates of the i^{th} particle are (x_i, y_i, z_i), its mass is m_i and total mass M is

  $M = m_1 + m_2 + \ldots + m_n$

  then the x-coordinate of the CM is;

$$x_{cm} = \frac{1}{M}(m_1 x_1 + m_2 x_2 + \ldots + m_n x_n)$$

Or in shorthand;

$$x_{cm} = \frac{1}{M}\sum_i^n m_i x_i$$

Also

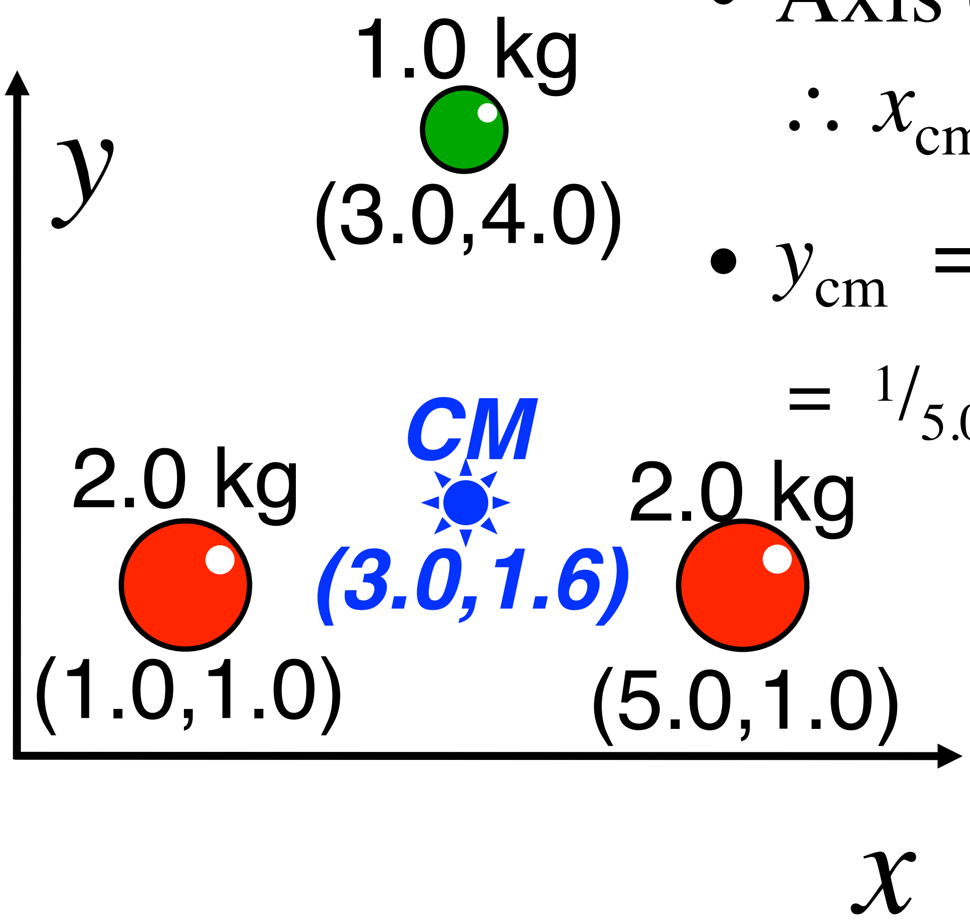
$$y_{cm} = \frac{1}{M}\sum_i^n m_i y_i$$

&

$$z_{cm} = \frac{1}{M}\sum_i^n m_i z_i$$

CM Example

- Find CM of these 3 particles; *2D only – don't need z*
  All distances in m



- Axis of symmetry @ $x = 3.0$
  $\therefore\ x_{cm} = 3.0$ m
- $y_{cm} = {}^{1}/_{M} \sum m_i y_i$
  $= {}^{1}/_{5.0}\,(2\times1 + 2\times1 + 1\times4) = {}^{8}/_{5} = 1.6$ m

$\therefore$ CM is at x=3.0, y=1.6

CM for Solid Bodies

- As before, if an object has any axes of symmetry then the centre of mass must lie on these axes of symmetry.
- Imagine the solid object is divided into a large number n of tiny pieces with masses $m_1, m_2, \ldots m_n$ (where total mass $M = m_1 + m_2 + \ldots + m_n$). Then the x-coordinate of CM is;

$$x_{cm} = \frac{1}{M}\sum_{i}^{n} m_i \, x_i$$

- *Psst! Strictly speaking you should take the limit as the $n \rightarrow \infty$ so the sum becomes an integral*

$$x_{cm} = \frac{1}{M}\int x \, dm$$

Newton's Laws & CM

All three laws apply to CM of a system.

