Lecture 10

Work, Power
and
Potential energy

Pre-reading: KJF §10.1 and 10.2

Compared to the amount of energy required to accelerate a car from rest to 10 kmh^{-1}, the amount of energy required to accelerate the same car from 10 kmh^{-1} to 20 kmh^{-1} is

1. the same
2. twice as much
3. three times as much
4. four times as much

What is Energy?

Energy is needed to do useful work.

Energy can move things, heat things up, cool them down, join things, break things, cut things, make noise, make light, and power our electronics, etc.

Energy can be changed from one form to another.

kinetic energy

gravitational potential energy

elastic potential energy

thermal energy

Energy

kinetic energy – energy of motion

$$K = \tfrac{1}{2} mv^2$$

potential energy – stored energy

$$U = mgh \qquad \text{(gravity)}$$

Kinetic energy and potential energy added together are called *Mechanical Energy*.

Law of Conservation of Energy

Energy cannot be created or destroyed
(i.e. it is "conserved")

It can only be changed from one form to another

OR

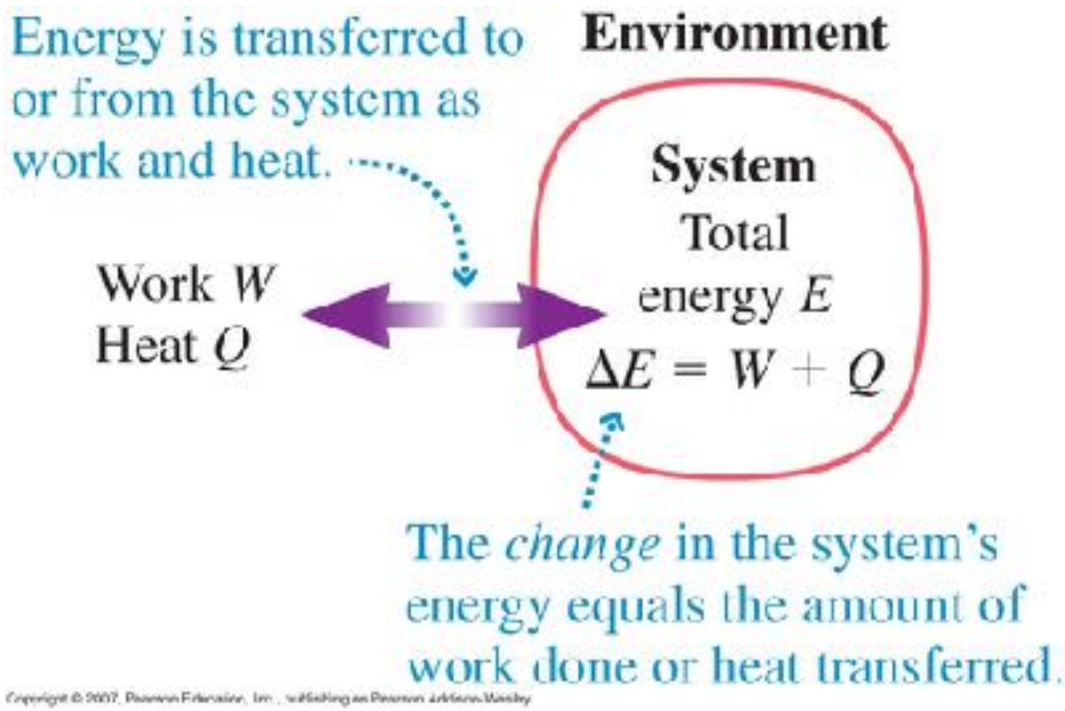
In an isolated system — one where there is no energy transfer into or out of the system — the total energy E_{tot} is conserved.

WORK

KJF §10.4

What is Work?

Work is the process of transferring energy from the environment to a system, or from the system to the environment, by the application of forces.



What is Work?

"Doing work" means using a force to

- *transfer energy* from one object to another, or
- *convert energy* from one form to another.

Work (W) is equal to the amount of energy transferred or converted by the force.

Work is a scalar. S.I. unit is also the joule (J).

