Lecture 2: Radiation processes

Senior Astrophysics

2018-03-07

Outline

1. Radiation processes
2. Emission
3. Absorption

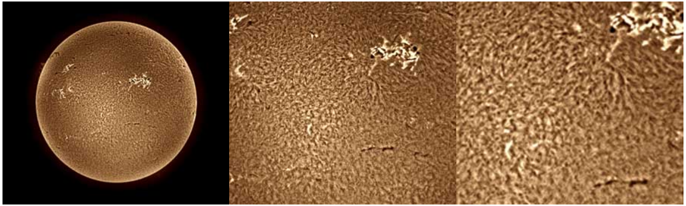
Equation of radiative transfer

Q: How does the intensity of radiation change in the presence of emission and/or absorption?

Recap: Brightness

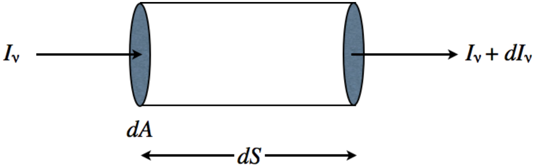
Recall: *brightness* doesn't change along the beam.

- **Consequence 1**: Brightness is independent of distance: you don't need to change your camera settings to photograph the Sun from different distances.
- **Consequence 2**: Brightness is the same at the source and at the detector: you can think of brightness in terms of energy flowing out of the source or as energy flowing into the detector.



Emission

- Consider radiation travelling through a medium which is itself emitting radiation, so we get **more** energy out the other end



- Define **emission coefficient** j_ν to be amount of energy emitted per unit time per unit volume, per unit solid angle, per unit frequency interval:

$$dE = j_\nu \, dV \, d\Omega \, dt \, d\nu$$

where $dV = dA \times dS$ beam of cross-section dA travelling a distance dS

- The **increase** in specific intensity is therefore

$$dI_\nu = j_\nu ds$$

or

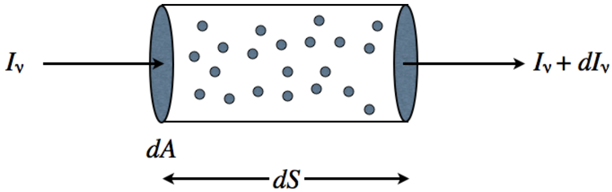
$$\frac{dI_\nu}{ds} = j_\nu$$

— the change of brightness for pure emission.
- If we know j_ν, calculate the change in specific intensity as radiation propagates through the gas by adding up the contributions to the emission all along the path

$$I_\nu(s) = i_\nu(s_0) + \int_{s_0}^{s} j_\nu(s')ds'$$

Absorption

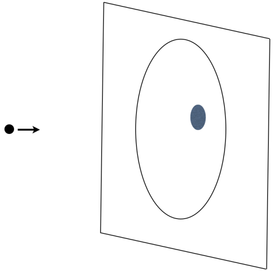
- If radiation travels through a gas which absorbs or scatters radiation, then the energy in the beam will be **reduced**



- **Optical depth** describes how much absorption occurs when light travels through an absorbing medium.

Cross-sections

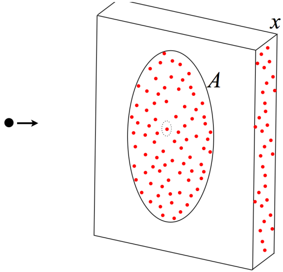
Use concept of **cross-section** (from particle physics).
Problem: if you are firing projectiles which you cannot see at targets which are much smaller than atoms, how do you make sense of what you detect on the other side of the target?
Imagine firing point-like projectiles at an area A which includes a solid target of area σ.



Probability of hitting target $= \sigma/A$

Cross-sections

Now consider a beam of particles incident on a sheet made of nuclei with n nuclei/unit volume, thickness x, area A hit by beam.
Say if a particle gets "close enough" to a nucleus, a reaction takes place.
Define the *effective area* σ for this reaction



- Total number of nuclei in $A = n \times Ax$
- Effective area available for reaction $= (nxA)\sigma$ m^2
- **Probability** that this reaction will take place $= (nxA\sigma)/A = nx\sigma$
  Note that $nx =$ projected surface density; also called *column density* $\equiv$ density integrated over length of column

Cross-sections

Units: cross section (area) measured in m^2 (cm^2)
Most convenient sub-multiple is 1 barn (b) $= 1 \times 10^{-28}$ m^2

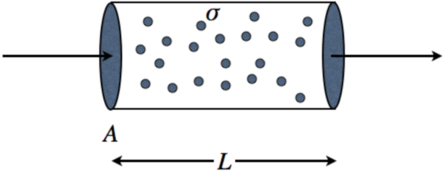
- 1 barn is a "large" cross section for nuclear physics
- More common: mb ($= 10^{-3}$ b), μb ($= 10^{-6}$ b), nb ($= 10^{-9}$ b),

Example: Neutrinos

Neutrinos have a very low cross-section for interaction with matter:
$\sigma = 10^{-47}\,\mathrm{m}^2$ for interaction with a nucleon.
What thickness of lead (density $\rho = 11,400\ \mathrm{kg\,m^{-3}}$) does a neutrino have to travel through for it to have a 50% chance of interaction?