For a distributed system (i.e. either many particles or a complex body),

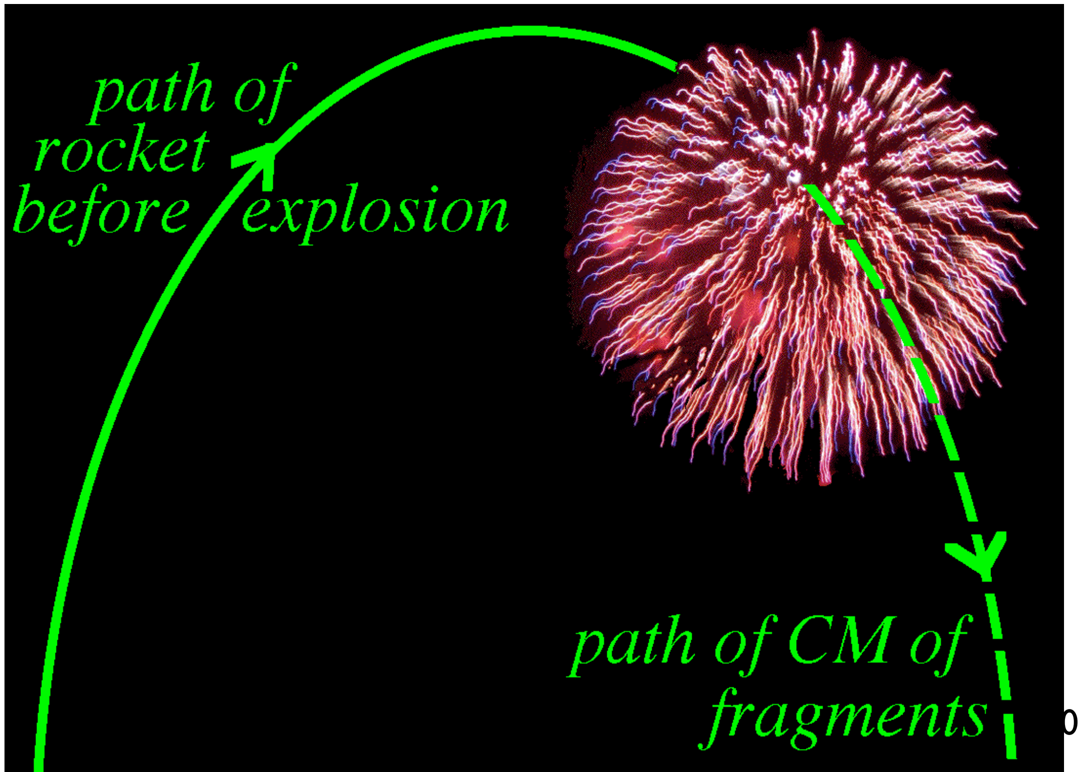
$$\Sigma \underline{F}_{\text{ext}} = M\, \underline{a}_{\text{cm}}$$

where M is the mass of the system, $\underline{a}_{\text{cm}}$ is the acceleration of the CM and $\Sigma \underline{F}_{\text{ext}}$ is the net external force

$\Sigma \underline{F}_{\text{ext}}$ means the sum of forces **external** to the system (not counting internal forces between particles or within a body)

CM and motion

If an object breaks up due to internal forces, the CM will continue to follow the same trajectory (if subject to the same external forces)



Imagine a fireworks rocket on the moon (so we can ignore air resistance)

