Lecture 13

Collisions
and
Review of material

Pre-reading: KJF §9.5

Please take an evaluation form

COLLISIONS

KJF §9.5, 10.7

Conservation of momentum

Recall from our discussion of momentum (Lecture 9), that if there are no external forces acting on the system, then

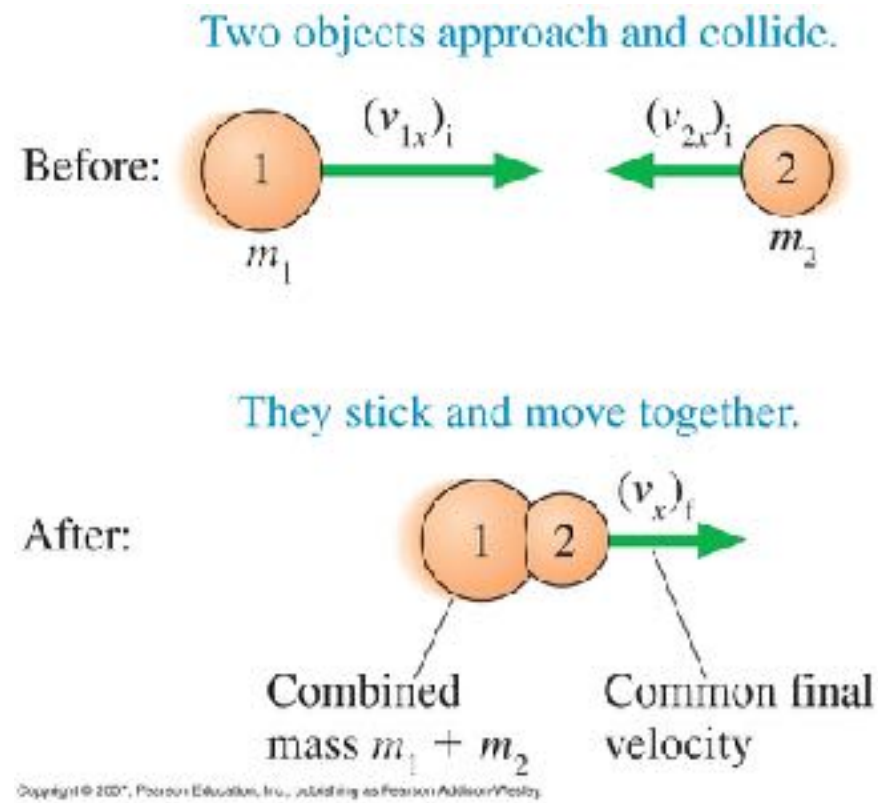
$$\underline{p}_{\text{initial}} = \underline{p}_{\text{final}}$$

i.e. **momentum is conserved**

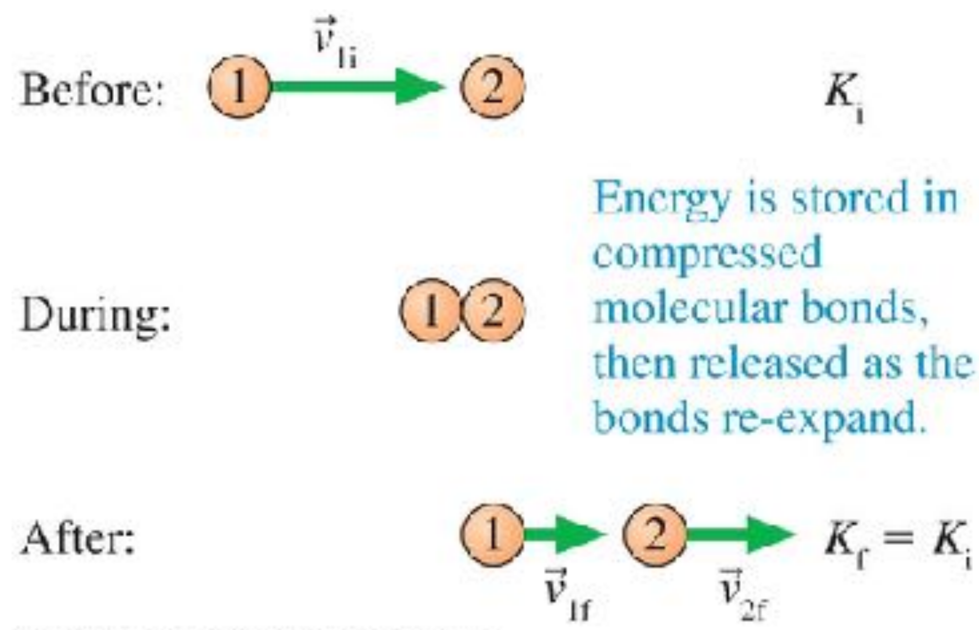
We are going to use this law to study collisions.