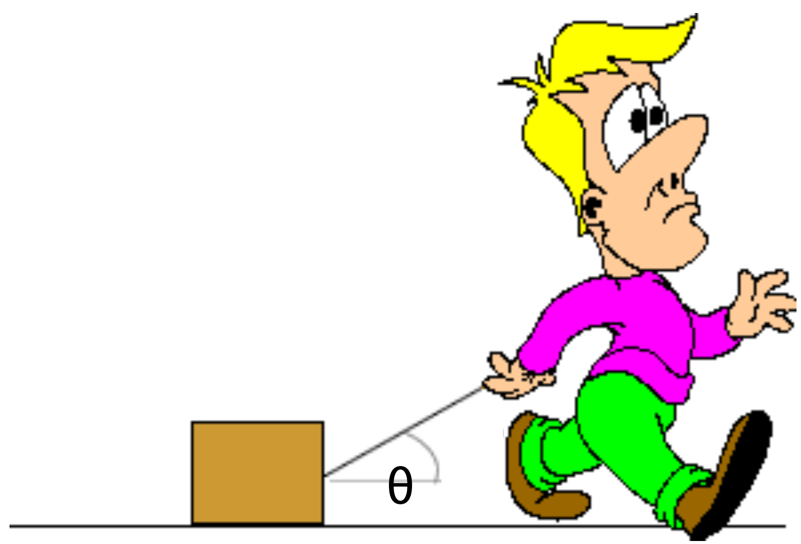
If force is constant then

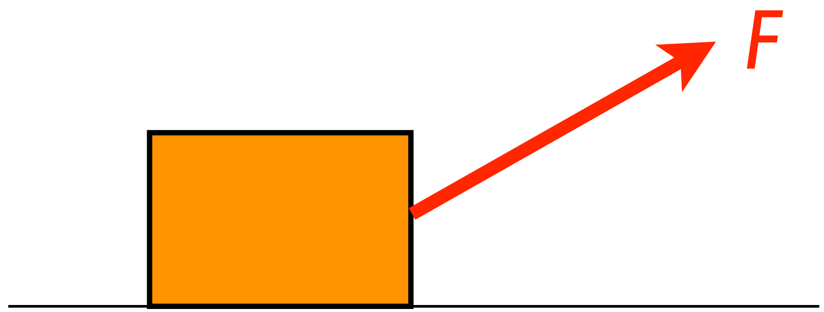
$$W = Fs \cos \theta$$

where F is applied force, s is object's displacement while the force is applied and θ is angle between applied force and displacement.

Work and Kinetic Energy (1)

Consider a single constant force F acting on a body causing a change in kinetic energy only. Suppose it moves in the x–direction and the force acts at angle θ to that direction.





If body moves displacement $d = x_f - x_i$. From the motion equations;

$v_f^2 = v_i^2 + 2a_x d$ where $d = x_f - x_i$

i.e. $2a_x d = (v_f^2 - v_i^2)$

But the K.E. is changing; $\Delta K = K_f - K_i$

$= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}m \times 2a_x d$

Work and Kinetic Energy (2)

Therefore $\Delta K = \tfrac{1}{2}\, m \times 2a_x d = m a_x d = F_x\, d$

$= F \cos\theta\, d$

But our definition of work was "change in energy" and in this case the only change is in kinetic energy.

