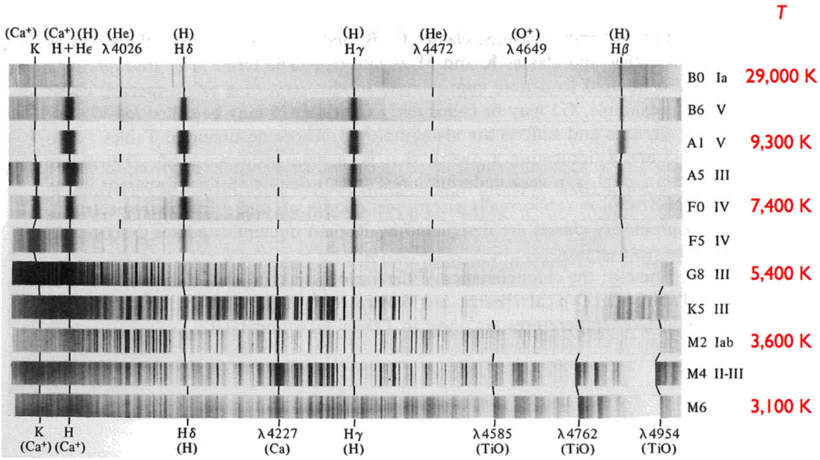
Lecture 4: Absorption and emission lines

Senior Astrophysics

2018-03-13

Outline

Motivation



Representative stellar spectra illustrating the Harvard spectral sequence. From Novotny (1973).

Motivation

Even H (simplest atom) has huge numbers of possible pairs of energy levels with different ΔE and hence different ν. How do we know which ones we will see?

- At particular T, some levels have higher probability of being occupied than others
- Probability of some transitions is greater than others
- Not all transitions are possible (**selection rules**) — e.g. a photon carrying angular momentum cannot be emitted by a transition between two states with zero angular momentum.

Boltzmann equation

So to understand what lines we see, we have to understand how many atoms are in each energy state. This calculation is difficult in general, but can be done if we assume the gas is in **local thermodynamic equilibrium** (LTE). As gas atoms collide, they gain and lose energy, so the distribution of speeds produces a definite distribution of electrons among the atomic orbitals. Orbitals of higher energy are less likely to be occupied by electrons, just as particles are less likely to have high speeds in the Maxwell-Boltzmann distribution.

Boltzmann equation

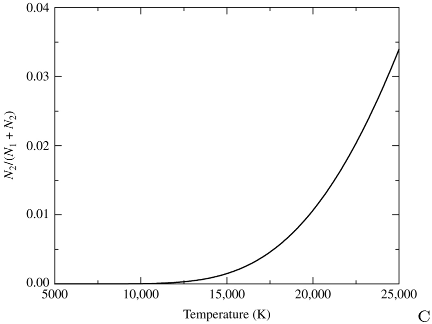
- If n_1 is the number density of atoms in the ground state, then the number density of atoms in an excited state with energy E above the ground state is given by
$$\frac{n_E}{n_1} = e^{-E/kT}$$
- Hence, at temperature T, the populations n_1 and n_2 of any two energy levels are
$$\frac{n_b}{n_a} = \frac{g_b}{g_a} e^{-(E_b - E_a)/kT}$$
where g_a and g_b are the statistical weights of the two levels, which allows for the fact that some energy levels are degenerate. For hydrogen,
$$g_n = 2n^2$$

Example: Hydrogen

- At what temperature will equal numbers of atoms have electrons in the ground state ($n = 1$) and the first excited state ($n = 2$)?

Example: Hydrogen

- Plot the relative occupancy of the ground and first excited state, $n_2/(n_1 + n_2)$, as a function of temperature.



CO fig. 8.7

- **Why** does the strength of the Balmer lines peak in A stars, with $T = 9250$ K, instead of 85,000 K?

