Lecture 2: Astrophysical Acceleration Processes I

Particle acceleration mechanisms can be classified as *dynamic*, *hydrodynamic* and *electromagnetic*, although there is no clear distinction between these because the dynamics of charged particles are ultimately governed by electromagnetic fields, which pervade the entire Universe. Nevertheless, in some cases it is clear that magnetic fields, whilst present, are not dynamically important. We will now consider a standard astrophysical particle acceleration process known as *Fermi acceleration* which can operate effectively in the absence of magnetic fields. Fermi made the first serious attempt at explaining the power law nature of the cosmic ray spectrum. He noted that if cosmic rays are injected steadily into a localised acceleration region where they gain energy at a rate that is proportional to their energy while at the same time their escape from the region is an energy-independent Poisson process, then the stationary particle distribution will always be a power law. Fermi also argued that the most efficient acceleration mechanism would be a stochastic one.

2.1 Some Kinetic Background Theory

High-energy astrophysical sources contain plasmas, which consist of charged particles. These charged particles can have a distribution of momenta $\mathbf{p}$ and this distribution can be a function of 3D space $\mathbf{x}$ and time t. The distribution function is normally given by the parameter

$$f(\mathbf{x}, \mathbf{p}, t) \qquad \text{particle distribution function} \tag{1}$$

This is also sometimes referred to as a *phase space density* because f gives the number of particles in the 6-dimensional phase space defined by $\mathbf{p}$ and $\mathbf{x}$ at a given t. The *number density* of particles is the number of particles per unit volume at a particular location $\mathbf{x}$ and is defined by

$$n(\mathbf{x}, t) = \int f(\mathbf{x}, \mathbf{p}, t)\, \mathrm{d}\mathbf{v} \qquad \text{particle number density} \tag{2}$$

where $\mathrm{d}\mathbf{v} = dv_x dv_y dv_z$. Particle conservation requires that the rate of change of the number of particles per unit time per unit volume is equal to the flux of particles across the surface of the volume (in the absence of sources or sinks). This conservation law is given by the following:

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \frac{\partial}{\partial \mathbf{p}} \left(\frac{d\mathbf{p}}{dt} f \right) = 0 \qquad \text{Vlasov equation} \tag{3}$$

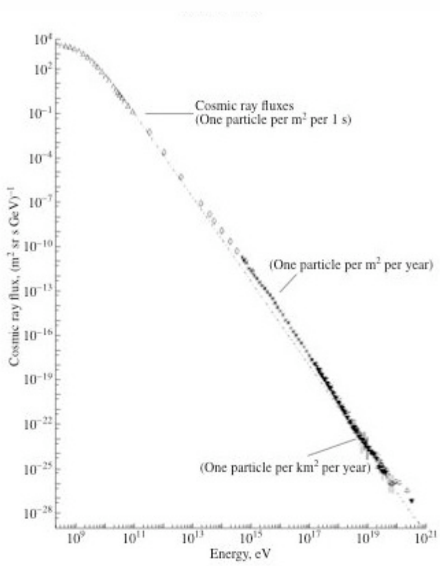
which is sometimes also referred to as the *collisionless Boltzmann equation* because Coulomb collisions are assumed to be negligible. If electrostatic interactions between particles are important, then inclusion of a Coulomb collision term gives the Boltzmann equation. Under steady-state conditions, the Boltzmann equation gives the following well-known solution to the particle distribution function:

$$f(\mathbf{p}, \mathbf{x}) = n \left(\frac{m}{2\pi kT} \right)^{3/2} \exp\left(-\frac{|\mathbf{p} - \langle \mathbf{p} \rangle|^2}{2mkT} \right) \qquad \text{Maxwellian distribution} \tag{4}$$

This is the distribution function for particles in thermal equilibrium. The mean kinetic energy is $\varepsilon = \frac{1}{2}kT$ per particle per degree of freedom.

In many powerful astrophysical sources, large amounts of energy appear to be dissipated into the ambient medium over a relatively short timescale. If the gas density is high, then Coulomb collisions between electrons and ions will efficiently thermalise the plasma. The dissipated energy ends up distributed equally amongst the particles, forming a Maxwellian distribution. This process is referred to as plasma *heating*.