Therefore the work done is

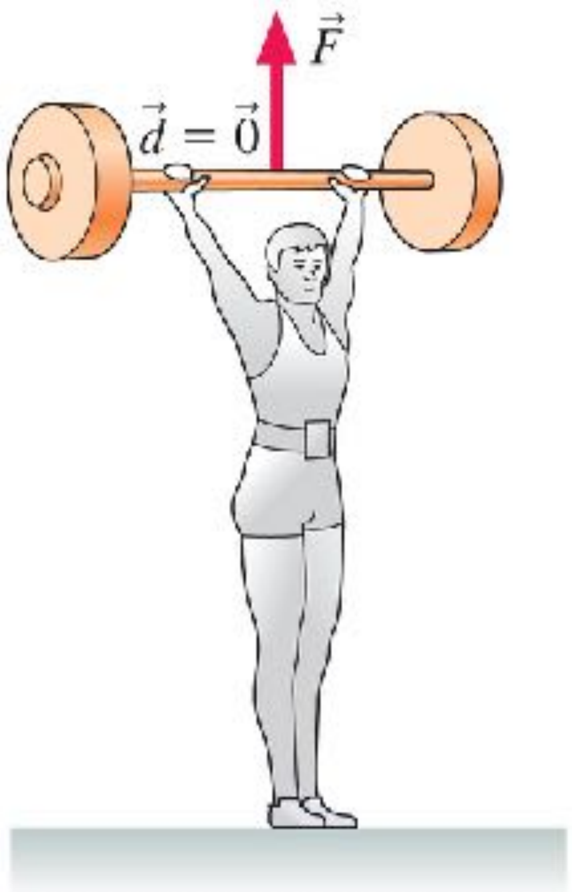
$$W = \Delta K = Fd\cos\theta$$

Sign Convention for Work

- **Work can be negative**! (i.e. Energy will be transferred *from* the body). ("*signed scalar*")
- If the applied force is causing the object to increase in energy, then the work is positive, e.g. work done by legs walking upstairs.
- If the applied force is causing the object to decrease in energy, then work is negative, e.g. work done by legs walking downstairs.

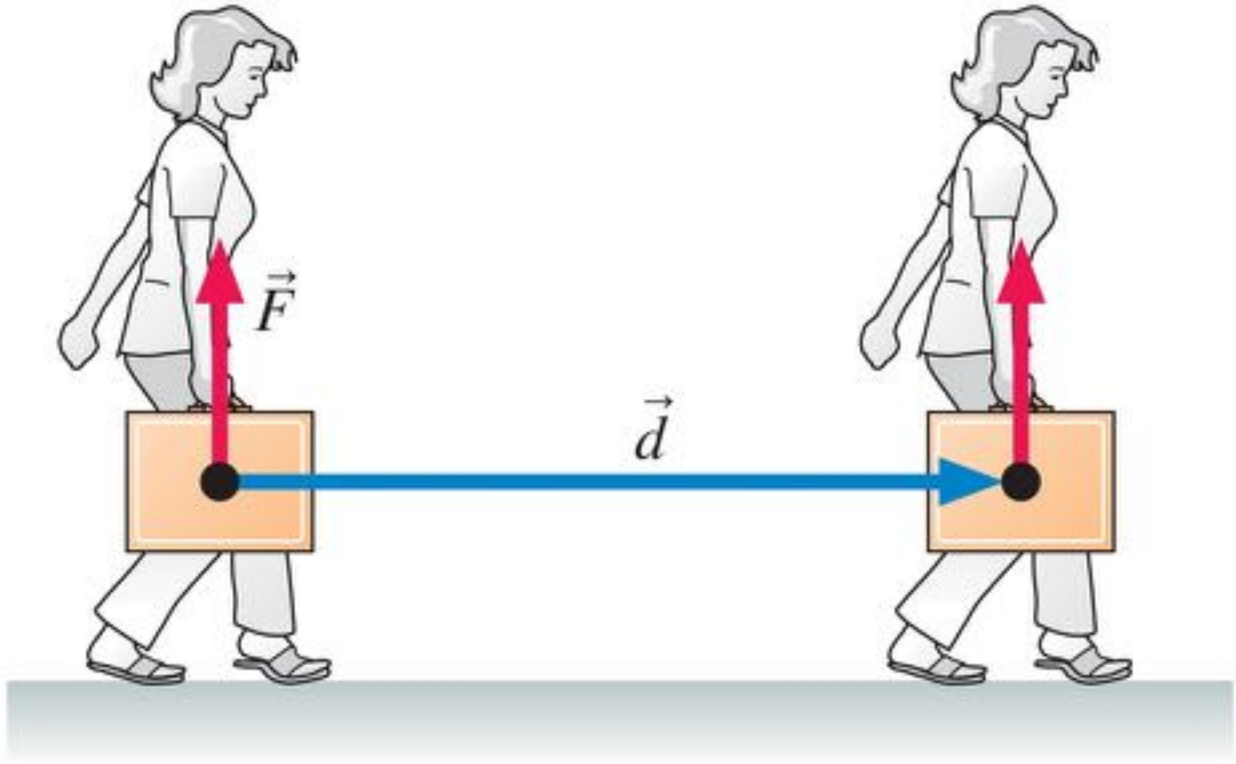
Forces that do no work

- If the object does not move, no work is done



Forces that do no work

- If the force is perpendicular to the displacement, then the force does no work



Work-Energy Theorem

Work is a scalar, i.e. the work done by the individual forces can be added together "arithmetically":

$$W = W_1 + W_2 + \ldots$$

Work-Energy Theorem: *(a special case of the law of conservation of energy)* The change in kinetic energy of a system equals the sum of work done by all the individual forces on the system:

$$\Delta K = \sum W$$

Strictly speaking this theorem applies to rigid ("non-squishy") objects

KJF §10.3

Example: KJF problem 10.11

A 20 g plastic ball is moving to the left at 30 m/s. How much work must be done on the ball to cause it to move to the right at 30 m/s?

