

Red variables in the OGLE-II data base – I. Pulsations and period–luminosity relations below the tip of the red giant branch of the Large Magellanic Cloud

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ABSTRACT

We present period–luminosity relations for more than 23 000 red giants in the Large Magellanic Cloud observed by the OGLE-II microlensing project. The OGLE period values were combined with the 2MASS single-epoch JHK_S photometric data. For the brighter stars we find agreement with previous results (four different sequences corresponding to different modes of pulsation in asymptotic giant branch stars). We also discovered two distinct and well-separated sequences below the tip of the red giant branch. They consist of almost 10 000 short-period ($15 < P < 50$ d), low-amplitude ($A_1 < 0.04$) mag red variable stars, for which we propose that a significant fraction is likely to be on the red giant branch, showing radial pulsations in the second and third overtone modes. The excitation mechanism could be either Mira-like pulsation or solar-like oscillations driven by convection.

Key words: stars: AGB and post-AGB – stars: late-type – stars: oscillations – stars: variables: other – globular clusters: general – distance scale.

1 INTRODUCTION

Low- and intermediate-mass stars (approximately $0.5\text{--}5 M_{\odot}$) enter late evolutionary stages when they become red giants on the first giant branch (RGB). Reaching the tip of the RGB (TRGB), rapid luminosity drop occurs after the helium flash. The stars then expand again, climbing the asymptotic giant branch (AGB), where they show strong pulsations (these are the Mira and semiregular variables) associated with enhanced mass-loss.

Recently, Ita et al. (2002) have reported the presence of many red variable stars in the Large Magellanic Cloud around the TRGB. Based on their time-series JHK photometry, they concluded that besides faint AGB variables, a substantial fraction is likely to be on the RGB. If confirmed, that would imply that global pulsations do occur in a less evolved stage than the AGB, which may have a strong impact on the pulsation-driven mass-loss history.

Earlier, Welty (1985) performed a photographic search for RGB and AGB variable stars in six globular clusters, and found relatively high amplitude variability ($A_B \leq 0.2$ mag) only near the tip of the RGB. From 16 yr of high-precision photoelectric photometry, Jorissen et al. (1997) found a minimum-variability boundary for red giants, concluding that all stars of M spectral types are intrinsic variable stars. The observed radial velocity jitter for some stars was interpreted as evidence for pulsationally induced variability. This result on the absence of stable red giants was confirmed by Eyser

& Grenon (1997), who analysed the *Hipparcos* Epoch Photometry data base. From theoretical point of view, a thorough study was published by Dziembowski et al. (2001), who presented a linear stability analysis of red giant models. One of their conclusions was that if turbulent convection was included into the models, the fundamental mode was strongly damped compared to higher overtones, i.e. the more realistic models tended to have observable amplitudes for overtone modes with orders $n \geq 5$. That was in stark disagreement with the observations of α UMa (Buzasi et al. 2000) and no definite conclusion was drawn on the nature of possible excitation mechanisms. Possibilities included self-excitation of unstable modes (referred to as Mira-like pulsations) and convection induced excitation of linearly stable modes (the so-called solar-like oscillations). To make a distinction between these, Dziembowski et al. (2001) suggested that the microlensing data of the Magellanic Clouds should be re-analysed to search for short-period and low-amplitude extensions of period–luminosity ridges (Wood et al. 1999) of red variables. This paper presents the results of such a search.

Microlensing surveys have stimulated work in *ensemble asteroseismology*, which deals with global properties of pulsating stars (see Szabados & Kurtz 2000, for reviews). AGB pulsators have been found to obey well-defined period–luminosity (PL) relations forming parallel ridges in the $\log P\text{--}K$ plane (Wood et al. 1999). Wood (2000) discussed the nature of MACHO red variables and concluded that the ridges correspond to fundamental, first, second and third overtone modes. His results have been recently confirmed by Cioni et al. (2001, 2003), Noda et al. (2002) and Lebzelter, Schultheis & Melchior (2002).

