Lecture 10: Stellar remnants

Senior Astrophysics

2018-04-10

Outline

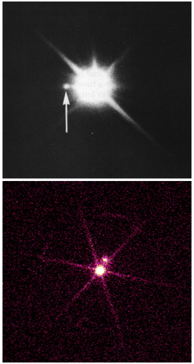
End states of stellar evolution

We have seen that the mass of the star determines the way it evolves, and hence also the final outcome of its evolution. Depending on the star's mass, a different remnant will be left behind.

M	Remnant
$< 0.5M_\odot$	$\tau >$ Hubble time
$0.5 < M/M_\odot < 8$	WD + planetary nebula
$\sim 8 < M/M_\odot < ?$	core collapse + SN $\rightarrow$ NS or BH (?)

White dwarfs

- First white dwarf discovered: Sirius B. Mass was known from the orbit: $M_B \sim 1.05 M_\odot$ (compared with $2.3 M_\odot$ for Sirius A)
- 1000 times fainter than Sirius A, but much hotter: $T \sim 27,000$ K compared with 9900 K. This implied a radius (from the Stefan-Boltzmann law) of $R \sim 5.5 \times 10^6$ m $= 0.008 R_\odot$, or about the size of the Earth.
- Estimate the density: $\rho \sim 10^9 \text{ kg m}^{-3}$!!
- It is estimated that there are about 35 billion white dwarfs in the Galaxy, possibly as many as 100 billion. They are by far the most common stellar remnant in the Galaxy ($\sim 97\%$ of evolved stars).



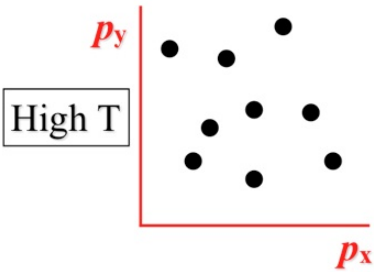
Optical (top) and X-ray (bottom) images of Sirius A and B, which are an A-type star and a white dwarf. The optical image is dominated by the main sequence star, the X-ray image by the white dwarf.

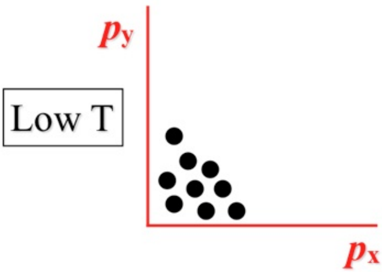
Degeneracy pressure

- White dwarfs are no longer producing energy in their interiors, so they cannot support themselves against gravity using gas pressure.
- Fowler and Chandrasekhar in the 1920s suggested they are supported by *degeneracy pressure*.

Degeneracy pressure

- For an ordinary classical gas, $P_{\text{gas}} \propto T \to 0$ as $T \to 0$. Also, the mean particle speed $v = \sqrt{2kT/m} \to 0$.
- Now, the gas particles have momentum $p_x = mv_x$, $p_y = mv_y$, $p_z = mv_z$, so as T decreases the particles concentrate near the origin in p_x, p_y, p_z–space:





Degeneracy pressure

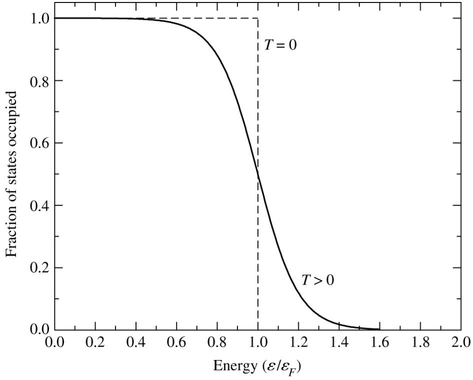
At low enough temperatures / high enough densities, the concentration of particles with similar (low) momenta would violate the Pauli exclusion principle, that no two electrons can occupy the same quantum state. This means that electrons in a dense, cold gas must have larger momentum than we would predict classically. Then, since the pressure depends on the momentum of particles

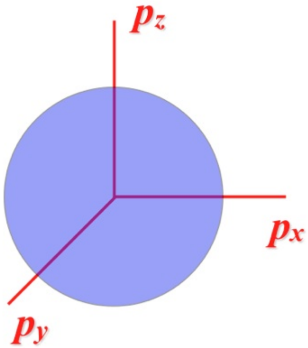
$$P = \frac{1}{3} \int_0^\infty n_p p v \, dp$$

this means that such gas has an extra source of pressure, which we call **degeneracy pressure**.