[0 J]

What is Power?

Power means the rate at which a force does work on an object.

Instantaneous power: $$P = \frac{\mathrm{d}W}{\mathrm{d}t}$$

$$\text{Average } P = \frac{\text{work done}}{\text{time taken}} = \frac{\Delta W}{\Delta t}$$

Also remember that $W = Fs\cos\theta$, so if F and θ are constant

$$P = \frac{\Delta W}{\Delta t} = F\frac{\Delta s}{\Delta t}\cos\theta = Fv\cos\theta = \underline{\mathbf{F}} \bullet \underline{\mathbf{v}}$$

Units

Units: watt (W) = joule.second^{-1} (J.s^{-1})

[1 horsepower (hp) = 746 W]

Note! Do not confuse the unit watt (W) with the algebraic symbol for work (*W*).

The 1st is the unit for power P, the 2nd represents an energy and has joules (J) as its unit.

	Algebraic Symbol	**Unit Symbol**
Energy, Work	*K, U, W…*	J
Power	*P*	W

Problem

The loaded cab of an elevator has a mass of 3.0×10^3 kg and moves 210m up the shaft in 23s at constant speed.

At what average rate does the force from the cable do work on the cab?

[2.7×10^5 W]

Problem 2 (KJF example 10.17)

Your 1500 kg car is behind a truck travelling at 90 km h^{-1} (25 m s^{-1}). To pass it, you speed up to 120 km h^{-1} (33 m s^{-1}) in 6.0 s.

What engine power is required to do this?

[58 kW]

Gravitational Potential Energy

Stored energy due to height in a gravitational field:

$$\text{G.P.E. or } U = mgh$$

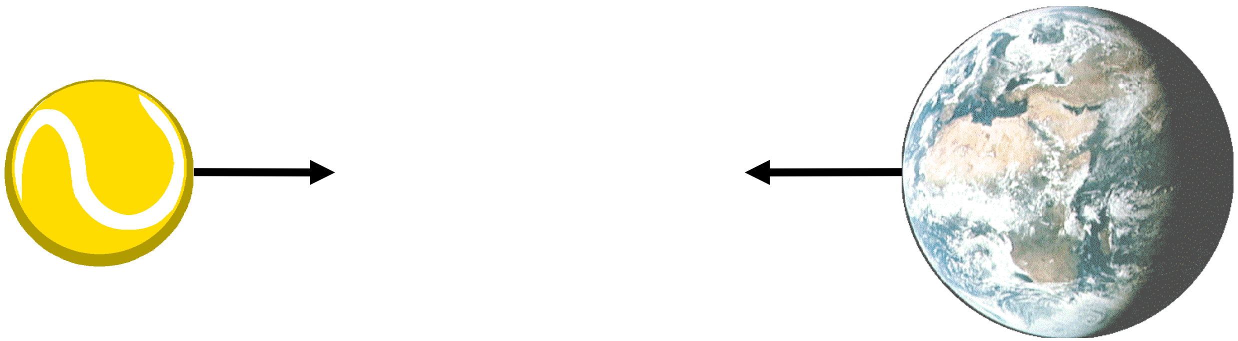
where m is mass (kg) and h is height above the origin level (m).

The origin position ($h = 0$) can be freely chosen
U is always *relative* to some reference level or position.

Example: A 1.0 kg mass is held 10 m above the ground. Find its G.P.E. relative to the ground.

[$U = 1.0 \text{ kg} \times 9.8 \text{ ms}^{-2} \times 10 \text{ m} = 98 \text{ J}$]

Gravitational Potential Energy



Two bodies are attracted because of gravity caused by their mass, so we have to do work W to push them apart.