We distinguish between two cases:

- *inelastic* collisions, where energy is lost in the collision



- *elastic* collisions, where energy is not lost in the collision



Elastic and Plastic

Elastic means that an object deformed by an external force rapidly returns to its original shape when the force is removed.

> Work done deforming the object is reversible.
> Little or no thermal energy generated. e.g. rubber band, steel spring, super ball

Plastic (or **inelastic**) means that an object deformed by an external force is permanently deformed even after the force is removed.

> Work done deforming the object is irreversible. All or most of work done is converted to thermal energy. e.g. wet clay, plasticine

Most substances will stretch or bend elastically until they reach their "elastic limit", beyond that they deform plastically (or just break!).

Inelastic Collisions

Kinetic energy **is not** conserved during the collision (i.e. some KE converted to heat, or sound, or deformation). BUT Momentum is conserved during collision.

∴ only one equation to solve:

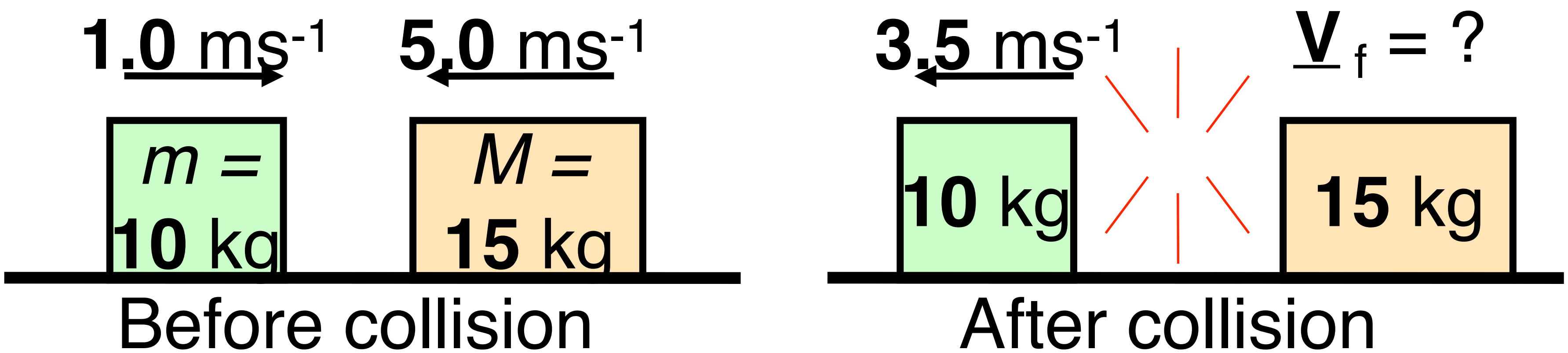
$$\underline{p}_{\text{initial}} = \underline{p}_{\text{final}}$$

In a perfectly inelastic collision, objects stick together after collision → treat the two objects as a single object after collision: $\underline{p}_{\text{final}} = (m_1+m_2)\,\underline{v}_{\text{final}}$

- Most collisions are inelastic.
- "Perfectly inelastic collisions" usually involve easily deformed objects e.g. wet clay.