In this course, we will focus on particle *acceleration*, rather than particle heating. This refers to energy dissipation processes in which a minority of particles end up with the majority of the available energy. This may occur when the dissipation mechanism is fast enough and the plasma diffuse enough that there is not enough time for thermalisation (via Coloumb collisions) to redistribute the energy equally amongst the particle population. The resulting particle energy distribution is *nonthermal* and is usually a power-law: $n(\varepsilon) \propto \varepsilon^{-p}$. The cosmic ray spectrum provides direct observational evidence that efficient cosmic particle accelerators exist. The challenge is to find a specific acceleration mechanism that can explain the power law slope of this spectrum: $p \approx 2.5$.



Cronin, Gaisser & Swordy, Sci. Am. 276, Jan. 1997, p. 44

2.2 Fermi Acceleration

In 1949, Enrico Fermi proposed the first serious acceleration mechanism in an astrophysical context. He proposed that galactic cosmic rays are accelerated in the interstellar medium as a result of many collisions with massive, magnetised clouds which act as a scattering center, like cosmic billiard balls (see movie at `spacephysics.ucr.edu`).

Consider a particle of momentum $\mathbf{p}$ and energy ε that collides with a massive cloud moving with a random velocity $\mathbf{V}$. In the centre of momentum frame of the cloud, the component of the particle's initial relativistic three-momentum parallel to the direction of $\mathbf{V}$ is

$$p'_{\|i} = \Gamma \left(p_{\|i} - \frac{\varepsilon_i V}{c^2} \right)$$

where $\Gamma = (1 - V^2/c^2)^{1/2}$ is the Lorentz factor of the cloud. In this frame, the collision is elastic, so the final parallel component of the particle's 3-momentum is $p'_{\|f} = -p'_{\|i}$ and it's final energy is equal to its initial energy:

$$\varepsilon'_f = \varepsilon'_i = \Gamma(\varepsilon_i - V p_{\|i})$$

Transforming back to the lab (observer) frame, the final energy of the particle is

$\varepsilon_f = \Gamma(\varepsilon'_f + p'_{\|f} V) = \Gamma(\varepsilon'_i - p'_{\|i} V)$. Substituting the expressions for ε'_i and $p'_{\|i}$ given above then yields

$$\varepsilon_f = \Gamma^2 \left[\left(1 + \frac{V^2}{c^2} \right) \varepsilon_i - 2 \left(\mathbf{V} \cdot \mathbf{p}_i \right) \right]$$

writing $\mathbf{p}_i = \varepsilon_i \mathbf{v}_i / c^2$, the observed change in particle energy is thus

$$\Delta\varepsilon = \varepsilon_f - \varepsilon_i = 2\Gamma^2 \left(\frac{V^2}{c^2} - \frac{\mathbf{V} \cdot \mathbf{v}_i}{c^2} \right) \varepsilon_i \tag{5}$$

Note:

1. For head-on collisions, $\mathbf{V} \cdot \mathbf{v}_i < 0$ and the particle gains energy.
2. For overtaking collisions, $\mathbf{V} \cdot \mathbf{v}_i > 0$ and the particle loses energy.

So after many collisions, is there a nett energy gain? Yes, because there is a higher rate of head-on collisions than overtaking collisions.

Consider N scattering centres (clouds) per unit volume with collisional cross section σ. The rate of encounters for a given direction of $\mathbf{V}$ (but with $\Gamma \simeq 1$) is

$$R \simeq N\sigma|\mathbf{v} - \mathbf{V}| \simeq N\sigma v \left(1 - \frac{\mathbf{v}\cdot\mathbf{V}}{v^2}\right)$$

which demonstrates that head-on enounters ($\mathbf{V}\cdot\mathbf{v}_i < 0$) are indeed more frequent than overtaking encounters ($\mathbf{V}\cdot\mathbf{v}_i > 0$). The average energy gain per unit time follows by averaging $R\Delta\varepsilon$ over all possible directions of $\mathbf{V}$:

$$\langle\frac{d\varepsilon}{dt}\rangle \simeq N\sigma v\langle\left(1 - \frac{\mathbf{v}\cdot\mathbf{V}}{v^2}\right)\Delta\varepsilon\rangle$$