When we do this, energy is transferred to the system — called gravitational potential energy, U. Near earth's surface $U = mgh$

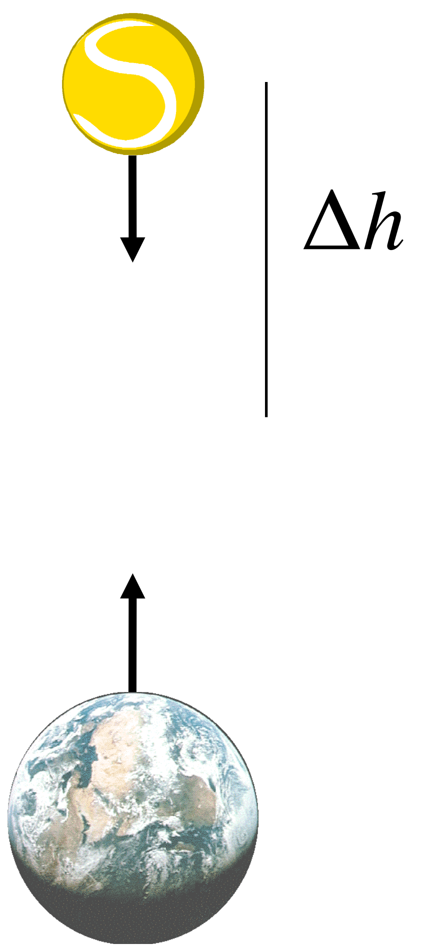
U is a property of the whole system *e.g.* system is earth + ball, but usually we simplify and say "the ball has potential energy"

KJF §10.6

Work done by weight (gravity) on the ball as it falls a distance h towards earth is