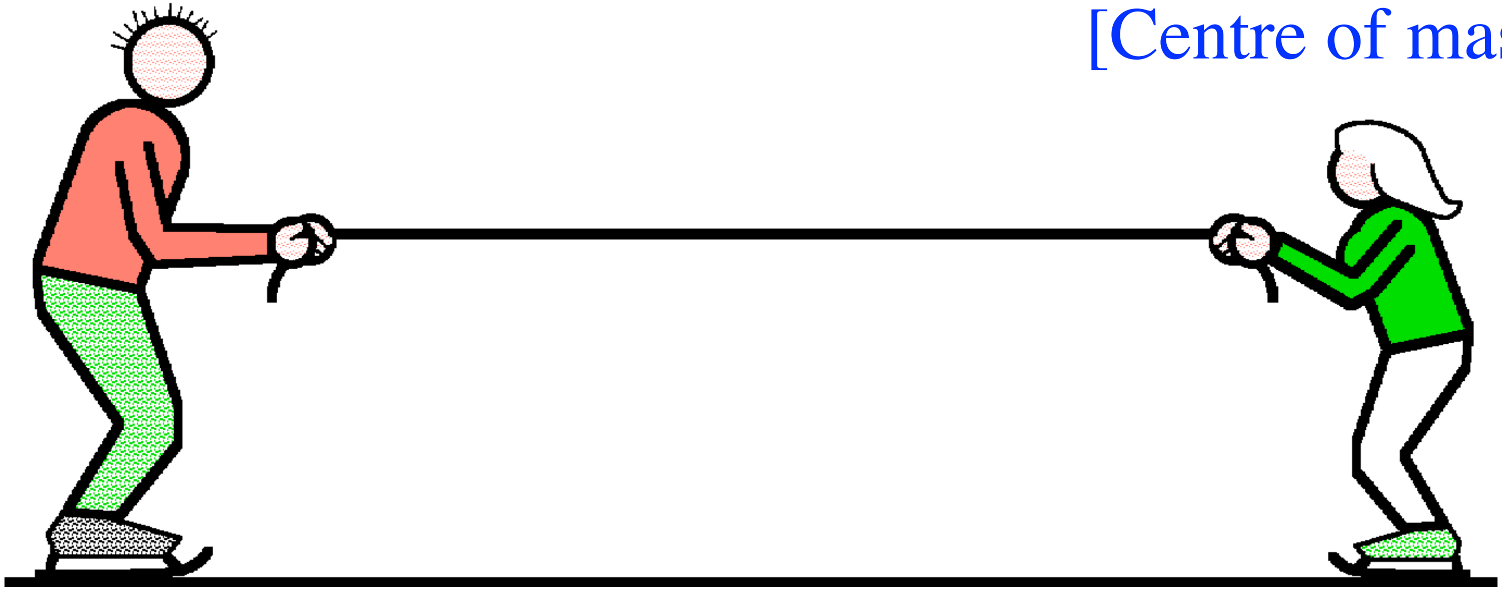
The CM of all the exploded fragments will complete the parabola since they are all still subject to the external force of gravity

Problem

Two ice skaters, Renfrew (65 kg) and Prunella (40 kg), stand initially at rest on an ice rink holding a rope of length 10m and negligible mass. Starting from the ends of the rope, they pull themselves along the rope (which remains under tension) until they meet.

Where will they meet? How far will Prunella move?
(Assume no friction)

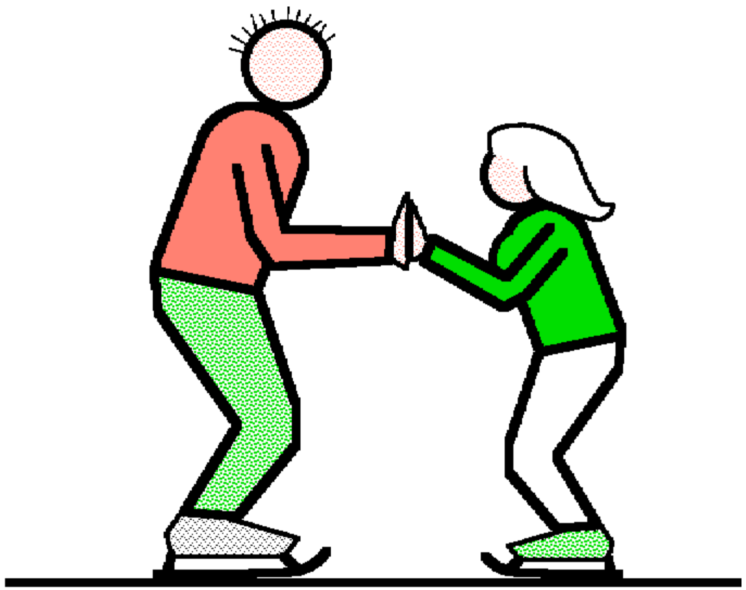
[Centre of mass; 6.2m]



Problem

- Renfrew (65 kg) and Prunella (40 kg), now stand on the ice rink initially at rest with the palms of their hands in contact. Prunella is offended by Renfrew's bad breath, so she pushes him away, so that they fly apart.