Absorption: Microscopic version

Consider light rays passing through a cylinder of absorbers with number density n. Each absorber has cross-section σ, and the cylinder has length L and area A:

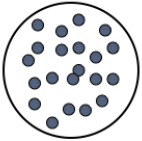


- Volume of the cylinder is $V = LA$
- Number of absorbers $= nLA$

Absorption: Microscopic version

Provided the absorbers don't shadow each other, then the fractional area blocked is

$$\sigma_{\rm total} = nLA\sigma$$



So the fraction of light blocked is

$$f_{\rm abs} = \frac{\sigma_{\rm total}}{A} = nL\sigma$$

We define this to be the **optical depth**: $\tau = nL\sigma$.

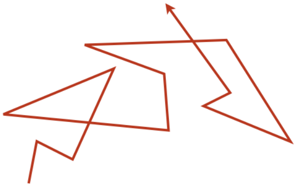
- if τ is small ($\tau < 1$) $\Rightarrow$ **optically thin**
- if τ is large ($\tau > 1$) $\Rightarrow$ **optically thick**

Absorption: Microscopic version

The quantity $\tau = 1$ has a geometrical interpretation: setting $\tau = 1$, the typical distance a photon will travel between interactions is

$$L = \frac{1}{n\sigma}$$

the **mean free path** of the photon: the typical length travelled by a photon before absorption.



Absorption: Microscopic version

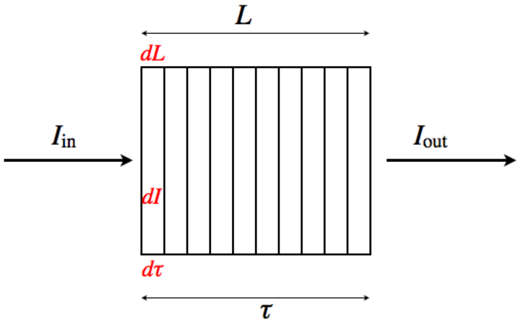
What if the absorbers do shadow one another?

- Think of an optically thick layer as a series of optically thin layers:

- In each layer, the change in intensity dI is the incoming intensity times the depth of that layer:

$$dI = -(I)(n\sigma dL) = -I d\tau$$

so $$\frac{dI}{I} = -d\tau$$



Absorption: Microscopic version

Integrate this:

$$\int_{I_{\rm in}}^{I_{\rm out}} \frac{dI}{I} = -\int_0^\tau d\tau$$

$$\text{so} \qquad \ln \frac{I_{\rm out}}{I_{\rm in}} = \tau$$

$$\text{or} \qquad I_{\rm out} = I_{\rm in} e^{-\tau}$$

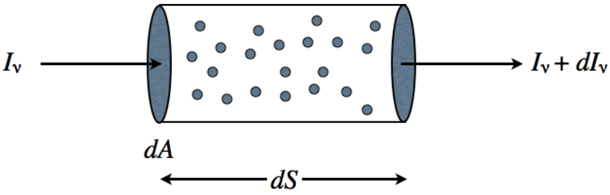
i.e. exponential decrease in light intensity.
Note that if $\tau \ll 1$, $e^{-\tau} \approx 1 - \tau$, so

$$I_{\rm out} = I_{\rm in}(1 - \tau)$$

i.e. fraction absorbed is τ, as before.

Absorption coefficient and opacity

Using the same notation as before:



$$\frac{dI_\nu}{I_\nu} = -n\sigma_\nu ds \equiv d\tau_\nu$$

In general σ will be a strong function of wavelength (frequency) $\Rightarrow \sigma_\nu$.

Absorption coefficient and opacity

Derived quantities:

- $\alpha_\nu = n\sigma_\nu$ is the **absorption coefficient** (units m^{-1}).
- $\kappa_\nu = \alpha_\nu/\rho$ is the **opacity** (units of $\mathrm{m}^2\ \mathrm{kg}^{-1}$) i.e. the cross-section of a kg of gas

The opacity of a gas is a function of its composition, density and temperature, and depends strongly on frequency.

Absorption

- σ_ν often depends on frequency

apod.nasa.gov/apod/ap090924.html

Example: Thomson scattering

- Free electrons will scatter (not absorb) photons through the process of **Thomson scattering**. The cross-section for Thomson scattering is independent of frequency:

$$\sigma_T = 6.65 \times 10^{-29} \mathrm{m}^2$$

- The opacity is therefore

$$\kappa_T = \frac{n_e}{\rho} \sigma_T$$

Example: Thomson scattering

- If the gas is completely ionised, then n_e is proportional to the density, and n_e/ρ depends only on the composition (how many electrons there are per ion). For pure hydrogen, $n_e/\rho = 1/m_H$, so the opacity for fully ionised hydrogen is

$$\kappa_T = \frac{\sigma_T}{m_H} = 3.98 \times 10^{-2}\,\mathrm{m^2 kg^{-1}}$$

- The Thomson cross-section has a very small value, much smaller than e.g. H cross-section for photoionisation. So electron scattering is only important when the electron density is high, and when the gas is completely ionised.

Pure absorption

- Re-arrange previous equation:

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu$$

  for pure absorption

- **Note** difference from emission equation

$$\frac{dI_\nu}{ds} = j_\nu$$

  for pure emission

- How much radiation we lose through absorption depends on **how much we already have** — i.e. multiplicative instead of additive.

Next lecture

- The radiative transfer equation
- Optical depth
- Blackbody radiation