Derivation of degeneracy pressure

Consider the simplest case, of a gas of electrons at zero temperature, so all the quantum states up to some momentum p_{F} are occupied, but no states higher than p_{F} are occupied.





Derivation of degeneracy pressure

- Consider a group of electrons $N_p \delta p$ with momenta in the range $p + \delta p$. The volume of momentum space occupied by these electrons is the volume of a shell of radius p, thickness δp: $4\pi p^2 \delta p$.
- Heisenberg's principle says that the number of electrons in a volume of phase space h^3 must be at most 2, so the number of quantum states, per unit volume, with momenta in the range $p + \delta p$ is

$$\begin{aligned} n_e(p)\,dp &= \frac{2}{h^3} 4\pi p^2\,dp & \qquad p \leq p_{\mathrm{F}} \\ &= 0 & \qquad p \geq p_{\mathrm{F}} \end{aligned}$$

Derivation of degeneracy pressure

The total number of electrons per unit volume is found by integrating over all possible momenta:

$$n_e = \int_0^\infty n_e(p)\,dp = \frac{8\pi}{h^3}\int_0^{p_\mathrm{F}} p^2\,dp = \frac{8\pi p_\mathrm{F}^3}{3h^3}$$

Rearrange this to find the maximum (or Fermi) momentum:

$$p_\mathrm{F} = \left(\frac{3h^3 n_e}{8\pi}\right)^{1/3}$$

Derivation of degeneracy pressure

- Hence the pressure of this degenerate gas of electrons is

$$
\begin{aligned}
P &= \frac{1}{3}\int_0^{\infty} v\,p\,n_e(p)\,dp \\
&= \frac{1}{3}\int_0^{p_{\mathrm{F}}} \left(\frac{p}{m_e}\right) p\,\frac{2}{h^3} 4\pi p^2\,dp
\end{aligned}
$$

or

$$
P_{\mathrm{deg}} = \frac{8\pi}{15 m_e h^3} p_{\mathrm{F}}^5 = \frac{h^2}{20 m_e}\left(\frac{3}{\pi}\right)^{2/3} n_e^{5/3}
$$

the **degeneracy pressure**.

- Note that this depends only on **fundamental constants** and the **density**.

Degeneracy pressure

- Degeneracy pressure, non-relativistic:

$$P_{\mathrm{deg}} = \frac{h^2}{20 m_e} \left(\frac{3}{\pi}\right)^{2/3} n_e^{5/3}$$

- For a relativistic degenerate gas ($p \gg m_e c$; $v \to c$ and $p \to \infty$), the corresponding pressure is

$$P_{\mathrm{deg}} = \frac{1}{8} \left(\frac{3}{\pi}\right)^{1/3} hc\, n_e^{4/3}$$

Degeneracy pressure

We need to convert the electron density n_e to mass density ρ: For each H atom (mass m_{H}) there is one electron, while for heavier elements there is $\sim \frac{1}{2}e^-$ for each m_{H}. Thus

$$n_e = \frac{\rho X}{m_{\mathrm{H}}} + \frac{\rho(1-X)}{2m_{\mathrm{H}}} = \frac{\rho(1+X)}{2m_{\mathrm{H}}}$$

where X is the hydrogen fraction.

Degeneracy pressure

Thus we get

non-relativistic

$P_{\rm deg} = K_1 \rho^{5/3}$

where

$$K_1 = \frac{h^2}{20 m_e} \left(\frac{3}{\pi}\right)^{2/3} \left(\frac{1+X}{2m_{\rm H}}\right)^{5/3}$$

relativistic

$P_{\rm deg} = K_2 \rho^{4/3}$

$$K_2 = \frac{hc}{8} \left(\frac{3}{\pi}\right)^{1/3} \left(\frac{1+X}{2m_{\rm H}}\right)^{4/3}$$

Most importantly, the pressure does **not** depend on temperature; it depends only on the density and chemical composition.

