Lecture 11: Binary stars

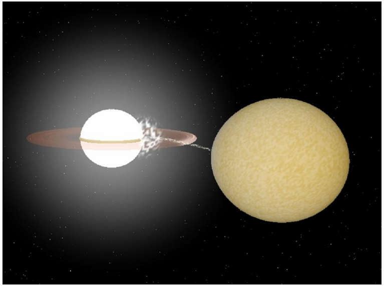
Senior Astrophysics

2018-04-11

Outline

Binary stars

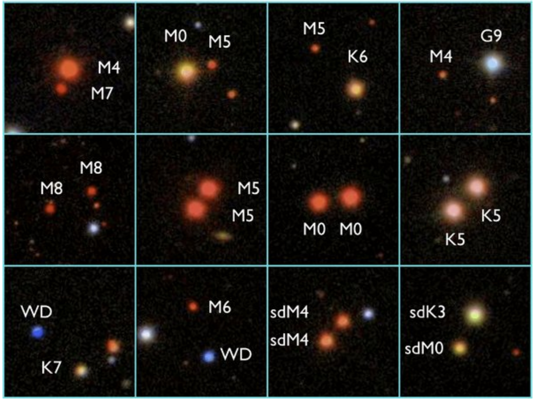
- More than half of all stars in the sky are in multiple-star systems, with two or more stars in orbit around a common centre of mass.



- Shortest period: HM Cancri, two white dwarfs with $P = 5.4$ minutes, $a = 8\,R_{\text{Earth}}$

Binary stars

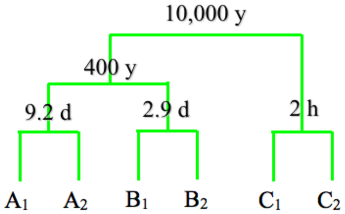
- Longest period: ??
  - Proxima Cen: $P = 500,000$ y
  - Common proper motion systems: bound?

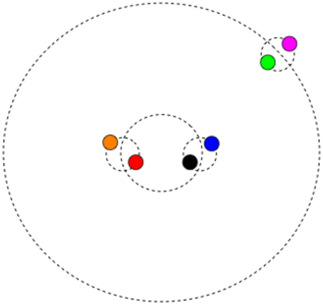


Common proper motion pairs found in the SLoWPoKES survey. From Dhital et al. 2010, AJ 139 2566

Multiple systems

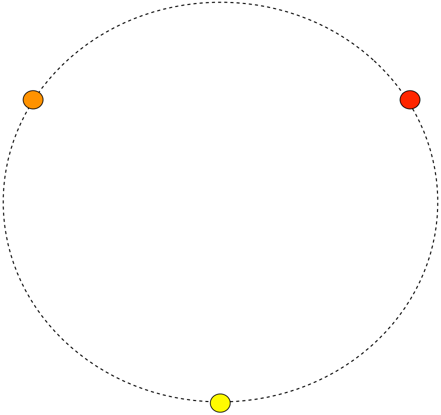
Multiple star systems are also common, containing three or even more stars. For example, Castor (α Geminorum), is in fact a sextuple star system, consisting of an inner pair of binaries, with a *third* binary orbiting around the inner pair of binaries. Multiple systems are always **hierarchical**.





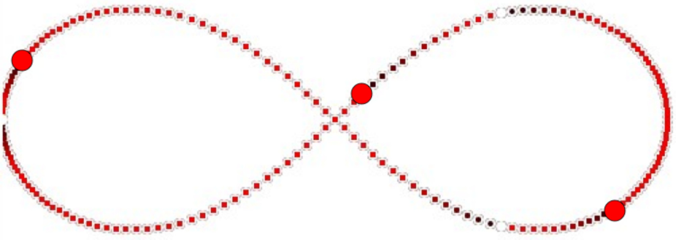
Other configurations

Other orbits are theoretically possible, e.g. the Lagrange configuration, with three objects orbiting in an equilateral triangle



Other configurations

Recently even more bizarre orbits have been found to be stable.