$$W = Fs\cos\theta = mg \times \Delta h \times \cos 0° = mg\Delta h$$

$i.e.$ work done by weight (gravity)
= minus the change in U

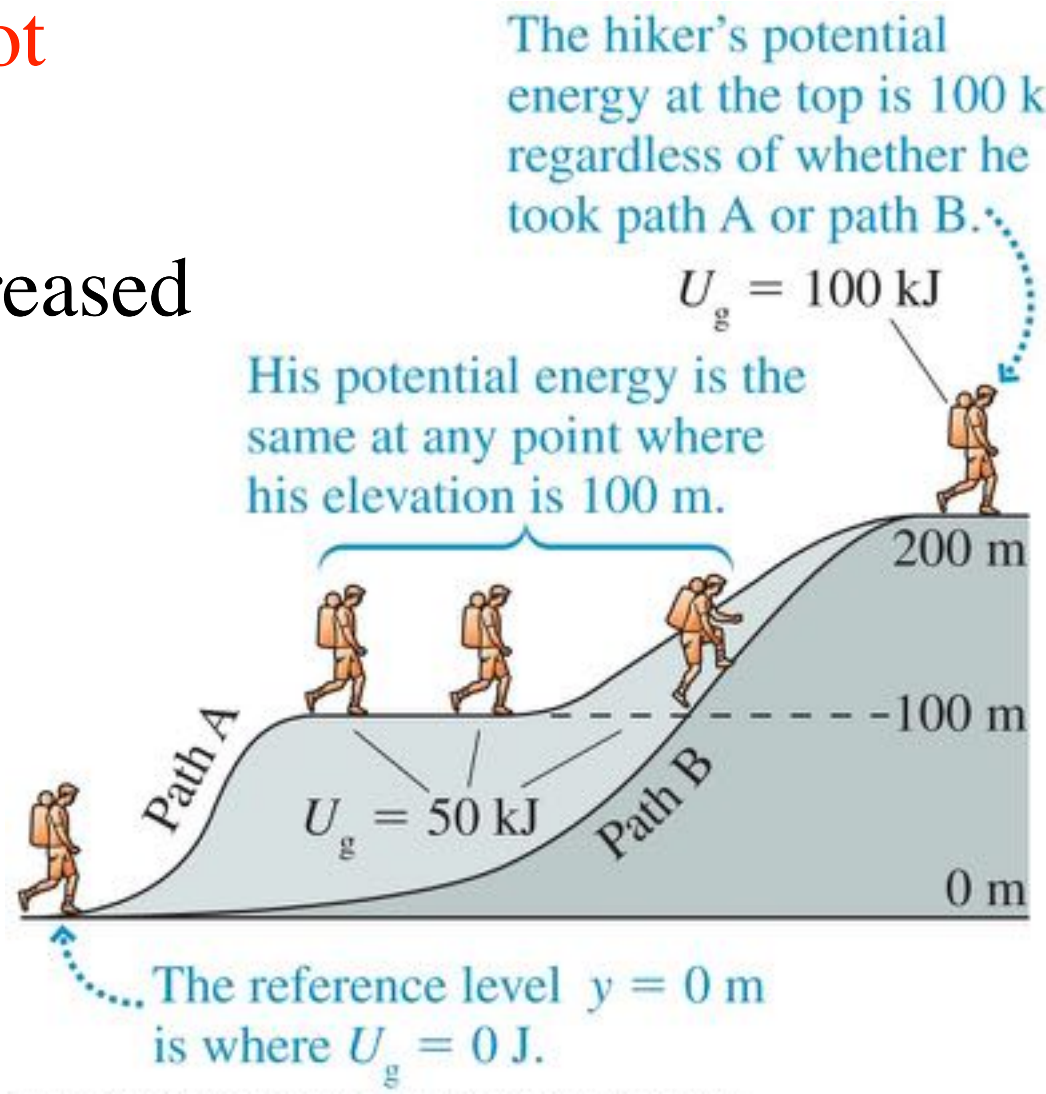


Only the change in height is significant
⇒ we are free to choose any reference level we like where U=0

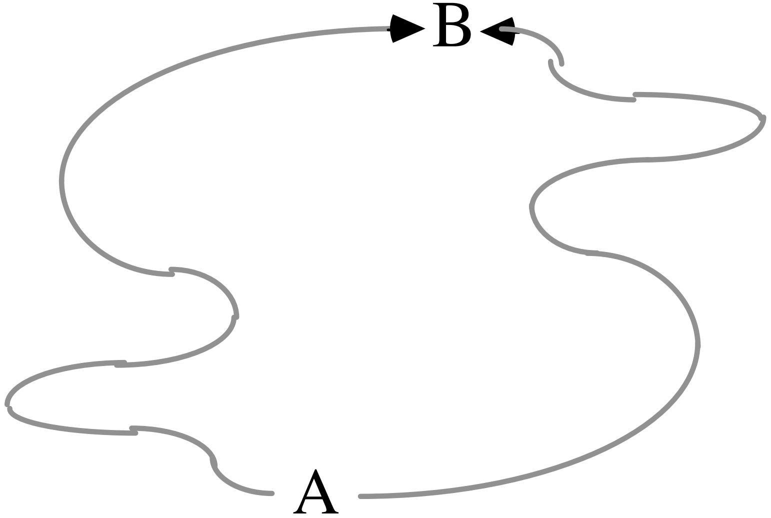
PE is independent of path

Because gravitational potential energy depends only on the height of the object above the reference level, the potential energy does not depend on the path taken.

In both cases the G.P.E increased by the same amount: mgh



Conservative Forces



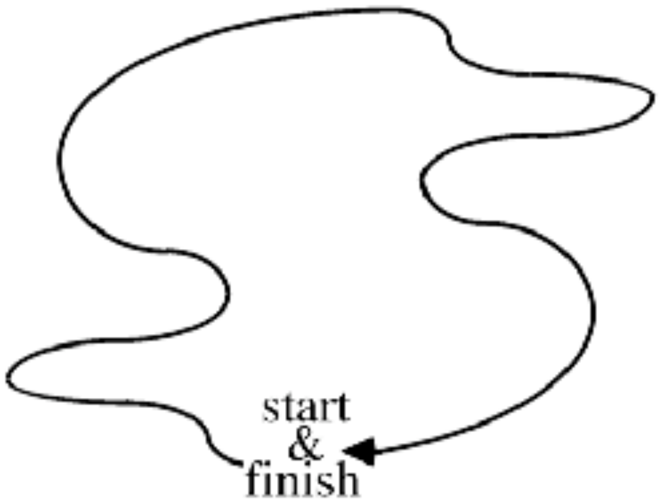
If work done by a force in moving an object from A to B does NOT depend on the path taken, we call it a conservative force (e.g. gravity, ideal springs).

An object moving under the influence of a conservative force always conserves mechanical energy:

$$ME = K_i + U_i = K_f + U_f$$

EVERY conservative force has "potential energy" associated with it (e.g. gravity → gravitational P.E., spring force → elastic P.E.).

Forces like friction which dissipate energy instead of storing it are non-conservative forces.



If an object moves in a closed loop under the influence of a conservative force, the total work done is ZERO.

Work done in moving an object from A → B under the influence of a conservative force is the exact negative of work done in going from B → A, i.e. reversible

$$W_{AB} = -W_{BA}$$

e.g. Work done by gravity going upstairs and back downstairs again.

NEXT LECTURE

Conservation of mechanical energy

Read: KJF §10.6