Degeneracy pressure

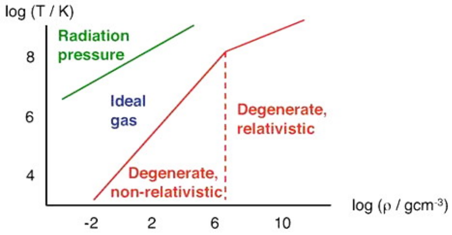
Degeneracy is important for “cold” gases: when the Fermi momentum p_{F} is much larger that the (classical) momentum from thermal motion

$$m_e v = (2m_e E)^{1/2} = (3m_e kT)^{1/2}$$

i.e. when

$$kT \ll \frac{h^2 n_e^{2/3}}{m_e}$$

So degeneracy depends on density as well as temperature.
(Note that “cold” gases can have quite high T!)



Mass-radius relation

Since white dwarfs are supported by degeneracy pressure, we can find the relation between the mass M_{wd} of a white dwarf and its radius R_{wd} by setting our crude estimate for the central pressure of a star (from lecture 5) equal to the degeneracy pressure: Recall from lecture 5 that we used the equation of hydrostatic equilibrium and assumed the star had constant density to derive an expression for the central pressure

$$P_c \sim \frac{3GM^2}{4\pi R^4}$$

Set this estimate equal to the degeneracy pressure

$$\frac{3GM^2}{4\pi R^4} = K_1 \rho^{5/3} = K_1 \left(\frac{M}{\frac{4}{3}\pi R^3} \right)^{5/3}$$

Mass-radius relation

Hence

$$R \propto M^{-\frac{1}{3}}$$

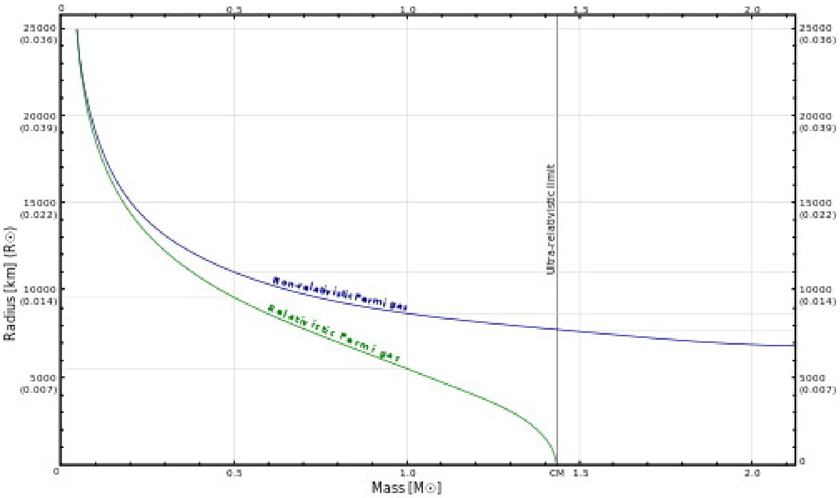
i.e. more massive white dwarfs are smaller.
This in turn implies that for a white dwarf

$$MV = \text{constant}$$

Because the white dwarf is supported by electron degeneracy pressure, the only way to provide more support for a larger star is to confine the electrons more closely. In fact, the mass–volume relation implies that $\rho \propto M^2$.

Mass-radius relation

- However, the higher the density, the more relativistic the electrons become. The electron speed approaches c ($p_{\mathrm{F}} \gg m_e c$), so there is less pressure available to counteract the mass, so massive white dwarfs have *smaller* radii than predicted by the mass-volume relation. In fact, as the mass increases the radius goes to *zero* for a finite value of the mass: there is a maximum mass that can be supported by electron degeneracy pressure.
- This limiting mass is called the Chandrasekhar mass.