However, it's hard to see how they could form!
[see `www.maths.manchester.ac.uk/~jm/Choreographies/`]

Distribution

- The distribution of masses and periods in binaries is very uncertain. The best guess for stars in the neighbourhood of the Sun is that at least half have at least one companion.
- However, this may not be true for stars much more or less massive than the Sun: there is evidence that only 40% of M stars have a binary companion. Conversely, between 70% and 100% of O and B stars appear to have companions. However, the masses of the companion stars are strongly skewed towards also being massive: there are few massive stars with low mass companions.

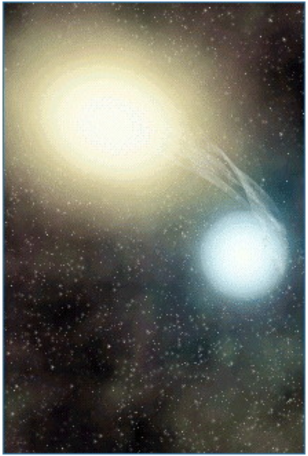
Interacting binaries

- If the two stars are far enough apart, they will have little or no effect on each other.
- However, if they are close, then stellar evolution can have a major effect as each star's radius changes with time.
- The observed properties of some binaries are inexplicable without taking this into account.

  e.g. some compact binaries, containing a white dwarf, have orbital periods $P < 2$ hours, implying orbital separations $a < R_\odot$.

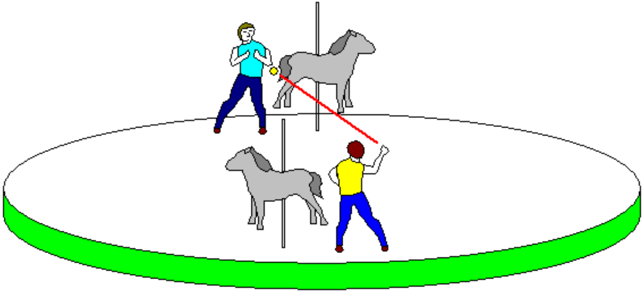
The Algol paradox

e.g. Algol, which consists of a main-sequence B star with $M = 3.5M_\odot$, plus a giant K star, with $M = 0.81M_\odot$.



- How can the less massive star be further advanced in its evolution?
- By the end of this section of the course, you will be able to answer this.

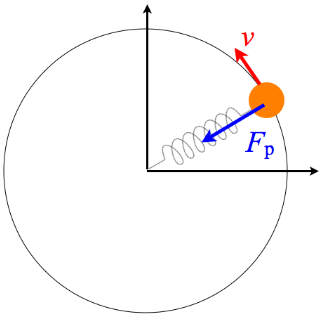
Gravity in a rotating reference frame: Motivation



from https://www.uwgb.edu/dutchs/EarthSC102Notes/102TheOceans.HTM

Gravity in a rotating reference frame: Motivation

Object rotating at ω (= stationary in rotating frame):



rotating frame

Gravity in a rotating reference frame: Motivation

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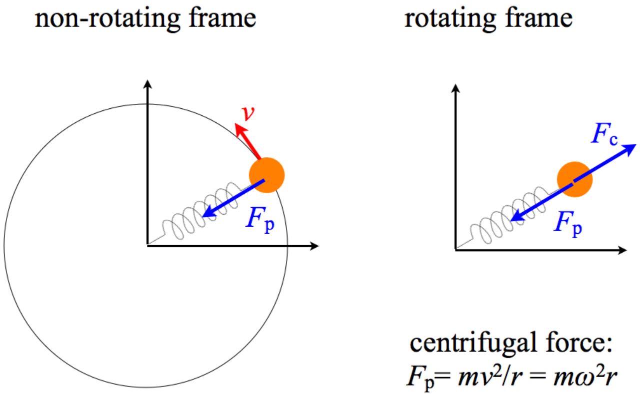
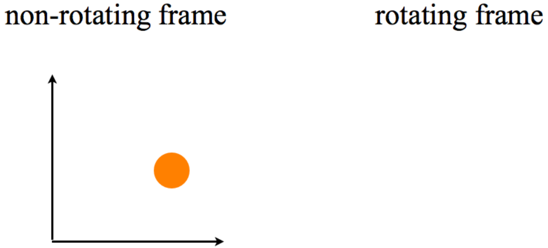


Gravity in a rotating reference frame: Motivation

Stationary object (= rotating at $-\omega$ in rotating frame):