Saha equation

- Clearly another factor we need to consider: the relative number of atoms in different stages of ionisation. At equilibrium, we have, for every atomic species, the following balance

$$X \rightleftharpoons X^+ + e^-$$

Saha equation

- Let ξ_i be the ionisation energy needed to remove an electron from an atom in the ground state, taking it from ionisation stage i to stage $i+1$; e.g. $\xi_1 = 13.6$ eV for hydrogen, going from neutral hydrogen (H I) to ionised hydrogen (H II).
- However, if the electron is not **in** the ground state, then less energy is needed, so an average must be taken over all orbital energies, taking into account the different ways the atom can arrange its electrons with the same energy: the **partition function** Z_i, which depends on the Boltzmann factor.

Saha equation

- Saha showed that the ratio of the number of atoms in stage $(i+1)$ to the number of atoms in stage i is

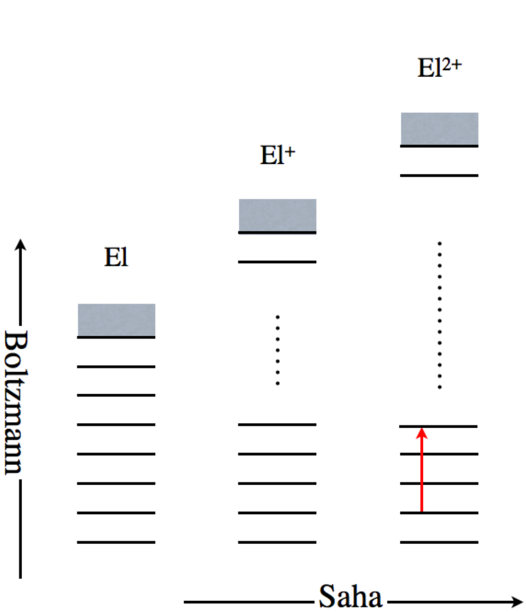
$$\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{n_e Z_i} \left(\frac{2\pi m_e kT}{h^2} \right)^{3/2} e^{-\xi_i/kT}$$

  where
  - N_i is the number density of ions in ionisation state i
  - n_e is the number density of electrons
  - m_e the electron mass
  - ξ_i the ionisation energy from the ground state in ionisation state i

Saha equation

- Z_i and Z_{i+1} are the **partition functions** of ionisation states i and $i+1$: the weighted sum of the number of ways the atom can arrange its electrons with the same energy

$$Z = \sum_{j=1}^{\infty} g_j e^{-(E_j - E_1)/kT}$$

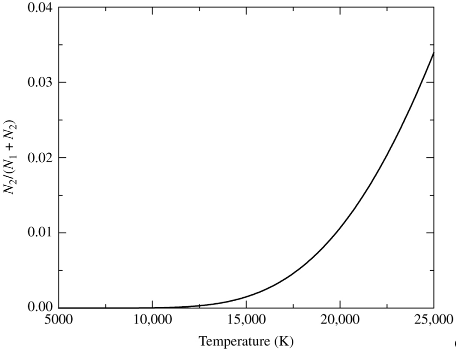

El^{3+}
El^{2+}
El^{+}
El
Boltzmann
Saha

Combining the Saha and Boltzmann equations

- At equilibrium, both the Saha and Boltzmann equation will apply. With many ions and atoms present, the solution of the resulting equation is extremely difficult, unless you make assumptions like a pure H atmosphere and ignoring all but the lowest energy levels. Nowadays the solution is calculated numerically for more realistic compositions and parameters.
- We will investigate solutions to these equations in the first computational lab; here are some results you will be finding.

Combining the Saha and Boltzmann equations

- *Recall*: The Boltzmann equation tells us how many electrons are in a given energy level at a given temperature.
- For hydrogen:



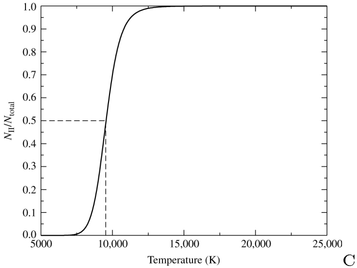
CO fig. 8.7

Combining the Saha and Boltzmann equations

- BUT we *also* need to know how many neutral atoms there are at T, for which we need the Saha equation.
- We need to calculate the fraction of atoms that are ionised