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This Letter is the first in a series of papers dealing with an analysis of *I*-band observations obtained by the OGLE-II project (Zebun et al. 2001). Here we report on pulsating red giants in the Large Magellanic Cloud and the existence of distinct PL relations for a large group of red variables below the TRGB. As shown below, the most reasonable explanation is the presence of global oscillations in a mixture of AGB and RGB stars, probably in the second and third overtone modes. A comparison between the Large and Small Magellanic Clouds will be presented in a subsequent paper.

2 DATA ANALYSIS

The second phase of the Optical Gravitational Lensing Experiment (OGLE-II; Udalski, Kubiak & Szymanski 1997) spanned 4 yr, from 1997 to 2000. About 7 deg² were observed repeatedly in the Magellanic Clouds, yielding about 6×10^9 photometric measurements for about 20×10^6 stars. The observations were reduced with a modification of the difference image analysis method (Alard & Lupton 1998; Wozniak 2000), which has been proved to be superior for detecting tiny photometric variations in crowded fields (the adopted definition of candidate variable stars was outlined by Wozniak 2000). The *I*-band light curves typically contain 400 points per star. The error of individual magnitude measurements for the brightest stars ($I < 16$ mag) is a few mmag, so that fairly low-amplitude changes can be studied. More than 68 000 variable stars were listed in the OGLE-II catalogue of variable stars (Zebun et al. 2001) and it forms the basis of this study.

There is one major advantage of using the OGLE-II data base. It consists of *I*-band observations, so that red variables had relatively higher flux levels in the original data than in ‘bluer’ bands (such as, for example, the MACHO red band). Consequently, deeper limiting magnitudes and better photometric precision is expected. On the other hand, there are two disadvantages. The first is the reduced photometric amplitude of red giant pulsators in the *I* band. Because of that, one can expect difficulties in the low-amplitude regime. The other drawback is the time-span of observations: a typical OGLE-II light curve consists of 400 points obtained over 1200 d, which is less than half of the MACHO time-span. As a result, period determination becomes uncertain for periods longer than a few hundred days.

We have found that, despite the smaller amplitudes in the *I* band, high-quality light curves enabled detection of periodicities of low-amplitude changes. For stars around $I = 15$ – 16 mag, 1 per cent variability could be safely analysed. For periods of less than a year, period determination seems to be well established. Spurious periods occurred at integer multiples of a year, and they can be easily distinguished from real periods. For shorter periods, OGLE-II data are well suited to infer accurate cycle lengths, and the possibility of studying small amplitude stars outweighs the disadvantages.

In order to reduce the effects of interstellar extinction and allow a direct comparison with previous results, we focused on $\log P - M_K$ relations. Our analysis consisted of the following steps.

(i) We made a cross-correlation of the 52 937 OGLE-II variable stars in the LMC with the 2MASS All-Sky Point Source Catalog;¹ with a search radius of 1 arcsec we found 32 062 stars with JHK_S magnitudes.

(ii) Next, we excluded the negligible number (7) of duplicates (i.e. when two stars were found within 1 arcsec of the OGLE coordinates).

(iii) Red giants were selected by the $J - K_S$ colour; as a safe threshold we chose 0.9 mag (according to standard lists of Hawarden et al. 2001, that corresponds to early M spectral type). The final sample consisted of 23 494 stars.

(iv) We performed a period search on OGLE light curves by an iterative Fourier analysis. First, we calculated the discrete Fourier transform (the examined frequency range was between 0 and 0.066 d^{-1} with a frequency step of $6 \times 10^{-7} \text{ d}^{-1}$). After finding the highest peak in the spectrum, we subtracted a sine wave from the data with that amplitude and the optimal phase. Then the whole procedure was repeated until a four-component fit was reached. We kept only those frequencies larger than $8 \times 10^{-4} \text{ d}^{-1}$ ($\sim 1/T_{\text{obs}}$) and with semi-amplitudes larger than 5 mmag.

(v) For each star, the resulting data base contains the OGLE identifier (made of the J2000.0 coordinates), periods, semi-amplitudes, phases, mean *I* magnitude and 2MASS JHK_S single-epoch magnitudes.