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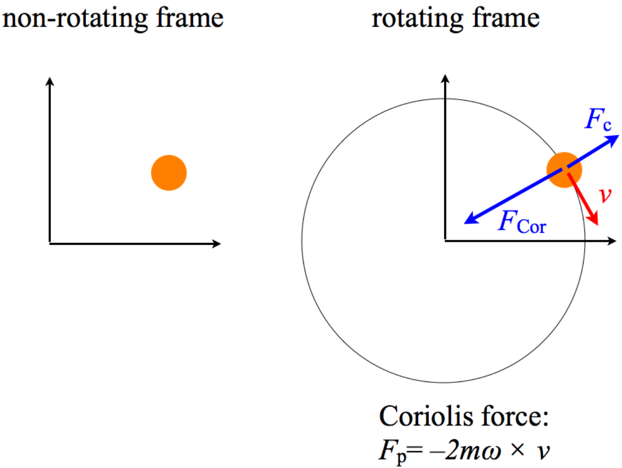


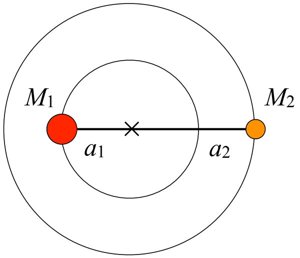
Gravity in a rotating reference frame

- Consider two stars orbiting their mutual centre of mass in x, y plane with angular velocity $\omega_1 = \frac{v_1}{a_1}$, $\omega_2 = \frac{v_2}{a_2}$, where v_1 and a_1 are the orbital speed and distance from the CoM for star 1, etc.

- From Kepler's 3rd law,

$$\left(\frac{P}{2\pi}\right)^2 = \frac{a^3}{G(M_1 + M_2)}$$

where $a = a_1 + a_2$.



Rotation properties

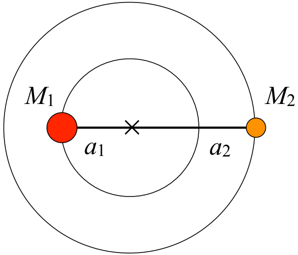
- Centre of mass distances are related by

$$a_1 = \frac{M_2}{M_1 + M_2} a, \quad a_2 = \frac{M_1}{M_1 + M_2} a$$

so $\quad \frac{a_1}{a_2} = \frac{M_2}{M_1}$

(simple CoM arguments)

- velocities: $v_i = \frac{2\pi a_i}{P}$ for each star



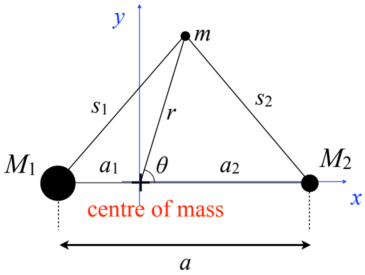
Gravity in a rotating reference frame

- We choose a *rotating* coordinate frame with period P and the CoM at the origin; then the stars are at rest at $-a_1$ and a_2.
- We introduce a centrifugal force ("push") to balance the mutual gravitational force ("pull").

- The centrifugal force on a particle with mass m at a distance r from the origin is

$$\mathbf{F}_c = m\omega^2 r\hat{\mathbf{r}}$$

in the outward radial direction.



Potential energy

- Now look at the gravitational potential energy: the work done in moving a particle from $r = \infty$ to r from a mass M is

$$U_g = \int_r^\infty G\frac{Mm}{r^2}dr = -G\frac{Mm}{r}$$

where we have assumed U_g goes to zero at infinity. We need to introduce a fictitious "centrifugal potential energy", setting ΔU_c to be the work done in moving from $\mathbf{r}_i$ to $\mathbf{r}_f$:

$$\Delta U_c = U_f - U_i = -\int_{r_f}^{r_i} m\omega^2 r\, dr = -\frac{1}{2}m\omega^2(r_f^2 - r_i^2)$$

Since our choice of zero-point is arbitrary, set $U_c = 0$ at $r = 0$, so

$$U_c = -\frac{1}{2}m\omega^2 r^2$$

Potential energy

- So for a particle with test mass m in the plane of the orbit, the effective potential energy in this frame is

$$U = -G\left(\frac{M_1 m}{s_1} + \frac{M_2 m}{s_2}\right) - \frac{1}{2} m\omega^2 r^2$$

Divide by the test mass m to obtain the **effective gravitational potential**, Φ:

$$\Phi = -G\left(\frac{M_1}{s_1} + \frac{M_2}{s_2}\right) - \frac{1}{2}\omega^2 r^2$$

the effective potential energy per unit mass.

