PHYS 5020
Computation and Image Processing
Lecture 1: Computation – Monte Carlo Overview

Thursday 2 August 2012
The Monte Carlo technique

Monte Carlo (MC) is a numerical method that uses random sampling of probability distributions to simulate stochastic processes in nature, e.g.

- stockmarket fluctuations
- population studies
- weather forecasting
- radiation transport
- **stochastic** processes (sometimes also referred to as random or probabilistic processes) depend on physical parameters that vary according to a probability distribution.
- Instead of a single outcome, there are multiple possible outcomes, some more probable than others.
- This is in contrast to **deterministic** processes, which have a single possible outcome.

**Example:** Brownian motion (a.k.a. random walk)
MC is a numerical integration technique:

\[ I = \int_{x_1}^{x_N} f(x) \, dx \approx N \delta x \frac{1}{N} \sum_{i=1}^{N} f(x_i) = \Delta x \hat{f} \]

\( \hat{f} = \text{sample mean} \) of integrand, obtained by randomly sampling \( N \) points \( x_i \) in the integration domain and evaluating \( f(x_i) \)

for complicated functions, just randomly sample larger uniform area \( A \) enclosing \( \int f \, dx \) and reject points \( (x_i, y_i) \) s.t. \( y_i > f(x_i) \)
sample mean, \( \hat{f} \), is only approximation to expectation value, \( \langle f \rangle \) (most probable value of \( f(x) \))

more random points \( \implies \) more accuracy, but longer computation time

particle transport problem involves integration of particle interaction probability functions \( P \) over \( x, y, z, t \) and over all directions \( \Omega_1, \Omega_2, \Omega_3 \) \( \implies \) multidimensional integration (c.f. Lec. 3)
Monte Carlo radiation transport

Radiation transport processes are fundamentally probabilistic in nature. There are 4 main components of a complete MC model for radiation transport:

1. interaction cross-section **physics data**

2. ray-tracing algorithms for **particle transport**

3. **geometry**, boundary and materials specifications

4. **data analysis** tools
Brief history

- MC first conceived in 1940’s by Von Neumann (+ Ulam, Fermi, Metropolis et al. at LANL) to estimate nuclear fission yields for Manhattan Project.

- MC techniques then developed to model neutron & photon transport in nuclear reactors (e.g. Kahn H, 1950, Random sampling (Monte Carlo) techniques in neutron attenuation problems: I, *Nucleonics* 6 27).
current MC codes also model charged particle and electron transport (larger no. of interactions per particle)
» computationally prohibitive to simulate all electron interactions – e.g. $\sim 10^4$ interactions in Al from 0.5 MeV to 1 keV.

» condensed history technique: multiple electron interactions grouped together to give the same macrophysical result (Berger MJ, 1963, Monte Carlo calculation of the penetration and diffusion of fast charged particles, Methods in Computational Physics, vol. 1, eds. B Alder, S Fernbach & M Rotenberg)

» subsequent rapid developments by high-energy physicists simulating electron-photon showers

Modern applications in the physical sciences

1. High Energy Physics – e.g. LHC ATLAS detector
2. Astrophysics – e.g. X-ray detector satellite payloads and telescope data analysis

- X-ray Multi-Mirror mission (XMM)
  - Launch December 1999
  - Perigee 7000 km
  - Apogee 114000 km
  - Flight through the radiation belts

- Chandra X-ray observatory, with similar orbit, experienced unexpected degradation of CCDs
- Possible effects on XMM?

![Image of X-ray satellite with detectors and data analysis graphs]

Cygnus X-1

$E_F_R \text{ (erg cm}^{-2} \text{s}^{-1})$

$10^{-8}$

$10^{-9}$

$10^{-10}$

$E \text{ (keV)}$

1

10

100

1000

$10^4$
Overview of Medical Physics Applications

Radiotherapy physics

- external/internal sources and dosimetry
- phantom simulations
- treatment planning
Diagnostic radiology

- detection systems
- physical quantities
- radiation protection
Nuclear medicine

- detectors
- imaging correction
- absorbed dose
Available Codes

- **EGSnrc** – Electron Gamma Shower, developed by National Research Council, Canada:  
  [irs.inms.nrc.ca/software/egsnrc](irs.inms.nrc.ca/software/egsnrc)

- **BEAMnrc** – also developed by the NRC, but for modelling medical linear accelerator beams:  
  [irs.inms.nrc.ca/software/beamnrc](irs.inms.nrc.ca/software/beamnrc)

- **Geant4** – software toolkit developed by international consortium (CERN, ESA, Fermilab, SLAC, KEK, ...):  
  [geant4.web.cern.ch/geant4](geant4.web.cern.ch/geant4)

- **MCNPX** – developed by Los Alamos National Laboratory:  
  [mcnp-green.lanl.gov](mcnp-green.lanl.gov)

- **FLUKA, PENELPOE, GATE, SimSET, MABOSE, .....**
Dosimetry modelling

EGSnrc:

20 MeV electrons in water
For more examples, see
www-personal.umich.edu/bielajew/NewStuff/EWarchive.html
The Monte Carlo technique

Brief history

Applications

Medical Physics

Applications

Overview

Codes

Dosimetry

Linac beam

Imaging

Geant 4 Medical applications

PET, SPECT

Radiotherapy with external beams, IMRT

Hadrontherapy

Nanodosimetry

Brachytherapy

Radiation protection

Courtesy of B. Mascialino et al., INFN Genova

Courtesy of P. Cirrone et al., INFN LNS

Courtesy of S. Guatelli et al., INFN Genova

Courtesy of L. Beaulieu et al., Laval

Courtesy of GATE Collaboration
Linac beam modelling

BEAMnrc model of Varian linac at RPAH RadOnc:
Percentage depth dose curves in water:
RPA Varian 6 MV linac beam (EGSnrc/BEAMnrc)
6 MV beam profiles in water: RPA Varian linac
5 × 5 cm², 15 × 15 cm²
Imaging modelling

Example: Positron Emission Tomography (emission and detection of annihilation gammas)
PET detector ring (gold) and water phantom with 511 keV gammas (green) modelled in Geant4.