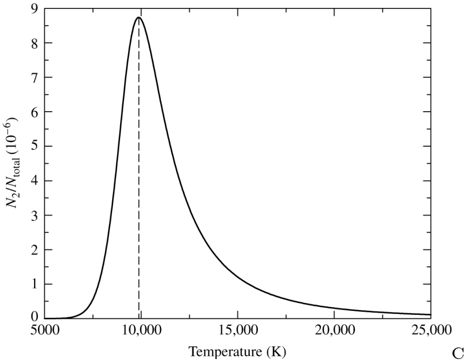
$$\frac{N_{H+}}{N_H} = \frac{N_{H+}}{(N_{H^0} + N_{H+})}$$

- Find: 50% of H is ionised at T $\simeq$ 9600 K



CO fig. 8.8

- So even though **more** of neutral H has electrons in $n = 2$ state at higher temperatures, there is almost no neutral H in the gas at higher temperatures! Combined effect of Boltzmann and Saha equations:

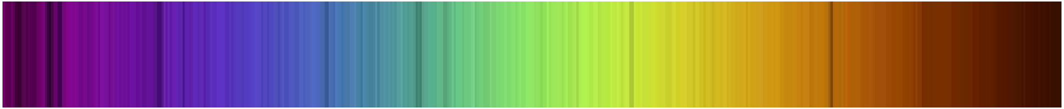


CO fig. 8.9

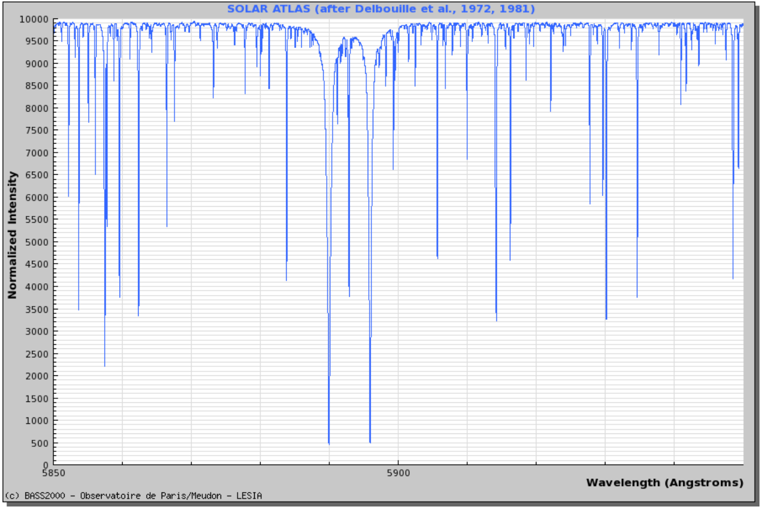
- Hence Balmer lines attain their maximum intensity at $T = 9500$ K

Absorption and emission line spectra

Some stars e.g. the Sun, show dark absorption lines superimposed on a bright continuum.



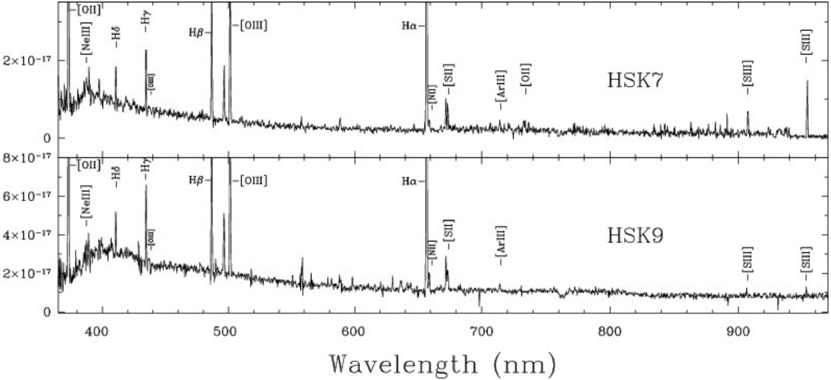
http://bass2000.obspm.fr/solar_spect.php?step=1



Region of the solar spectrum in the region of the strong Na D doublet.

http://bass2000.obspm.fr/solar_spect.php?step=1

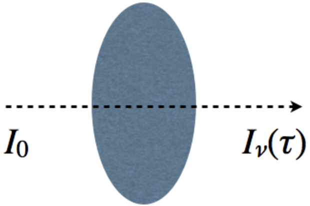
Other stars (also nebulae, quasars etc) show emission line spectra, with spectral lines typically **stronger** than the continuum.



