Lecture 11

Conservation of mechanical energy

Pre-reading: KJF §10.6

Conservation of Mechanical Energy

Under the influence of conservative forces only
(i.e. no friction or drag etc.)

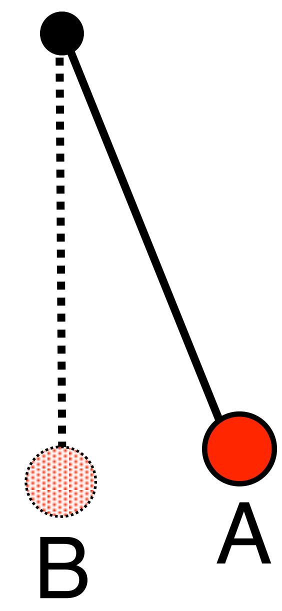
$$\text{M.E.} = K + U = \text{constant}$$

Note that U and K can include such things as elastic potential energy, rotational kinetic energy, etc.

Example: simple pendulum or slippery dip
(if friction & air resistance are negligible).

KJF §10.7

Simple Pendulum



The system is (pendulum + earth).

F_T (tension in string) is always perpendicular to motion so does no work.

Weight (gravity) does all the work.

Because gravitational force is conservative, if drag & friction negligible then,

$$\text{M.E.} = K + U = \text{constant, i.e. } (K + U)_A = (K + U)_B$$

Example: Tarzan

Tarzan who weighs 688N swings from a cliff at the end of a convenient vine that is 18m long. From the top of the cliff to the bottom of the swing he descends by 3.2m.

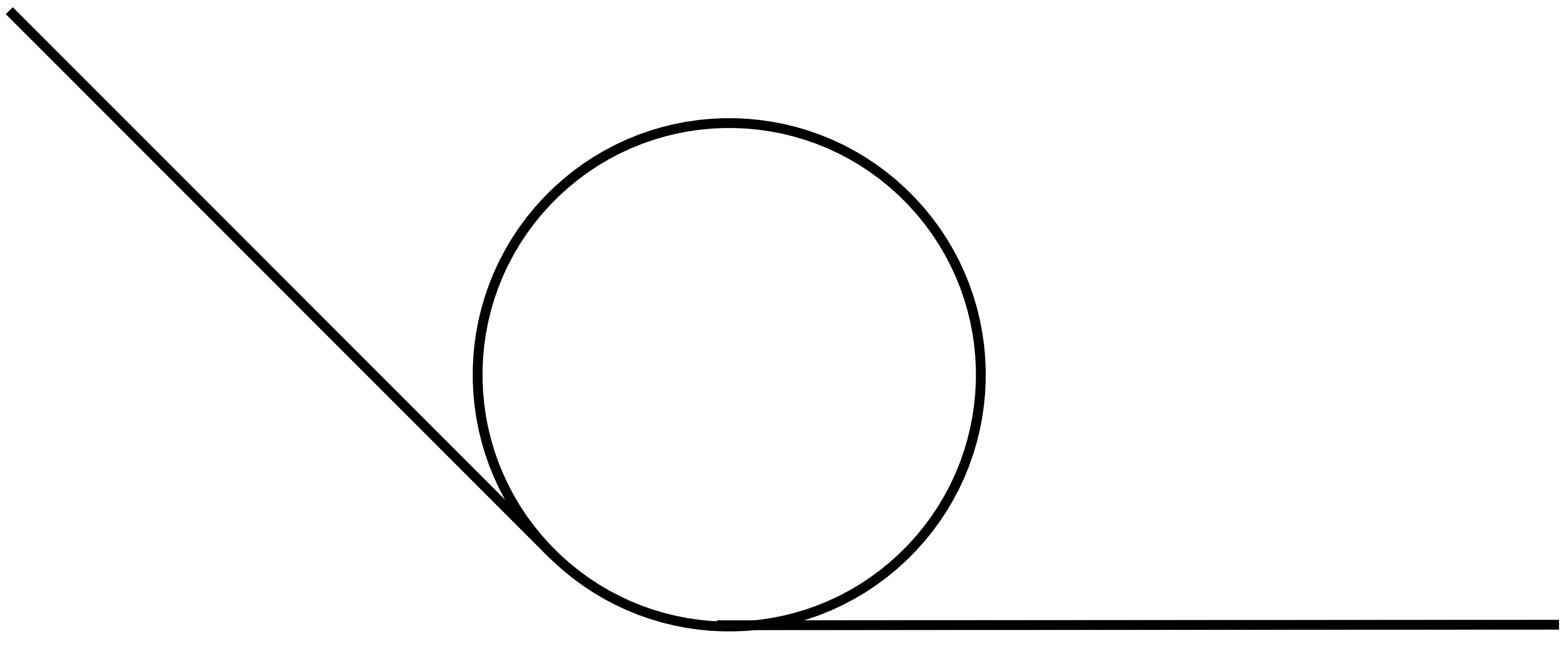
(a) What is his speed at the bottom of the swing? Neglect air resistance.

(b) The vine will break if the force on the vine exceeds 950N. Does it break at the bottom of the swing?

[7.9 m.s–1, no]

Loop the loop

What height does the ball have to start at to make it through the loop?



Non-Conservative Forces

These are dissipative forces such as drag and friction.

Mechanical energy is not conserved when non-conservative forces are acting because friction (and other dissipative forces) convert work or ME directly into thermal energy.

Thermal energy is just the sum of all the kinetic & potential energies of the molecules of a body.