There are some necessary simplifications in this procedure enforced by the large number of stars. However, it is exactly this factor that enables good statistics. In overlapping regions between previous works and our study (brighter stars with larger amplitudes), the agreement is excellent. For instance, Cioni et al. (2001, 2003) made very careful, object-to-object control of light curves before studying statistics, and their results are very well reproduced by our data. The tight constraint on the coordinates has made it very likely that infrared counterparts have been unambiguously identified. Random checks of period determination showed that below 200 d, periods are likely to be accurate to a few per cent. Five per cent period error ($\delta \log P = 0.02$) is smaller than the intrinsic period jitter well known in local semiregular (SR) variables (Kiss et al. 1999). For longer periods, there are unavoidable groups of spurious periods at integer multiples of a year, which are easily recognizable as vertical strips in the period–luminosity plane. We consider our data to be reliable up to about 500–600 d, for which at least two cycles were covered by the observations.

3 DISCUSSION

We present the resulting PL relations in Fig. 1. The outstanding features of the diagram: (i) there is a sudden drop of stellar density at $K_S = 12.05$, located exactly at the TRGB (see Cioni et al. 2000, for details); (ii) above the TRGB, there are four sequences (two partly overlapping) identified by Wood (2000) as stars pulsating in fundamental (F), first (1O), second (2O) and third (3O) overtone modes (in his notation, these were sequences C, B and A); (iii) below the TRGB, there are two distinct and well-separated sequences (R_2 and R_3), slightly shifted relative to 2O and 3O; (iv) there is also a continuation of 1O below the TRGB (R_1), less populated by an order of magnitude than R_2 and R_3 ; (v) there are two long-period sequences (extending well below the TRGB), one of which has been explained by Wood (2000) as binary variables ($L_1 = E$ in Wood 2000) and the other being long secondary periods of ambiguous origin ($L_2 = D$, see also Olivier & Wood 2003). Similar ridges and distributions have also been found for the SMC (Kiss & Bedding, in preparation), which means we did not find strong dependence on metallicity, at least over the range covered by the LMC and SMC. In the following, we discuss some general properties with special emphasis on variables below the TRGB. In order to reveal some hidden features in Fig. 1, we have examined the luminosity function (LF), colour–magnitude diagram (CMD) and the dependence on amplitude (including colour information).

¹ <http://irsa.ipac.caltech.edu>

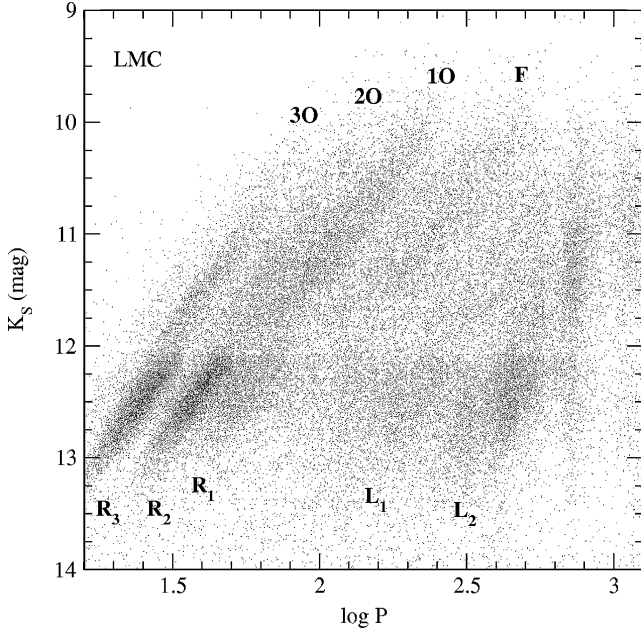


Figure 1. PL relations for red giants in the LMC (62 591 periods for 23 494 stars). This figure is available in colour in the on-line version of the journal on *Synergy*.