(Magrini et al. 2005 adsabs.harvard.edu/abs/2005A%26A...443..115M)

Spectra of ionised gas around young stars

- Why the difference? Why do we sometimes see absorption and sometimes emission?
- Start from the result we derived earlier, for intensity of radiation after passing through a cloud of gas in thermal equilibrium:



$$I_\nu(\tau_\nu) = I_0 e^{-\tau_\nu} + B_\nu(1 - e^{-\tau_\nu})$$

Optically thin source

- Suppose there is no radiation entering the cloud: $I_0 = 0$, so

$$I_\nu(\tau_\nu) = B_\nu(1 - e^{-\tau_\nu})$$

- If the cloud is very optically thin, τ_ν small, then

$$e^{-\tau_\nu} \approx 1 - \tau_\nu$$

so

$$I_\nu(\tau_\nu) \approx B_\nu(1 - 1 + \tau_\nu) = \tau_\nu B_\nu$$

Optically thin source

Recall that the optical depth τ_ν is related to the absorption coefficient via

$$\tau_\nu = \alpha_\nu \Delta s$$

for constant α. Which means that

$$I_\nu = \tau_\nu B_\nu \propto \alpha_\nu B_\nu$$

The radiation intensity from the cloud is large at frequencies where the absorption coefficient is large

Optically thin source

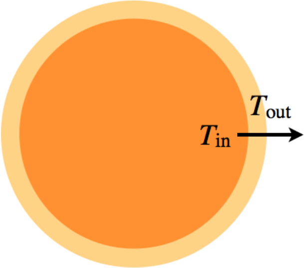
For a hot gas, α_ν is large at the frequencies of the spectral lines
$\rightarrow$ For an optically thin medium (such as a nebula), we expect an **emission line** spectrum with large intensity at the frequencies where α_ν is large

Optically thick source

- Next, consider an optically thick source:
- As we saw, the radiation in the interior is described by the Planck function.
- Radiation escaping from the source will be modified because the temperature (and thus the Planck function) **varies** along the path

Optically thick source

- *Example*: consider a star with two temperatures, the interior at $T_{\rm in}$ and the exterior at $T_{\rm out}$.



Radiation starts from the inner layer as blackbody radiation at temperature $T_{\rm in}$ and escapes through an atmosphere of optical depth τ_ν and temperature $T_{\rm out}$.

Optically thick source

- Use the same solution as before to describe the intensity of the radiation:

$$\begin{array}{lcccc} I_\nu(\tau_\nu) & = & I_0 e^{-\tau_\nu} & + & B_\nu(1-e^{-\tau_\nu}) \\ \downarrow & & \downarrow & & \downarrow \\ \text{escaping} & & B_\nu(T_{\rm in}) & & B_\nu(T_{\rm out}) \\ \text{radiation} & & & & \end{array}$$

- Valid provided that all the gas is in thermal equilibrium (LTE)

Optically thick source

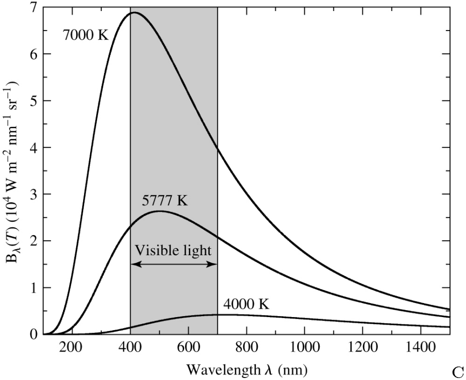
- Assume that optical depth of outer layer is small and use approximate expansion for the exponential as before:

$$\begin{aligned} I_\nu(\tau_\nu) =& B_\nu(T_{\rm in})e^{-\tau_\nu} + B_\nu(T_{\rm out})\left[1 - e^{-\tau_\nu}\right] \\ \approx& B_\nu(T_{\rm in})[1 - \tau_\nu] + B_\nu(T_{\rm out}) \times \tau_\nu \\ =& B_\nu(T_{\rm in}) + \tau_\nu\left[B_\nu(T_{\rm out}) - B_\nu(T_{\rm in})\right] \end{aligned}$$