- How far will their common centre of mass move after Prunella has moved 6.2 m? *(Assume* *friction)*



CM in Sport

- CM does not have to be within the object
- By changing shape (e.g. high jumper) position of centre of mass can be changed
- In the "Fosbury flop" the high jumper bends his/her body so the CM passes under the bar
- Since the height of CM is lower than the bar, it requires less force to jump up to the bar

KJF §7.3

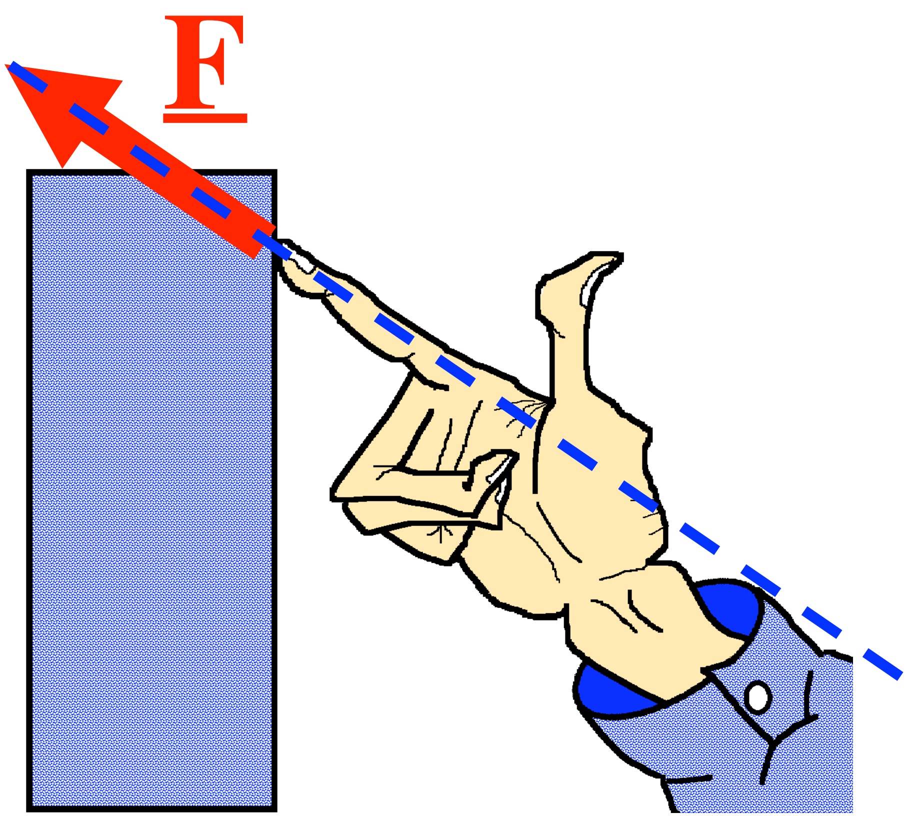
TORQUE

KJF §7.2

Line of action

Consider a force F exerted on an object at a particular point.

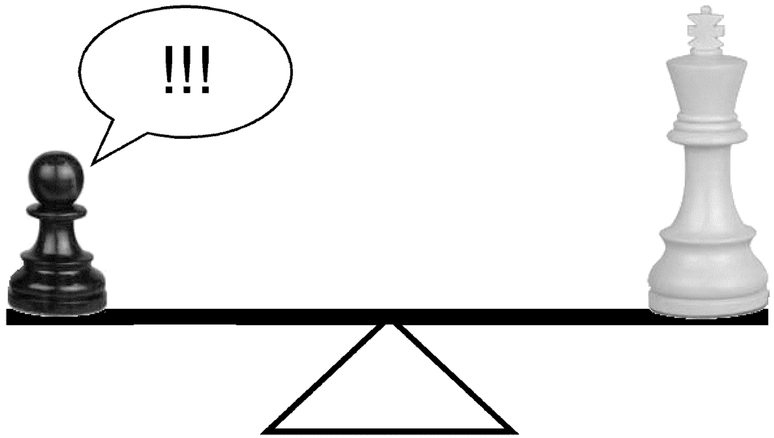
- Draw the force as an arrow vector originating on that point
- Extend the arrow in both directions off to infinity to form a line
- This is the "line of action" of the force F