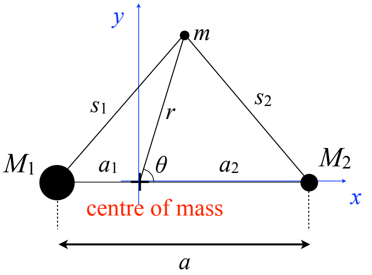
Potential energy

Note that

$$s_1^2 = a_1^2 + r^2 + 2a_1 r \cos\theta$$
$$s_2^2 = a_2^2 + r^2 - 2a_2 r \cos\theta$$

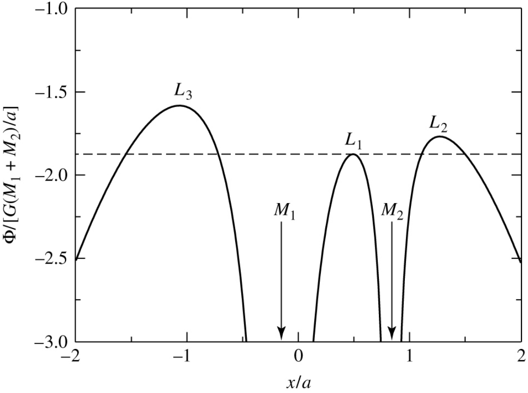
and from Kepler's laws,

$$\omega^2 = \left(\frac{2\pi}{P}\right)^2 = \frac{G(M_1 + M_2)}{a^3}$$



Lagrange points

- We can now evaluate the effective gravitational potential Φ at every point in the orbital plane.
- Along the x-axis, the potential has three peaks.



Lagrange points

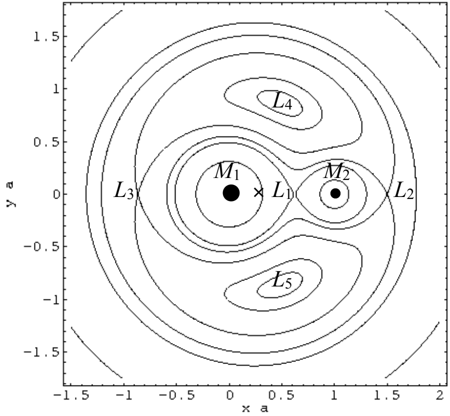
- Recall that the x-component of the force on a small test mass is

$$F_x = -\frac{dU}{dx} = -m\frac{d\Phi}{dx}$$

- The three peaks are Lagrange points, where there is no force on the test mass ($d\Phi/dx = 0$).

Equipotential surfaces

- Plotting Φ at every point in the orbital plane defines *equipotential surfaces*, which share the same value of Φ.



Equipotential surfaces

- Since the force is perpendicular to these lines, equipotential surfaces are level surfaces for stars. If a star changes its radius, it will expand to fill successively larger equipotential surfaces.
- Near to each star, these surfaces are nearly spherical and centred on each mass. Farther away, the surfaces distort into tear-drop shapes, until they touch at the inner Lagrange point L_1. Still further away, the surfaces become a dumbbell shape surrounding both stars.

Roche lobes

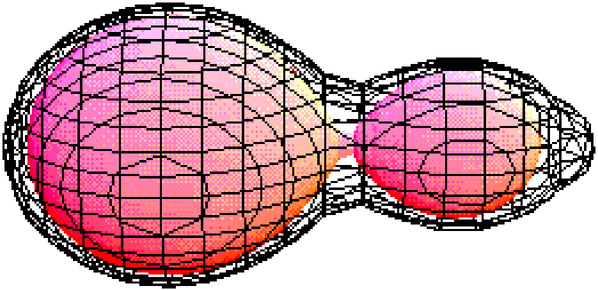
- The tear-drop shaped regions of influence around each star are called **Roche lobes**.
- The size of the Roche lobes depends on the mass ratio $q = M_2/M_1$ and the semi-major axis a.
- The relative size of the two Roche lobes depends only on the mass ratio.
- Paczyński (1967) found a useful approximation for the radius of the Roche lobe of the secondary star

$$R_{L2} = 0.46a \left(\frac{M_2}{M_1 + M_2} \right)^{1/3}$$

which is accurate to within 2% provided $q < 0.8$; this condition typically holds in e.g. X-ray binaries.