KJF §9.5

Inelastic Collision Examples

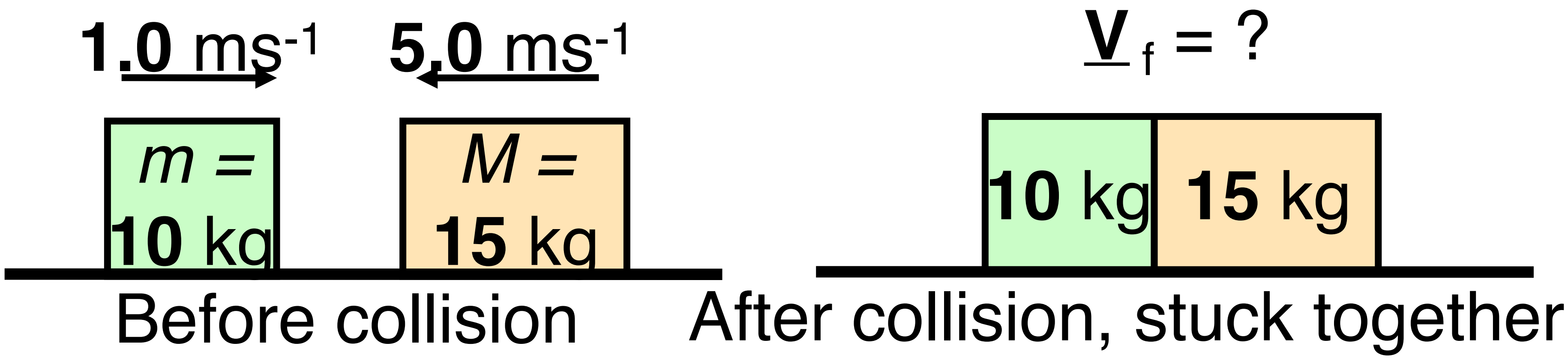


All motions are along x-axis on frictionless surface
+ to the right

Find V_f

[2.0 ms^{-1} to the left]

Perfectly Inelastic Collision



All motions are along x-axis on frictionless surface

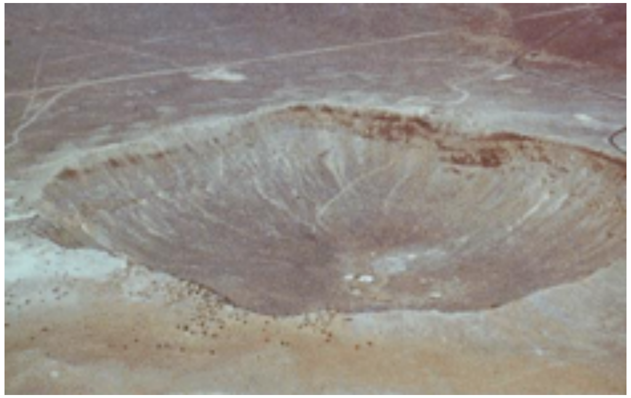
+ to the right

Find V_f

[2.6 ms^{-1} to the left]

Problem

Try this one at home



A crater in Arizona is thought to have been formed by the impact of a meteorite with the earth over 20,000 years ago. The mass of the meteorite is estimated at 5×10^{10} kg and its speed 7200 ms^{-1}. Mass of earth = 5.98×10^{24} kg.

Judging from a frame of reference in which the earth is initially at rest, what speed would such a meteor impart to the earth in a head-on collision? Assume the pieces of the shattered meteor stayed with the earth as it moved.

[$6{\times}10^{-11}$ ms^{-1}: approx 2mm per year]

Perfectly **Elastic** Collisions

Kinetic energy is conserved during the collision (no energy is lost to the surroundings or participants).