The LF (thick solid line in Fig. 2) is clearly bimodal, with two approximately Gaussian components. A least-squares fit gives $K_1 = 11.33 \pm 0.05$ mag, $K_2 = 12.54 \pm 0.01$ mag, $\sigma_1 = 1.81 \pm 0.14$ mag and $\sigma_2 = 0.82 \pm 0.03$ mag for the maxima and full-widths at half maxima, respectively. The boundary between them is at $K_S \approx 12.0$ mag, which is in excellent agreement with the TRGB value determined by Cioni et al. (2000). For this, two possible explanations exist. Alves et al. (1998) and Wood (2000) suggested that all variables below the TRGB are thermally pulsing AGB stars and their brighter cut-off just coincides with the TRGB. This has been questioned by Ita et al. (2002), who argued that a significant fraction is likely to consist of first-ascent red giants. To show what level of coincidence is required for the former interpretation, we compare our LF with the more complete sample of Cioni et al. (2000) (thin line in Fig. 2). Their data are foreground-subtracted DENIS mag-

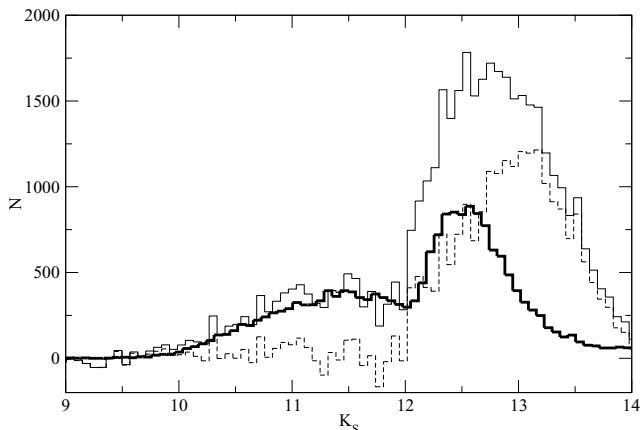


Figure 2. Stellar magnitude distributions for OGLE-II variables, (thick solid line), LMC foreground-subtracted sample from Cioni et al. (2000) (thin solid line) and the difference (dashed line). This figure is available in colour in the on-line version of the journal on *Synergy*.

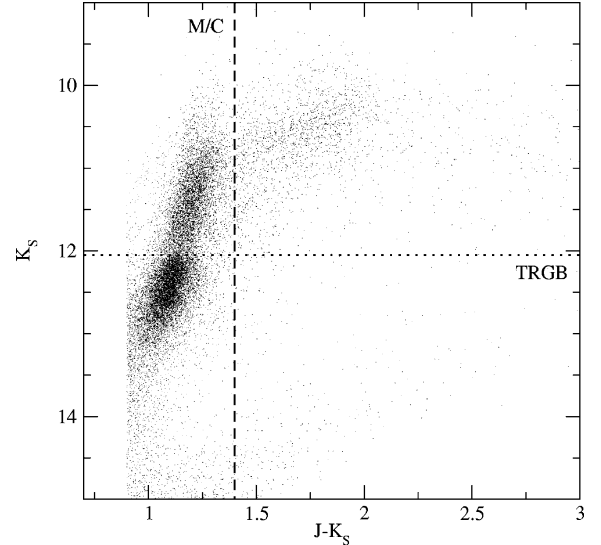


Figure 3. The colour-magnitude diagram. Dashed line shows the boundary between oxygen-rich and carbon stars, while dotted line represents the TRGB.

nitude distributions, which means we do not have information on the spatial distribution of stars in the sample, and so we did not attempt to correct for the different survey areas. Two features are worth noticing: (i) the bright AGB components of the two LFs are essentially the same, so that OGLE-II has detected practically all AGB stars above the TRGB; (ii) the rise at $K_S = 12$ occurs in exactly the same place in both LFs (to within 0.07 mag). The latter point is strong evidence that the OGLE variables below the TRGB contain substantial numbers of RGB stars. The colour distributions (see below) will give further empirical support to this argument.