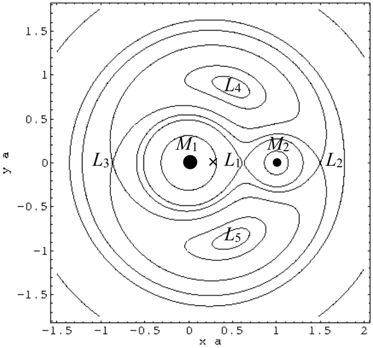
Roche lobes

- Keep in mind that equipotentials are defined in three dimensions, so Roche lobes are three-dimensional. Roche surfaces *exist* for any pair of stars; they define the region where each star's gravity is dominant.



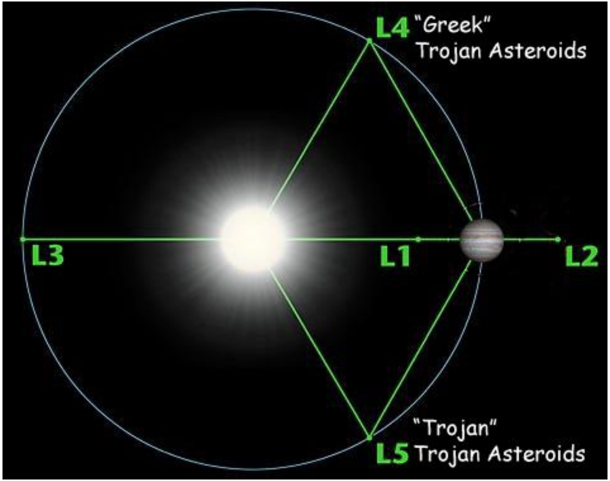
Lagrange points

- The five points labelled L_1–L_5 are the Lagrange points. These are the five places in the vicinity of two large orbiting masses where a small body can orbit at a fixed distance from the larger masses.
- L_1, L_2 and L_3 lie on the stars' line of centres, and are points of *unstable equilibrium* (saddle points of the potential).



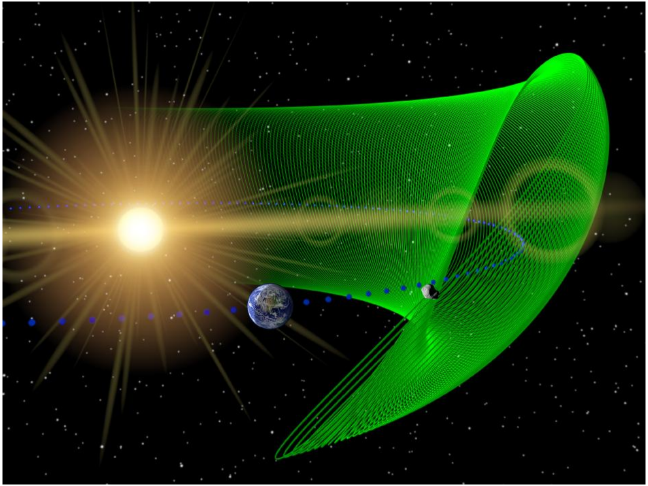
- L_4 and L_5 are local maxima, and hence unstable. However, for particular mass ranges of the large bodies, small objects will orbit around the L_4 and L_5 points.

Lagrange points



Location of the Trojan asteroids in the Jupiter-Sun system, from http://cseligman.com/text/asteroids/trojan.htm

Lagrange points



First Earth Trojan asteroid, discovered by the WISE satellite
http://www.nasa.gov/mission_pages/WISE/news/wise20110727.html

Next lecture

- Lab 3 on Friday in SNH Learning Studio: WTTS evolution of massive stars *(also at 10am today)*

then

- Next lecture: Accretion
  - Classes of binary stars
  - Accretion energy
  - The Eddington limit
  - Accretion disks