Vector Algebra Notes:
Adding Vectors & Finding Components

Section 1) Definition of Vectors:

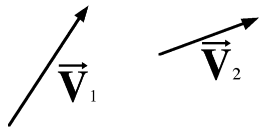
Read section 2.6 "Some Vector Algebra" (all references are to Hecht 2nd edition)
Do 2-13M, 2-71P

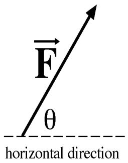
Quantities that need to be represented by magnitude (size) *and* direction are called "vectors" (*e.g.* displacement, velocity, acceleration and force). In printed text, vectors are usually represented by letters in **bold** font (**V**). They can also be represented with "tildes", "hats" or "arrows" above or below the letter;

(*e.g.* $\tilde{\mathbf{V}}$, $\bar{\mathbf{V}}$, $\vec{\mathbf{V}}$, $\underset{\sim}{\mathbf{V}}$, $\underline{\mathbf{V}}$ or $\underset{\rightarrow}{\mathbf{V}}$).

In this document, we will use examples of some of these notations.

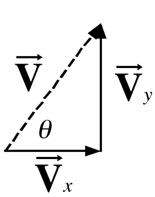
The magnitude of a vector is usually represented by unbolded italic text (*e.g.* V). Vectors can be represented in diagrams by arrows. The length of the arrow represents the vector's magnitude, and the direction the arrow points represents the vector's direction.





Usually, the direction is expressed as an angle θ to a known direction (*e.g.* horizontal).

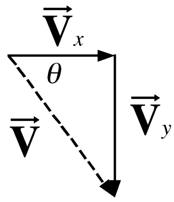
The *position* of the vector on the page is irrelevant (unless we are considering torque) and so the arrow representing it *can be moved to anywhere on the page* and it's still the same vector as long as the length and direction don't change.



Every vector ($\vec{\mathbf{V}}$ in this example) can be thought of as being the resultant sum of separate vector components, each one parallel to each of the three coordinate axes: x, y and z. Here we will only consider the x and y components, V_x and V_y.

Remember that when finding components, the original vector is always the hypotenuse of a right-angle triangle and the x and y components must be head-to-tail.

Remember that sometimes these components can be negative as is V_y is in this example (if we define "up" to be +ve). Don't forget to note carefully *which angle* in your triangle you will use in your trigonometry calculation.



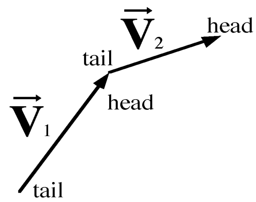
Section 2) Adding vectors:

Read section 2.8 "Components and Vector Addition"
Do 2-61P, 2-67P, 2-83P, 2-85P, 2-98P

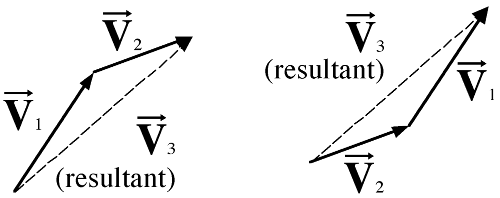
Method 1: Adding Vectors Geometrically

The geometrical method of adding vectors is most suitable for adding 2 vectors. In theory it works when adding 3 or more vectors but can become messy.

To add two vectors geometrically, you must arrange the arrows representing the two vectors so that they are joined together "head to tail", but without changing their lengths or directions.

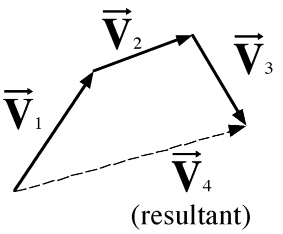


The sum or "resultant" of the two vectors is generated by drawing an arrow that starts at the beginning (tail) of the first vector and ends at the end (head) of the second. It does not matter in what order the original two vectors are - either way the resultant will have the same magnitude and direction.

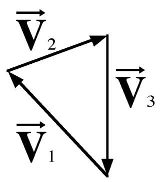


One must use trigonometry (sin, cos & tan *etc.*) to find the magnitude of the resultant (or Pythagoras' theorem if one of the angles is 90°) and the angle of the resultant to a known direction (*e.g.* horizontal).