  - First term is the initial radiation intensity
  - Second term is the **change** in intensity caused by the outer layer. Depends on frequency

$$I_\nu(\tau_\nu) = B_\nu(T_{\text{in}}) + \tau_\nu \left[B_\nu(T_{\text{out}}) - B_\nu(T_{\text{in}})\right]$$

Recall that the intensity of BB radiation increases at **all** frequencies as the temperature goes up



CO fig. 3.8

$\rightarrow$ the **sign** of the second term depends on whether $T_{\text{out}} > T_{\text{in}}$

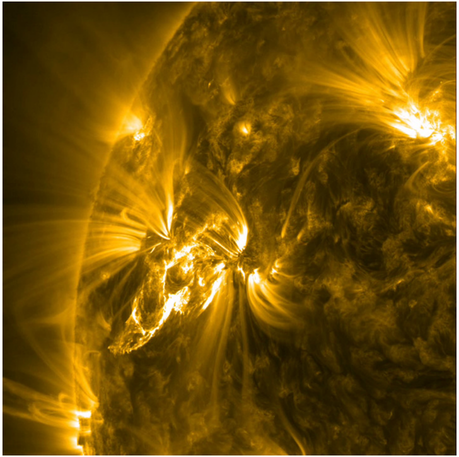
$$I_\nu(\tau_\nu) = B_\nu(T_{\rm in}) + \tau_\nu \left[B_\nu(T_{\rm out}) - B_\nu(T_{\rm in})\right]$$

1. $T_{\rm out} > T_{\rm in}$: second term is positive
   Escaping intensity is **larger** at frequencies where τ_ν is greatest (frequencies corresponding to spectral lines). Expect **emission** lines on top of the continuum
2. $T_{\rm out} < T_{\rm in}$: second term is negative
   Escaping intensity is **reduced** at frequencies where τ_ν is greatest (frequencies corresponding to spectral lines). Expect **absorption** lines superimposed on the continuum

- For the Sun, the temperature near the optical photosphere **decreases** outward (as energy is transported from the centre to the outside), so we are in the second regime: $T_{\rm out} < T_{\rm in}$.
- Expect to see an **absorption spectrum**, as observed

http://bass2000.obspm.fr/solar_spect.php?step=1

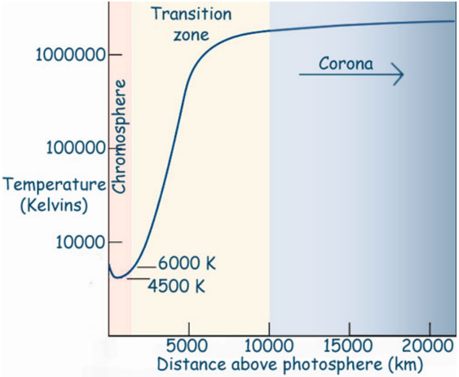
- In the X-ray and UV wavebands, however, we see strong emission from the Solar corona, so obviously the temperature there is much hotter than that of the photosphere.



UV emission from a large sunspot group from Jan 2013

www.sdo.gsfc.nasa.gov/gallery/main.php?v=item&id=179

- UV radiation comes from the region where T is increasing → emission lines
- Optical radiation comes from the region where T is decreasing → absorption lines



www.cseligman.com/text/sun/sunatmosphere.htm

Summary

- Saha and Boltzmann
  - Boltzmann equation describes distribution of atoms in different *energy* states
  - Saha equation describes distribution of atoms in different *ionisation* states
  - Combination produces lines at particular temperatures
- Emission line spectra
  - Optically thin gas with no background light
  - Optically thick gas in which T increases outward
- Absorption line spectra
  - Cold gas lies in front of a source of radiation at a higher temperature

Next lecture

- we begin Part II, Stellar structure
- The **following** session (Friday 12pm) will be held in

SNH Learning Studio 4003

where we will be exploring the Saha-Boltzmann equation.