The K_S versus $J - K_S$ CMD (Fig. 3) looks very similar to the one presented in Cioni & Habing (2003), which was based on a colour-selected sample. The overall agreement shows there is no significant difference between variability-selected and colour-selected samples. We show the stellar density drop estimated from Fig. 2 (a more sophisticated determination is beyond the scope of the present paper) and the conventional boundary between oxygen-rich and carbon-rich stars ($J - K_S = 1.4$ mag). Stars below the TRGB are well concentrated in the range $J - K_S = 0.95$ – 1.3 mag, so that there is no excess of some redder fainter population of stars, which might be associated with, for instance, obscured AGB stars (Van Loon et al. 1998) or low-mass carbon stars below the TRGB (Lattanzio 1989) recognizable from their extreme red colours.

In Fig. 4, we plotted six different slices of the (period, amplitude, K_S magnitude) data cube, where the amplitudes were calculated as twice the Fourier semi-amplitudes. To reveal more information, we also indicate three different $J - K_S$ colour ranges ($J - K_S = 0.9$ – 1.2 , 1.2 – 1.4 and >1.4 mag). For normal M-type stars (with no dust), $\delta(J - K) = 0.1$ mag corresponds to $\delta T_{\text{eff}} \sim 200$ K (Bessell et al. 1998), thus the chosen ranges enable a rough classification of stars as ‘hot’, ‘warm’ and ‘cool’ red giants (actually, the latter group is likely to consist of carbon stars). The exact temperature scale is not important for our purposes, because we are only interested in their distribution.

There are many interesting features in Fig. 4. First, there is a sudden disappearance of stars below the TRGB with increasing amplitude. For amplitudes larger than 0.04 mag, very few short-period ($P < 50$ d) stars remain, while for $A > 0.14$, almost every star