Of course momentum is conserved during the collision.

∴ two sets of equations are true simultaneously:

$$\Sigma K_{\text{initial}} = \Sigma K_{\text{final}}$$

$$p_{\text{initial}} = p_{\text{final}}$$

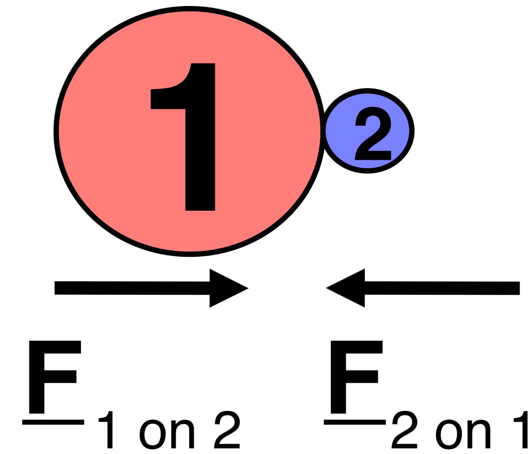
Solve equations simultaneously; quadratic (not in exam)

$$\text{e.g. } \tfrac{1}{2}\, m v_i^2 + \tfrac{1}{2}\, M V_i^2 = \tfrac{1}{2}\, m v_f^2 + \tfrac{1}{2}\, M V_f^2$$

$$\& \quad m v_i + M V_i = m v_f + M V_i$$

- *Usually involves sub-atomic particles or highly rigid objects e.g. steel or glass balls.*
- *If both objects are same mass, their velocities swap after perfectly elastic collision e.g. Newton's cradle*

Collisions and Impulse



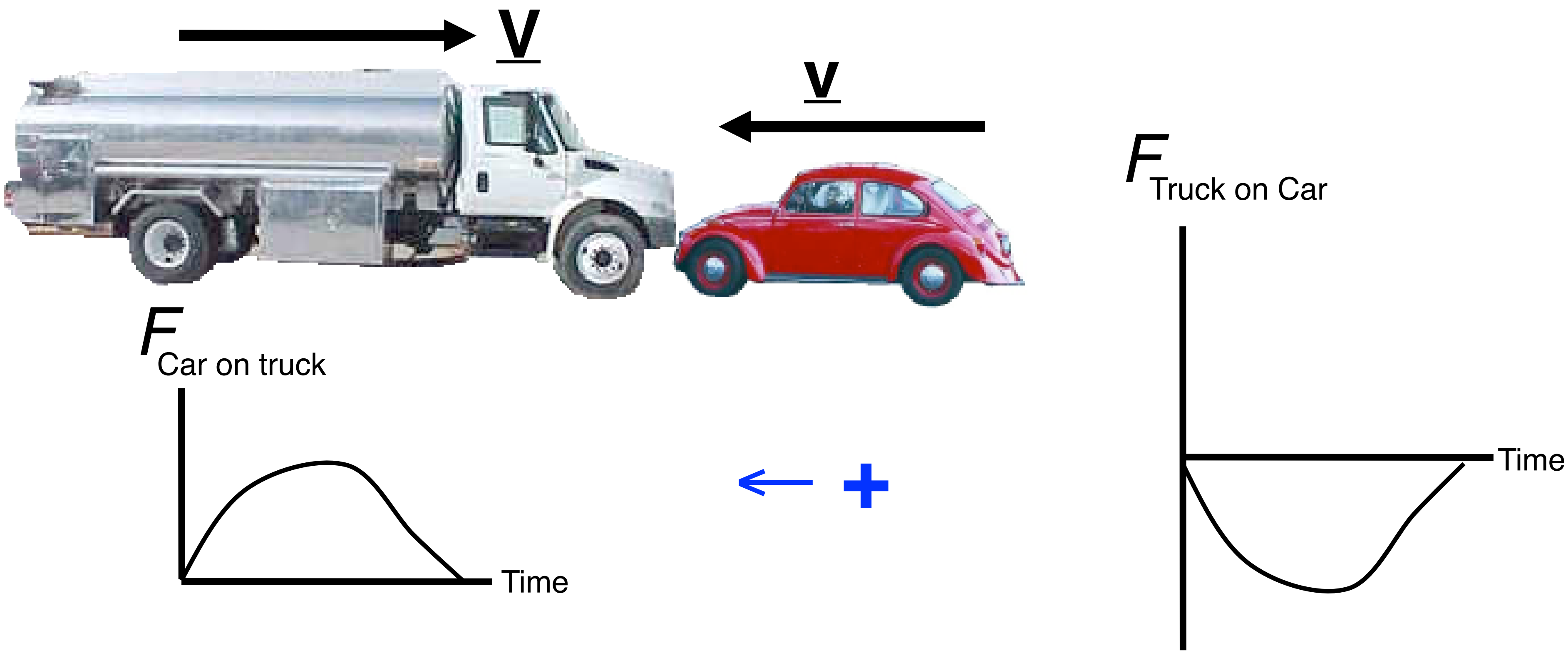
During collision, momentum is conserved – none is lost