Ultra-relativistic limit
Non-relativistic Fermi gas
Relativistic Fermi gas
Radius [km] (R⊙)
Mass [M⊙]
25000 (0.036)
20000 (0.039)
15000 (0.022)
10000 (0.014)
5000 (0.007)
0
0.5
1.0
CM 1.5
2.0

The Chandrasekhar mass

The limiting (Chandrasekhar) mass is the limit when the electrons are completely relativistic, so just equate the central pressure with the degeneracy pressure for relativistic electrons:

$$\frac{3GM^2}{4\pi R^4} = K_2\rho^{4/3} = K_2\left(\frac{M}{\frac{4}{3}\pi R^3}\right)^{4/3}$$

R cancels out, and we get an expression for M which depends only on m_{H} and fundamental constants. These constants — h, c and G — represent the effects of quantum mechanics, relativity and Newtonian gravitation.

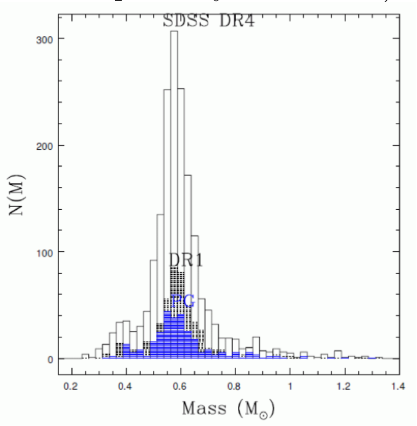
$$M_{\mathrm{Ch}} \sim \frac{3\sqrt{\pi}}{16}\left(\frac{\hbar c}{G}\right)^{3/2}\left(\frac{1+X}{2m_{\mathrm{H}}}\right)^2 = 0.44\,M_\odot$$

A more accurate calculation gives $M_{\mathrm{Ch}} = 1.44 M_\odot$.

The Chandrasekhar mass

The Chandresekhar mass is independent of the radius; in fact, it depends only on fundamental constants. In other words, a completely relativistic white dwarf has a unique mass. (Real white dwarfs are partially relativistic).

- This turns out to be the maximum mass a white dwarf can have.
- The measured masses of white dwarfs are strongly peaked at $M \sim 0.6 M_{\odot}$; the highest mass WD measured is $1.33 M_{\odot}$.



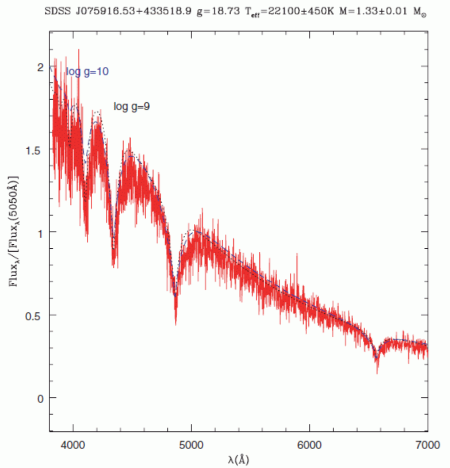


Figure 10. Spectrum of SDSS J075916.53+433518.9 with $g = 18.73$, $T_{\rm eff} = 22\,100$ K and two models, with $\log g = 9$ and 10. The higher $\log g$ fits the Hα line better, but the lower $\log g$ fits the higher lines better, where the SNR is smaller. This low SNR is typical for the stars closer to our upper cut-off of $g = 19$.

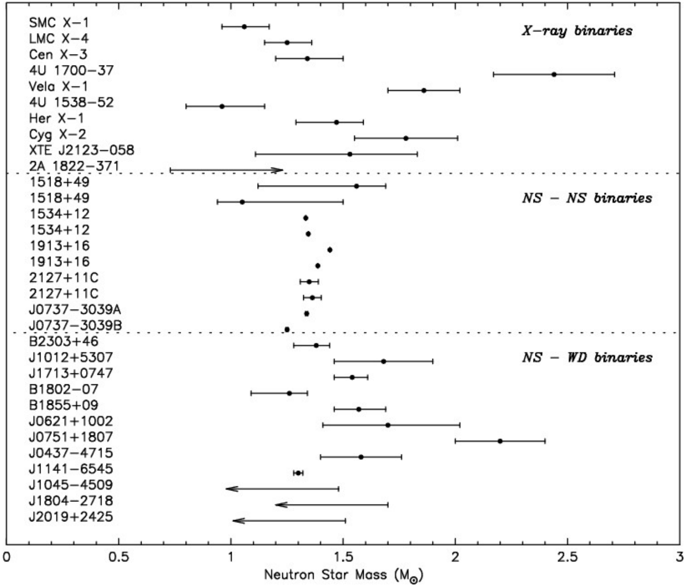
(g = log of surface gravity in cm s^{-2}); from Kepler et al. MNRAS 375 1315 (2007)