If you attempt to add more than 2 vectors by this method, the result is no longer a triangle and so the trigonometry becomes more complicated, so avoid this. It is still correct, but harder to do.

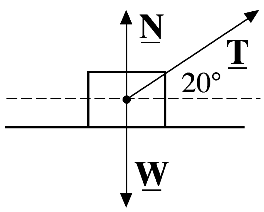


An exception is if the 3 vectors represent the forces on an object "in equilibrium" (*i.e. no net force*). In this case, the resultant force = 0, therefore the 3 vectors form a closed loop which of course will be a triangle and so you can use normal trigonometry to add them up.



Method 2: Adding x & y Components Separately

This method is the best one for adding 3 or more vectors. Of course it also works for 2 vectors.



(For revision on how to find components of vectors, read Section 3 of this document first)

Example; Find the resultant (net) force of all the forces acting on a block of weight W, being pulled by a string with tension T at 20° to the horizontal.

Choose a sign convention. (e.g. choose *up* to be +ve and *right* to be +ve.)

(Note; we cannot assume $|\underline{\mathbf{N}}| = |\underline{\mathbf{W}}|$ *because there is also a vertical component of* $\underline{\mathbf{T}}$ *here and the question doesn't even say if the object is in equilibrium.)*

Consider vertical (y) components first:

y component of $\underline{\mathbf{N}}$ (or N_y); $\quad N_y = +N$

y component of $\underline{\mathbf{W}}$ (or W_y); $\quad W_y = -W$

y component of $\underline{\mathbf{T}}$ (or T_y); $\quad T_y = +T \sin 20°$

$$\boxed{F_{y\,(\text{net})} = N_y + W_y + T_y = N - W + T \sin 20°}$$

Now consider horizontal (x) components:

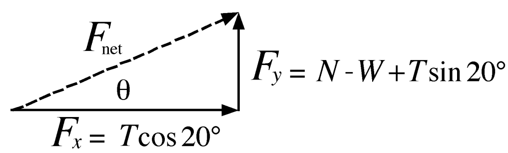
x component of $\underline{\mathbf{N}}$ (N_x); $\quad N_x = 0$

x component of $\underline{\mathbf{W}}$ (W_x); $\quad W_x = 0$

x component of $\underline{\mathbf{T}}$ (T_x); $\quad T_x = +T \cos 20°$

$$\boxed{F_{x\,(\text{net})} = N_x + W_x + T_x = 0 + 0 + T \cos 20° = T \cos 20°}$$

Finally, using Pythagoras' theorem, add the vertical and horizontal components to find $\underline{\mathbf{F}}_{\text{net}}$:



Substitute in the data supplied with the problem, calculate F_x and F_y then use Pythagoras' theorem to calculate the magnitude of $\mathbf{\underline{F}}_{net.}$

The direction θ of the *net force* is given by tan $\theta = {}^{F_y}/_{F_x}$. ***Don't*** assume that θ is equal to the angle of the string to the horizontal (20°).

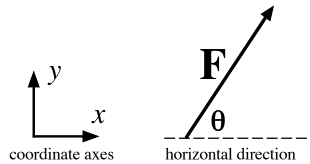
The same method works even if you use a rotated set of coordinate axes such that the now rotated X axis is parallel to another direction, such as an inclined plane. See the section below called "Situation 2: *X-axis is rotated from horizontal" for an explanation.*

Note: Vector Subtraction; $\underline{\mathbf{V}}_1 - \underline{\mathbf{V}}_2$ *is the same as adding* $\underline{\mathbf{V}}_1 + (-\underline{\mathbf{V}}_2)$. *The vector* $(-\underline{\mathbf{V}}_2)$ *is the same as the vector* $\underline{\mathbf{V}}_2$ *except that the arrow is pointing in the opposite direction.*

Section 3) Finding Components of Vectors:

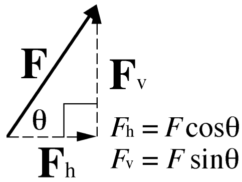
Situation 1; x-axis is horizontal

Find vertical (or x) and horizontal (or y) components of vector **F**.