KJF §7.2

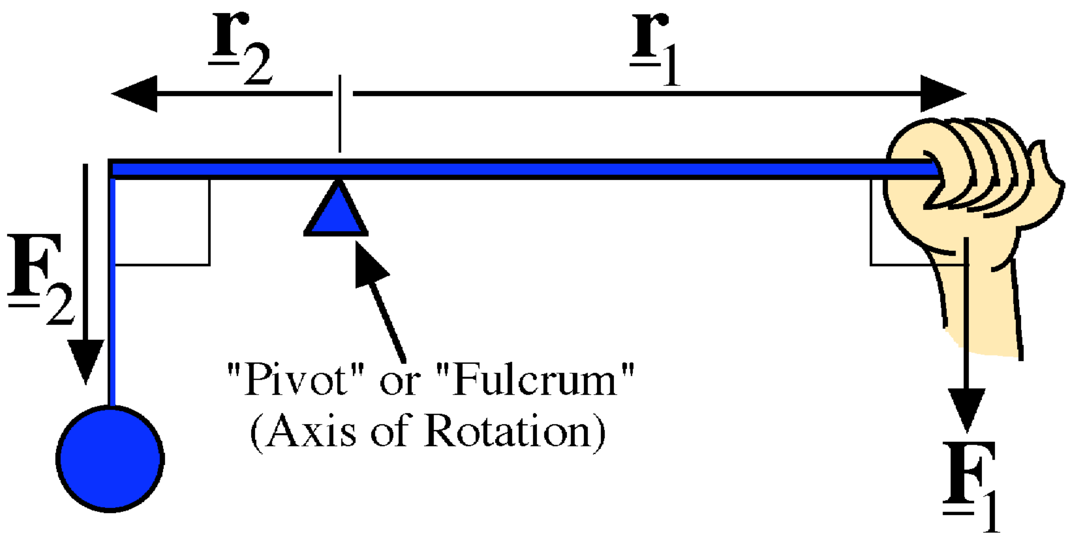
Translation vs Rotation

- If forces act on an object, translation might result
- If the lines of action of all forces on an object don't pass through a single point then rotation might also result.



- Where do the forces act on the seesaw?

Archimedes' Lever Rule

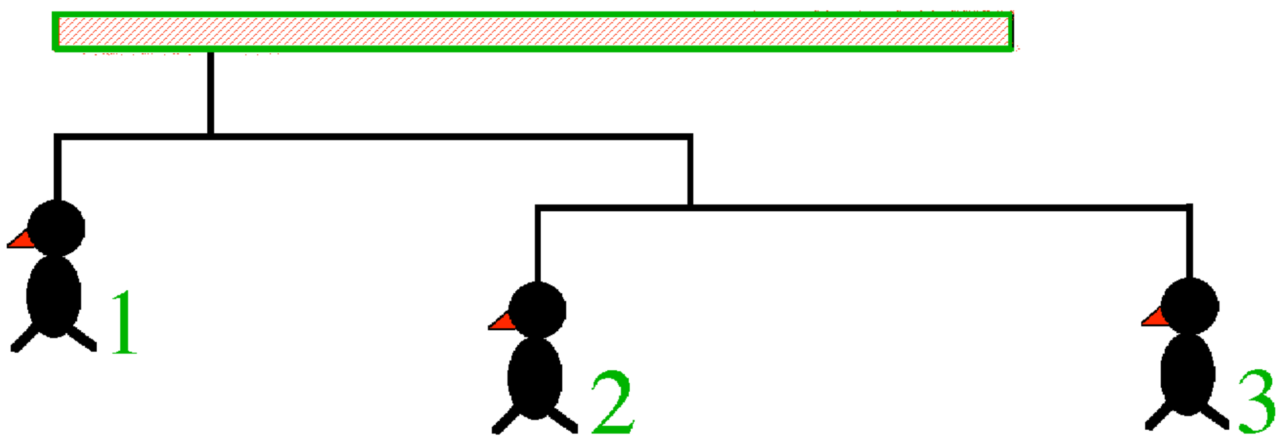


At equilibrium (and with forces 90° to lever):

$$r_1 F_1 = r_2 F_2$$

“Give me a place to stand and with a lever I will move the whole world”

Past Exam Question



A toy "mobile" of penguins hangs motionless. Each cross-bar is horizontal, of negligible mass & extended 3 times as far to the right of the supporting wire as to the left. Mass of penguin 1 is 9.6g.

What are the masses of penguins 2 & 3?

NEXT LECTURE

Torque and Equilibrium

Read: KJF §8.1, 8.2