$\therefore$ momentum **lost** by 1 = momentum **gained** by 2 *(or vice versa)*

$$\therefore\ \Delta \underline{p}_1 = -\Delta \underline{p}_2 \text{ i.e. } \underline{J}_1 = -\underline{J}_2$$

i.e. impulses are equal and opposite

Collision of Truck and Car



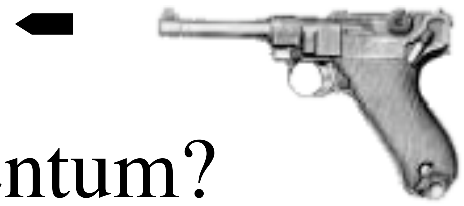
- Which has the greatest magnitude of change in momentum?
- Which has the greatest magnitude of change in velocity?
- Which vehicle is it safest to be **in** and why? *(Write it down!)*

The impulse approximation

During any collision, if there are no net external forces on the system

Momentum is absolutely conserved

But there usually are external forces on the system (e.g. weight force)



Can we use conservation of momentum?

KJF §9.3

If the external force is much smaller than the collisional (or explosive) forces, and the collision (or explosion) time is short, so during the collision (or explosion) we can ignore the momentum change due to net external force, then

Momentum is very nearly conserved during collisions or explosions even with external forces

(e.g. hitting nail with a hammer + gravity, recoiling gun + gravity)

This is called the impulse approximation.

Problem from 1996 Exam

A ball of mass 700g is fastened to a cord 800mm long and fixed at the far end at a support, and is released when the cord is horizontal. At the bottom of its path, the ball strikes a stationary 350g ball suspended from the same support with a cord 800mm long. The two balls stick together after the collision.

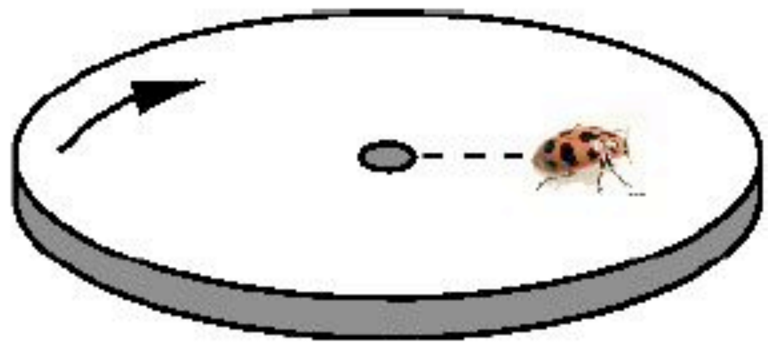
a) Calculate the speed of the falling ball just before it hits the stationary ball.

b) Calculate the speed of the two balls immediately after the collision.