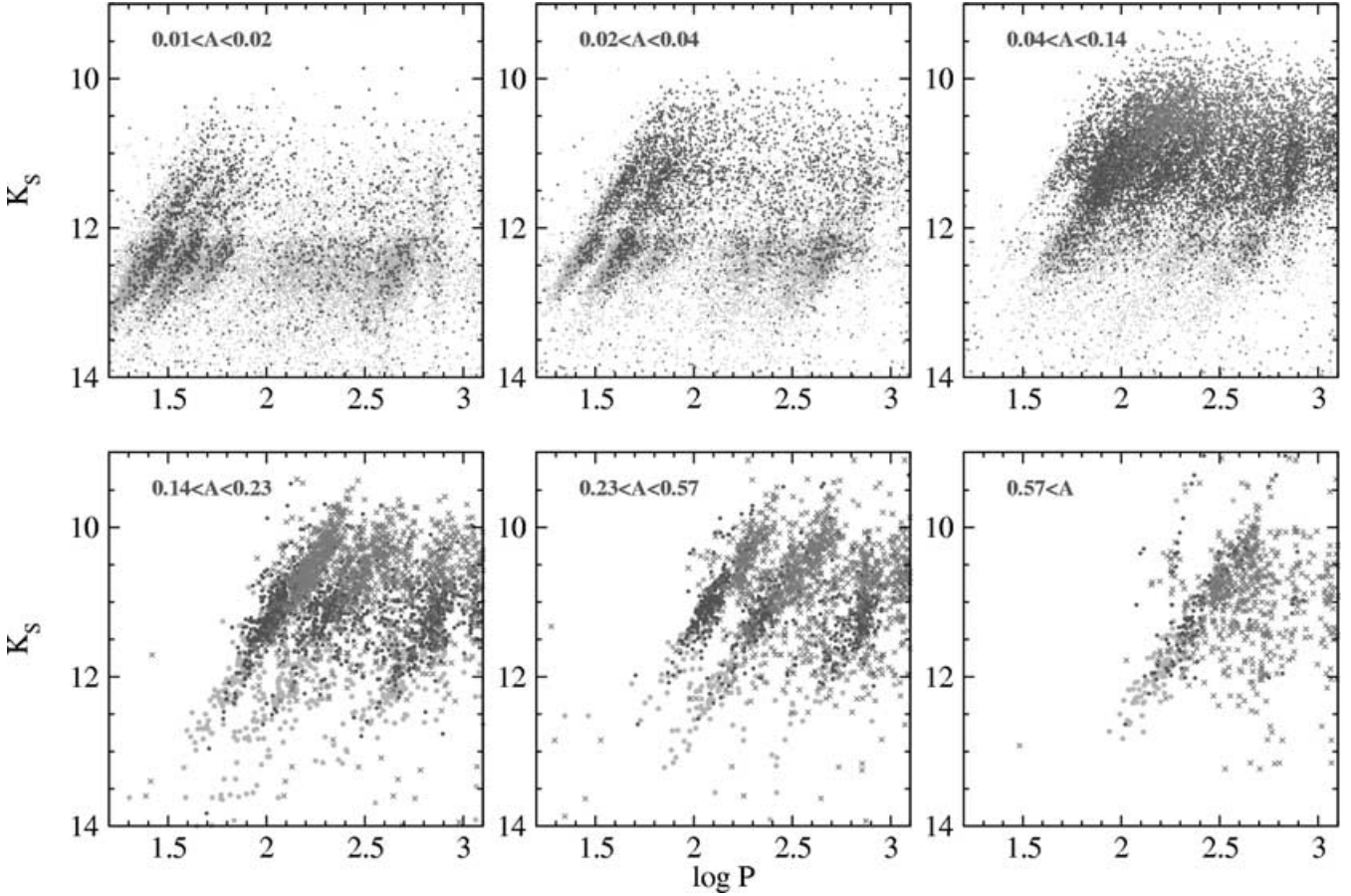


Figure 4. PL relations in the LMC as function of the full amplitude of modes. Three $J - K_S$ colour ranges were selected to plot in different shades of grey. (This figure can be seen in colour in the on-line version of the journal on *Synergy*.) Light grey (turquoise in the colour version): $0.9 < J - K_S \leq 1.2$; black (blue): $1.2 < J - K_S \leq 1.4$; dark grey (red): $J - K_S > 1.4$. The symbols in the lower panels are drawn larger for improved clarity.

fainter than the TRGB disappears. The lower three panels show very clearly the fundamental and first overtone sequences and hints of the long secondary period sequence. The upper panels are dominated by the higher overtone pulsators, so that there is a good correlation between the amplitude and mode of pulsation. Secondly, the colour distributions show the temperature differences within each mode of pulsation very well. The lower three panels are in good agreement with the expectations: for a given mode and assuming a narrow range of masses, the PL relation is equivalent to a density–luminosity relation, which implies a monotonic temperature variation along any particular sequence. The upper right-hand panel reveals that stars in the overlapping 1O and 2O ridges are easily distinguishable by their colours and the transition between them is quite sharp (1O contains very few black dots). Thirdly, ridges R_1 , R_2 and R_3 differ from the other ridges in that each contains both ‘hot’ (light grey) and ‘warm’ (black) red giants along their full extent, with a tendency for ‘warm’ stars to have slightly longer periods (i.e. the black dots are concentrated on the right-hand side of the ridges).

We interpret this last point as evidence for RGB stars mixing with thermally pulsing AGB variables. At the TRGB, RGB and AGB stars have very similar luminosities, with a tendency for RGB stars to have slightly lower temperatures. Recent evolutionary models of the Magellanic Clouds (Castellani et al. 2003) show that for stars with $1\text{--}2 M_{\odot}$, the temperature difference at constant luminosity is $\delta \log T_{\text{eff}} \approx 0.01$. This leads to $\delta \log R \approx -0.02$, or $R_{\text{AGB}}/R_{\text{RGB}} \approx$