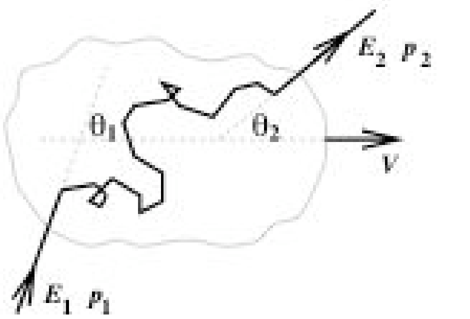
where $\Delta\varepsilon$ is given by (5) (omitting the 'i' subscript). To calculate the average quantities, we write $\mathbf{v}\cdot\mathbf{V} = vV\cos\theta$ and average over all $0 \le \theta \le \pi$, noting that $\langle\cos^2\theta\rangle = \frac{1}{3}$ and $\langle\cos\theta\rangle = 0$ for an isotropic velocity distribution. The result is

$$\langle\frac{d\varepsilon}{dt}\rangle \simeq \frac{8}{3}N\sigma v\frac{V^2}{c^2}\varepsilon \qquad (6)$$

and the mean energy gain per collision is

$$\boxed{\frac{\langle\Delta\varepsilon\rangle}{\varepsilon} \simeq \frac{\langle d\varepsilon/dt\rangle}{\langle R\rangle} \simeq \frac{8}{3}\frac{V^2}{c^2} \quad \text{2nd-order Fermi acceleration}} \qquad (7)$$

This is Fermi's famous initial result, demonstrating that a statistical nett gain in energy results from collisions of particles off scattering centers, with an average energy gain that is *second order in* V/c. This is a type of *stochastic acceleration*: the average systematic energy gain of a particle results from many small, non-systematic energy changes.



Interaction of a cosmic ray with an interstellar cloud moving at velocity V. Although 2nd-order Fermi acceleration can be simply described in terms of billiard ball collisions, the scattering process is somewhat more involved. As the particle enters the cloud, it scatters off irregularities in the internal magnetic field. So the particle's final energy and momentum are the result of many scatterings inside the cloud.

2.3 The Spectrum Due to 2nd-Order Fermi Acceleration

The result (6), *viz.* $\langle d\varepsilon/dt\rangle = \alpha\varepsilon$, where $\alpha = \frac{8}{3}N\sigma\frac{V^2}{c^2}$ implies that $\varepsilon(t) = \varepsilon_0 \exp(\alpha t)$, so the characteristic timescale of the acceleration process is $t_{\rm acc} \sim \alpha^{-1}$. The particles can also escape from the region where the acceleration takes place. Suppose this occurs on a timescale $t_{\rm esc}$. The flow of particle energy under the influence of stochastic Fermi acceleration can be described by a diffusion equation (formally derived from a Fokker-Planck approximation to the Vlasov equation, because the fractional energy-momentum changes in a single collision are small). Let $dn = n(\varepsilon,t)d\varepsilon$ be the number density of particles with energy in the range $(\varepsilon, \varepsilon + d\varepsilon)$. Then the evolution of the particle population can be described by

$$\frac{dn(\varepsilon,t)}{dt} + \frac{\partial}{\partial\varepsilon}\left[\langle\frac{d\varepsilon}{dt}\rangle n(\varepsilon,t) - \frac{\partial}{\partial\varepsilon}\left(Dn(\varepsilon,t)\right)\right] \approx -\frac{n}{t_{\rm esc}} + Q(\varepsilon,t) \tag{8}$$

The term in square brackets on the LHS is the mean particle energy flux; it is the nett difference between the rates of mean energy gain and energy diffusion (where D is an energy diffusion coefficient). The last term on the RHS of (8) is a source term describing injection of particles. If we neglect energy diffusion and the source term Q, and consider a steady-state

solution ($dn/dt = 0$), then we find

$$-\frac{d}{d\varepsilon}\left(\alpha\varepsilon n\right) - \frac{n}{t_{\rm esc}} \simeq 0$$

Differentiating and rearranging gives

$$\frac{dn(\varepsilon)}{d\varepsilon} \simeq -\left(1 + \frac{1}{\alpha t_{\rm esc}}\right)\frac{n}{\varepsilon}$$

and thus, $n(\varepsilon) \propto \varepsilon^{-p}$, which is a power-law particle distribution with spectral index $p = 1 + (\alpha t_{\rm esc})^{-1}$. Although this was the reason why stochastic Fermi acceleration was initially a very promising mechanism for astrophysical sources, observations indicate that $p \simeq 2.5$ over a broad range of different sources. This means that the combination of the parameters α and $t_{\rm esc}$ must be very fine-tuned, which is unlikely. Other effects have been considered, such as stochastic acceleration by MHD turbulence and plasma waves in general, but even so, a more favourable acceleration mechanism, particularly for ultra-relativistic particles, is Fermi's 1st-order acceleration which can occur naturally at astrophysical shocks.