Step 1) draw the vector, noting any angles given in problem

Step 2) draw in a vertical (or x component) and a horizontal vector (or y component) which add up to the original vector - NOTE: original vector is ALWAYS hypotenuse.



Step 3) find F_v & F_h using trigonometry.

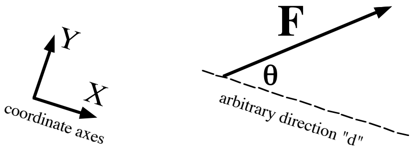
Notes:

The vertical component of a horizontal vector is always zero (and vice versa)

The two directions chosen for resolving components don't have to be "x" and "y". You could also use other sets of perpendicular directions such as "northerly" and "easterly".

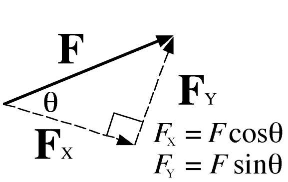
Situation 2; X-axis is rotated from horizontal

Find the components of vector **F** relative to an arbitrary direction, (called for example "**d**"). For example, **d** might be along an inclined plane.



Step 1) draw coordinate axes, rotated so the new "X" axis is parallel to the required direction "**d**". I've used capital X and Y to avoid confusion with the normal unrotated x and y axes.

(You can also call Y, the "perpendicular" component and X, the "parallel" component to the inclined plane)



Step 2) draw vector **F** and the required direction "**d**", noting any angles given in problem. Be

Step 3) draw in two vectors (the X and Y components) which add up to original vector, the X component lying *parallel* to the direction "**d**" and the other, the Y component, *perpendicular* to "**d**"

NOTE: original vector **F** is ALWAYS the hypotenuse.

Step 4) find F_X and F_Y using trigonometry

Notes:

Because F is the hypotenuse, ANY component of **F** *in any direction should always be less than or equal to* **F** *in magnitude.*

The component of any force in a direction ***perpendicular*** *to that force is always zero, e.g. the X component of a force acting in the Y direction is equal to 0.*

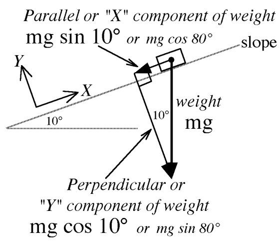
The component of any force in a direction ***parallel*** *to that force is always just the force itself, e.g. the X component of a force* **F** *acting in the X direction is equal to* **F**.

Example of Finding Components;

a) A 10.0 kg block is sliding down a frictionless inclined plane 10° to horizontal. Find the components of weight parallel and perpendicular to the plane.

b) Find the acceleration of the block.

c) What is the magnitude of normal force exerted by the plane onto the block?



a) Component of weight parallel to the plane (X component) is;

$$\text{mg sin } 10° = 10.0 \times 9.81 \times 0.174 = 17.0 \text{ N}$$

along the slope, downwards

Component of weight perpendicular to the plane (Y component) is;

$$\text{mg cos } 10° = 10.0 \times 9.81 \times 0.985 = 96.6 \text{ N}$$

perpendicular to the slope, downwards

b) No friction ∴ only force acting along slope is X component of weight $F_X = ma_X$ ∴ $a_X = F_X/m = {}^{\text{mg sin } 10°}/_{\text{m}} = 1.70 \text{ ms}^{-2}$

down the slope

c) As in *ALL* cases of sliding motion along a slope, there is no net force perpendicular (*i.e.* Y direction) to the slope (no Y motion ∴ no acceleration perpendicular to the slope). Therefore all forces in the perpendicular direction are balanced. The only forces in this direction are normal force $\underline{\mathbf{N}}$ and perpendicular component of weight, mg cos 10°. Take up to be +ve.

$$F_Y = 0 = N - \text{mg cos } 10° \therefore N = \text{mg cos } 10° = 96.6 \text{ N}$$