0.96 . For a given mode of pulsation, the period–density relation ($P\sqrt{M/R^3} = Q$) results in $\delta \log P = 1.5 \times \delta \log R \approx -0.03$, in good agreement with the period shift ~ -0.05 within R_2 and R_3 in the upper left panel of Fig. 4. On the other hand, the fact that R_2 and R_3 are apparent continuations of 2O and 3O ridges, suggests they are second and third overtone pulsators, a mixture of TPAGB and RGB stars. We also note that even TPAGB stars show a slight shift relative to 2O and 3O, which is in agreement with their higher temperatures in lower luminosity regions (Vassiliadis & Wood 1993). The apparent displacement of ‘hot’ and ‘warm’ red giants is therefore consistent with the assumption of the same mode and we conclude RGB stars do pulsate mostly in the second and third overtone modes (some penetration is also likely in R_1 , the faint continuation of the first overtone ridge).

Finally, we also note in the lower right-hand panel an interesting group of Mira stars located below the PL sequence. We identify them (Kiss, in preparation) with dust-enshrouded Mira stars (Wood 1998).

4 CONCLUSIONS AND FUTURE WORK

In this paper we presented the first results of a complex analysis of OGLE-II red variable stars in the LMC. Our most important result is the discovery of separate PL relations of a huge number of variable stars below the TRGB. We found 9617 stars with $K_S > 12.05$ mag

that have at least one period shorter than 50 d (that is, 41 per cent of the full sample). Alves et al. (1998), Wood et al. (1999) and Wood (2000) also detected such stars in the MACHO data (though their samples were more than an order of magnitude smaller) and they favoured thermally pulsing AGB explanation, instead of placing stars on the RGB. However, our sample is the largest available to date and the extreme number of variables below the TRGB can be hardly explained solely by accumulated TPAGB (or early AGB) stars at the TRGB. The revealed colour dependence, accompanied with the slight period shift, can be consistently explained by the mixture of AGB and RGB pulsators.

The existence of separate PL relations below the TRGB leads to the conclusion that RGB pulsations have remarkable astrophysical potential. On one hand, they extend applicability of asteroseismological considerations to a so-far neglected class of stars. On the other hand, their PL relations can serve as a powerful test of globular cluster (GC) distance scale. The brighter ends of the RGB luminosity functions (Zoccali & Piotto 2000) suggest that the number of upper-RGB stars in typical GCs can be from tens to hundreds, so that reliable statistics might be obtained within a reasonable timespan. This is provided by the short periods (15 to 50 d) and relatively large photometric amplitudes (0.01 to 0.04 mag in the I band). We will address the problem of separating RGB stars in R_2 and R_3 and derivation of useful relations for future GC observations in a forthcoming publication.

An interesting question is, where are the local counterparts of the RGB pulsators? One of the closest red giant stars is γ Crucis (M3III, $d = 27$ pc, ESA 1997), for which radial velocity data by Cummings (1999) suggested a pulsational time-scale of 13–16 d ($\log P \approx 1.17$). Its 2MASS $K_S = -3.258$ mag translates to $M_{K_S} = -5.41$ mag, or K_S (LMC) = 13.09 mag (adopting μ (LMC) = 18.50). This position is in excellent agreement with R_3 . Although this is only a single example, is it reasonable to predict similar behaviour for other local RGB stars, too.

With the presented properties, RGB pulsators bridge the gap between the AGB pulsators and low-amplitude K giant pulsators discovered in 47 Tucanae by Edmonds & Gilliland (1996). The emerging view of red giant (RGB and AGB) pulsations suggests that all stars do pulsate similarly, in the sense of obeying similar PL-relations. Only the mode distribution is changing over the varying physical parameters (and possibly evolutionary status). The majority of stars oscillates in high overtones, which are heavily perturbed by, e.g., the convection, leading to the ‘semiregular’ behaviour. However, they remain close relatives of the more regular Mira stars, with different modes of pulsation. In a future work we will tackle the problem of photometric mode identification exploiting the ensemble properties of red giant light curves to gain a better understanding of mode distributions in local variable stars.