[3.9ms^{-1}; 2.64ms^{-1}]

REVIEW

Question 3



A beetle sits at the centre of an old-fashioned record turntable spinning at a constant 45 revolutions per minute. The beetle spots a particular location on the edge of the turntable and begins to walk at a constant pace radially outward towards that location. (As far as the beetle is concerned it is walking in a straight line, while from our viewpoint, the beetle is spiralling outward.)

(a) Is the beetle accelerating? Briefly justify your answer.

(b) A centripetal force is acting on the beetle. What is the origin of this centripetal force?

(c) As the beetle moves from the centre towards the edge of the turntable, does it become more likely, less likely or remain equally likely that it will lose its grip and slide off the turntable? Briefly justify your answer.

(5 marks)

Question 4

Jack is in trouble! When he stood on the table at home, the table supported his weight. When Jack jumped from a ledge onto the same table, it broke.

(a) Draw the forces *acting on Jack* when he stood on the table. Do these forces represent an action-reaction pair? Explain your answer.

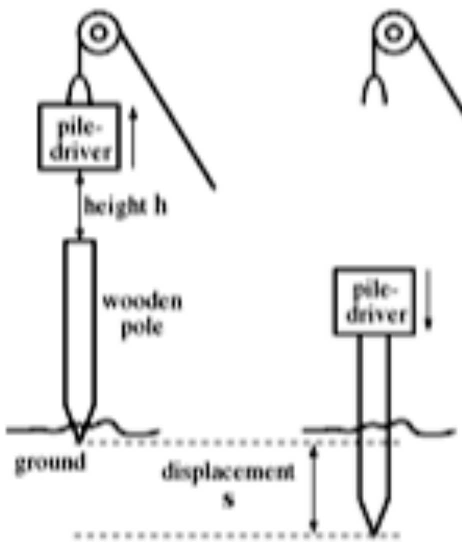
(b) Why did the table break when Jack jumped onto it? Explain your answer in terms of momentum, impulse and the forces acting when Jack was in contact with the table.

(c) After breaking one table, Jack found that he could safely jump onto a similar table if he placed a spongy mattress on top of it. Again, in terms of momentum, impulse and the forces acting, explain why this is possible.

(5 marks)

Problem from 2004 Exam

A "pile-driver" is a heavy weight which is used to drive poles into the ground. It is similar to a hammer driving a nail into wood, except that it is much larger. The pile-driver is lifted up by a cable and then dropped onto the end of a wooden pole several times until the pole is pushed to the desired depth into the ground.



Suppose a pile-driver of mass $M = 525$ kg is raised and then dropped (from rest) onto a wooden pole of mass of $m = 25.0$ kg. It collides with the pole after dropping through a height of $h = 2.00$ m.

(a) What is the speed of the pile-driver at the instant just before it hits the pole? (Ignore air resistance.)

6.26 ms^{-1}

The pile-driver collides with the pole without bouncing off it. As the pole is driven into the ground, the pile-driver remains in contact with the pole, the two objects moving downwards together at the same velocity.

(b) (i) Was the collision perfectly elastic, perfectly inelastic or neither? *Briefly* justify your answer.

(ii) *Just at the instant of collision*, was momentum conserved? Was energy conserved?

(c) Calculate the speed at which the pile-driver and the wooden pole move downwards at the instant *just after the collision*.

perfectly inelastic; momentum conserved, energy not; 5.98 ms^{-1}

The pole is now pushed into the ground, with the pile-driver still in contact with it. The pole penetrates the ground through a displacement $s = 0.750$ m and then stops.

You can assume that the soil in the ground exerts a constant upward force of friction on the pole as it penetrates.

(d) Calculate the magnitude of the friction force exerted by the ground on the pole + pile-driver system.

13.1 kN