Include this in our expression for conservation of energy:

$$K_i + U_i + W = K_f + U_f + \Delta E_{th}$$

KJF §10.3, eqn. 10.6

Work and Friction (1)

Example 1:

Block on horizontal surface slides to rest due to kinetic friction. Work done by friction is

$$\Delta \mathrm{ME} = \Delta K = -F_{\mathrm{k}} d$$

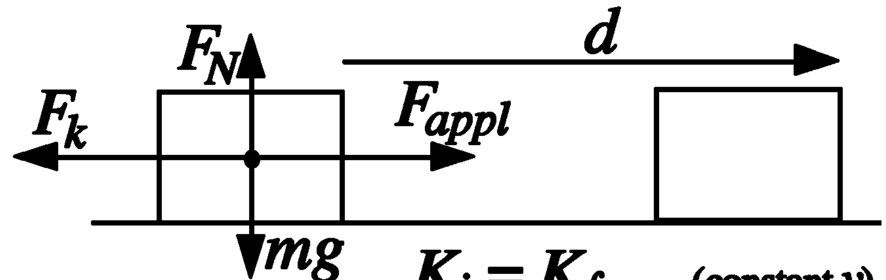
Example 2:

Block sliding along a horizontal surface at constant velocity. If work is done AGAINST friction by an applied force F_{app} and ΔK & $\Delta U = 0$ then;

The amount of thermal energy produced must be exactly equal to the amount of work done, in other words…

Work and Friction (2)

$W = F\, s \cos\theta$



$K_i = K_f$ (constant v)

$mgh_i = mgh_i$ (constant h)

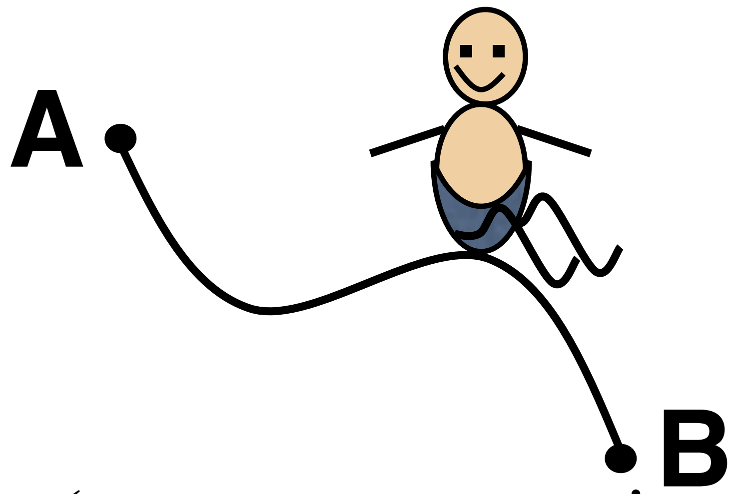
Force working AGAINST friction is $F_{appl} = -F_k$ *(why?)* but $F_k = \mu_k F_N$, $\cos\theta = 1$, and $s = d$, so the amount of thermal energy produced is

$$\Delta E_{th} = F_k\, d = \mu_k\, F_N\, d$$

Clearly, here work is not reversible. *(Why not?)*

Work done BY friction - same magnitude, opposite sign

Example: child on a slide with friction



Since friction (a non-conservative force) is acting,

- ME_A does not equal ME_B ,
- ME_B will be less, and
- $ME_A - ME_B$ = thermal energy produced.

Also note:

- Work done by gravity (or weight) is always = mgh
- When sliding, work done by normal force = 0 because cos 90° = 0

A 20 kg child, starting from rest, slides down a 3m high frictionless slide.

How fast is he going at the bottom?

[7.7 ms^{-1}]

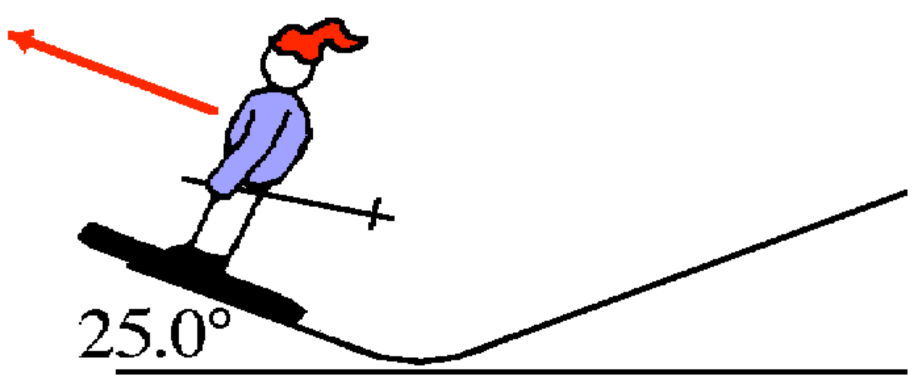
Now he slides down a slide with friction, and his speed at the bottom is 6.0 ms^{-1}. How much thermal energy has been produced by friction?

[228 J]

Problem: Skier

A 60 kg skier leaves the end of a ski jump ramp with a velocity of 24 ms^{-1} directed 25° above the horizontal. Suppose that as a result of air resistance the skier returns to the ground with a speed of 22 ms^{-1} and lands at a point down the hill that is 14m below the ramp.

How much energy is dissipated by air resistance during the jump?



[11 kJ]

Problem

During a rockslide, a 520kg rock slides from rest down a hillside 500m long and 300m high. Coefficient of kinetic friction between the rock and the hill surface is 0.25.

a) If the gravitational potential energy of the rock-Earth system is set to zero at the bottom of the hill, what is the value of U just before the slide?

b) How much work is done by frictional forces during the slide?

c) What is the kinetic energy of the rock as it reaches the bottom of the hill?

d) What is its speed then?

[1.53MJ, –0.510MJ, 1.02MJ, 63ms^{-1}]

NEXT LECTURE

Energy ILD