The OGLE-II data base provides an excellent opportunity to study general properties of red giant pulsators, being the most extensive sample presently available. There are a number of questions which may strongly benefit from utilizing the statistical power of OGLE-II data. They include, for instance, a very accurate relative distance modulus of the LMC and SMC, a robust comparison of pulsational properties in the Magellanic Clouds and the Galactic bulge (Wozniak et al. 2002) and finding evolutionary effects on pulsation. By analysing OGLE-II data, a real advance is expected in theoretical understanding of red giant stars, which is a major issue of the contemporary stellar astrophysics.

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REFERENCES

- Alard C., Lupton R. H., 1998, *ApJ*, 503, 325
 Alves D. et al., 1998, in Takeuti M., Sasselov D. D., eds, *Proc. of IAU JD24, Pulsating Stars – Recent Developments in Theory and Observations*. Universal Academic Press, Tokyo, p. 17
 Bessell M. S., Castelli F., Plez B., 1998, *A&A*, 333, 231
 Buzasi D. et al., 2000, *ApJ*, 532, L133
 Castellani V., Degl’Innocenti S., Marconi M., Prada Moroni P. G., Sestito P., 2003, *A&A*, 404, 645
 Cioni M.-R. L., Habing H. J., 2003, *A&A*, 402, 133
 Cioni M.-R. L., van der Marel R. P., Loup C., Habing H. J., 2000, *A&A*, 359, 601
 Cioni M.-R. L. et al., 2001, *A&A*, 377, 945
 Cioni M.-R. L. et al., 2003, *A&A*, 406, 51
 Cummings I., 1999, PhD thesis, Univ. Canterbury
 Dziembowski W. A., Gough D. O., Houdek G., Sienkiewicz R., 2001, *MNRAS*, 328, 601
 Edmonds P. D., Gilliland R. L., 1996, *ApJ*, 464, L157
 ESA, 1997, *The Hipparcos and Tycho Catalogues*, ESA SP-1200
 Eyer L., Grenon M., 1997, *ESA, SP-*, 402, 467
 Hawarden T. G., Leggett S. K., Letawsky M. B., Ballantyne D. R., Casali M. M., 2001, *MNRAS*, 325, 563
 Ita Y. et al., 2002, *MNRAS*, 337, L31
 Jorissen A., Mowlavi N., Sterken C., Manfroid J., 1997, *A&A*, 324, 578
 Kiss L. L., Szatmáry K., Cadmus R. R., Jr, Mattei J. A., 1999, *A&A*, 346, 542
 Lattanzio J. C., 1989, in Johnson H. R., Zuckerman B., eds, *Evolution of Peculiar Red Giants*. Cambridge University Press, Cambridge, p. 161
 Lebzelter T., Schultheis M., Melchior A. L., 2002, *A&A*, 393, 573
 Noda S., Takeuti M., Abe F. et al., 2002, *MNRAS*, 330, 137
 Olivier E. A., Wood P. R., 2003, *ApJ*, 584, 1035
 Szabados L., Kurtz D. W., eds, 2000, *ASP Conf. Ser. Vol. 203, The Impact of Large-Scale Surveys on Pulsating Star Research*. Astron. Soc. Pac., San Francisco
 Udalski A., Kubiak M., Szymanski M., 1997, *Acta Astron.*, 47, 319
 Van Loon J. Th. et al., 1998, *A&A*, 329, 169
 Vassiliadis E., Wood P. R., 1993, *AJ*, 413, 641
 Welty D. E., 1985, *AJ*, 90, 2555
 Wood P. R., 1998, *A&A*, 338, 592
 Wood P. R., 2000, *PASA*, 17, 18
 Wood P. R. et al., 1999, in Le Bertre T., Lèbre A., Waelkens C., eds, *IAU Symp. 191, Asymptotic Giant Branch Stars*. Astron. Soc. Pac., San Francisco, p. 151
 Wozniak P. R., 2000, *Acta Astron.*, 50, 421
 Wozniak P. R. et al., 2002, *Acta Astron.*, 52, 129
 Zebun K. et al., 2001, *Acta Astron.*, 51, 317
 Zoccali M., Piotto G., 2000, *A&A*, 358, 943

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