Neutron stars

- We have seen that a neutron star forms from the collapsing core of a massive star.
- If the mass of the remnant exceeds the Chandrasekhar mass $M_{\rm Ch}$, electron degeneracy pressure cannot support the star. Instead, neutron stars are supported by **neutron degeneracy pressure**: since neutrons are also fermions, their behaviour is analogous to the behaviour of electrons.
- We can estimate the radius of the neutron star using the same argument we used for an electron gas: a $1.4 M_\odot$ neutron star has $R \sim 10$–15 km, $\rho \sim 6 \times 10^{17}\,\mathrm{kg\,m^{-3}} \sim$ nuclear density. A neutron star is like a nucleus with mass number $A \approx 10^{57}$.

Neutron star mass limit

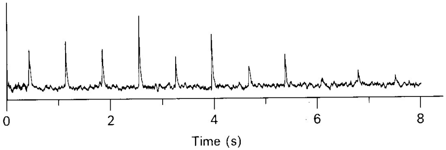
- There is also an upper limit to the mass of a neutron star, corresponding to the Chandrasekhar mass. Above this mass, neutron degeneracy pressure is unable to balance the neutron star's self-gravity. However, the exact value is not well known; the details depend on GR and the details of the equation of state. The upper limit is less that $\sim 3M_\odot$; above this limit, the neutron star will collapse to a black hole.
- Observed masses for neutron stars are typically very close to the Chandrasekhar mass; the most massive pulsar has a mass $1.97 \pm 0.04 M_\odot$ (Demorest et al. 2010).
- It is not clear if all neutron stars are born at the Chandrasekhar mass, or some are born massive.



van der Meer et al., A&A 473, 523 (2007)

Observations of neutron stars

- Since neutron stars are so small, how do you find them?
- Serendipitous discovery as pulsating radio sources by Jocelyn Bell in 1967.
- Rapid regularly spaced pulses with $P = 1.337\,\mathrm{s} \rightarrow$ source must be **small**.

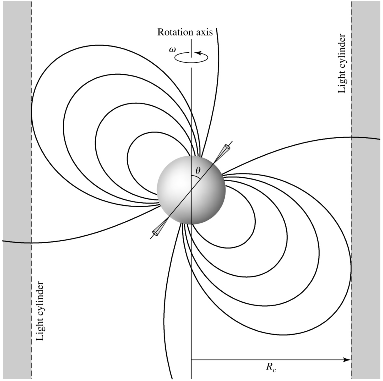


Individual pulses from PSR 0329+54, with P=0.714s

Pulsars are rotating neutron stars

- Pulsars are rapidly rotating NS with strong magnetic field. e– are accelerated along magnetic field lines, and radiation is beamed in the acceleration direction.

The magnetic field not aligned with rotational axis → get flash of radio waves once per spin period ("lighthouse model"). Only see pulsars beamed towards us (f = 10–20%?)



Black holes

- The theory of black holes is based entirely on General Relativity, and is beyond the scope of this course. However, we can make some simple arguments:
- In Newtonian gravity, the escape speed for a particle from an object with mass M and radius R is

$$v_{\mathrm{esc}} = \sqrt{\frac{2GM}{R}}$$

- This becomes greater than c at a radius

$$R_{\mathrm{S}} = \frac{2GM}{c^2} = 3\ \mathrm{km}\frac{M}{M_\odot}$$

- the **Schwarzschild radius**. Remarkably, this is the same expression as given by GR.

Finding black holes

- We can only find black holes via their gravity, so binary star systems are an ideal hunting ground. We will talk about binaries for the next few lectures.

Next lecture

- Binary stars
- Observed characteristics
- Interacting binaries
- Gravity in a rotating reference frame