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On the period history of χ Cygni

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Abstract. We discuss the light variability of the Mira variable χ Cygni, present a homogeneous set of times of minimum and maximum light since its discovery date in 1686, and analyse the variations of the pulsation period. There is a substantial cycle-to-cycle variation in the pulsations (standard error $\sim 6\overset{\mathrm{d}}{.}5$) with quite strong evidence for a linearly increasing period. Quasi-cyclic patterns in the $O-C$ diagrams are visible, but there is no compelling evidence that these apparent cyclic period changes are real since the residuals left after fitting a linear period increase plus intrinsic period scatter have properties consistent with being white noise. The Lombard frequency domain test for a changing period applied to the residuals also gives unconvincing significance levels. The low gradient of the luminosity change with period is compatible with the star undergoing a rise in luminosity when H shell-burning takes over.

Key words: stars: individual: HD 187796 – stars: oscillations – stars: AGB and post-AGB

1. Introduction

χ Cygni (HD 187796 = HR 7564 = BD+32 3593) is famous for having the largest visual amplitude amongst the well-observed Mira variables. In early July 1686, Gottfried Kirch planned observations of Nova 11 Vulpeculae (discovered 16 years earlier), and at this occasion he compared the surrounding field with Bayer's *Uranometria* atlas made in 1603. He could not find the star that was designed by Bayer as χ Cygni (even Flamsteed took the star 17 Cyg for χ Cygni) and, consequently, Kirch kept this region under close surveillance. On October 19, 1686 he found χ Cygni again as a star of about fifth magnitude. This appearance was the first reliable historical time of maximum of this variable, and χ Cygni is thus one of the oldest known variable stars.

From the discovery date till 1738 the star was mainly monitored by G. Kirch and Christfried Kirch, and–rather occasionally–also by Cassini and Halley. But the really significant observational contributions were made by Argelander and contemporaries (specifically J. Schmidt–who was one of the first who systematically monitored the star through its phase of minimum light–and who provided an almost continuous sequence of observations covering a time interval of nearly 40 years from 1845 to 1884). From the beginning of the 20$^{\mathrm{th}}$ century the star was observed almost uninterruptedly by different observers, particularly by amateur astronomers. Fig. 1 illustrates the evolution of the number of magnitude estimates as a function of time, and Fig. 2 shows the light curves available since 1865.

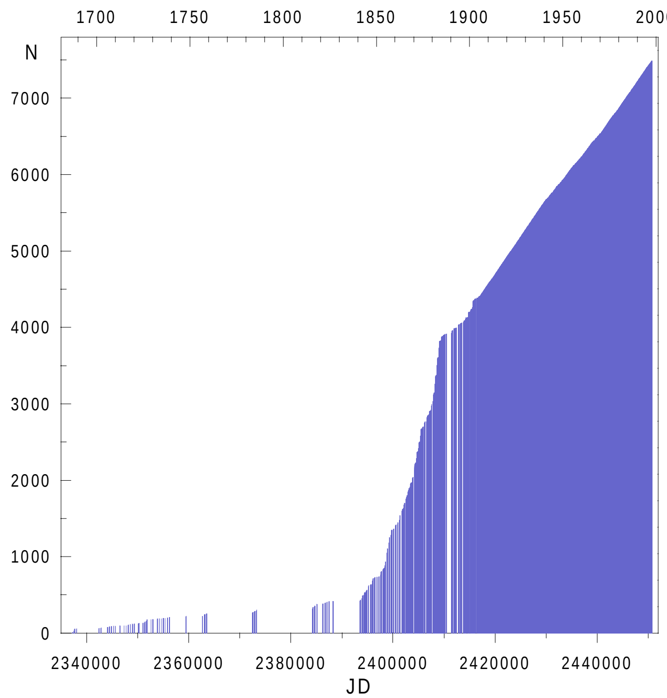

Fig. 1. Evolution of the number of magnitude estimates as a function of time.

The bulk of the historical data, transformed to the V scale of the Johnson UBV sytem, has been published by Sterken & Broens (1998) and Broens et al. (1998).

2. The period of χ Cygni

To Gottfried Kirch, χ Cygni was a regular variable with a period of $404\overset{\mathrm{d}}{.}5$ days, but in 1713 already Maraldi found a period of

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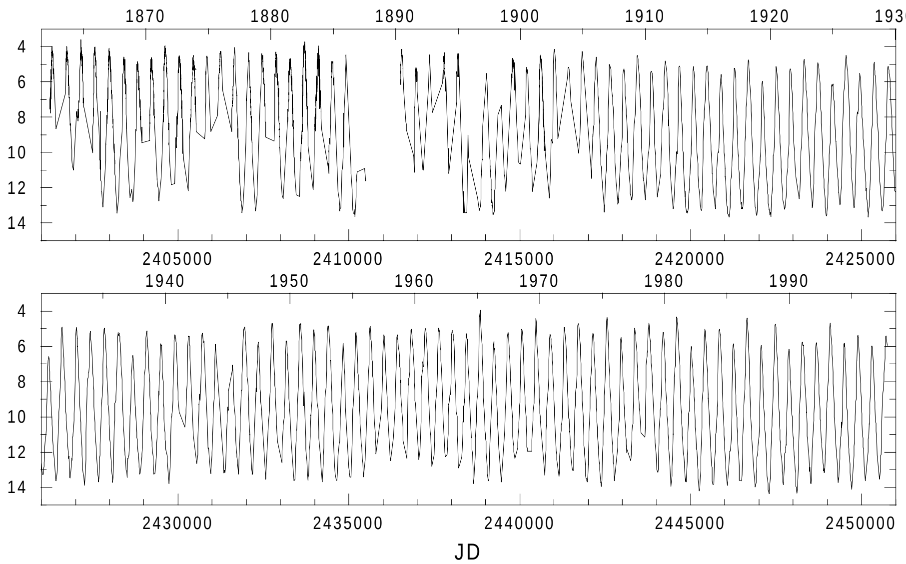

Fig. 2. Light curves of χ Cygni available since 1865. Note that the minima are not always well-covered (data from Sterken & Broens 1998, Broens et al. 1998).

405 days and pointed out that deviations of the cycle length of one month do occur, so that the time interval in between two maxima at some times would be 13 months and at other times 14 months. Indications that χ Cygni's period was lengthening, were also given by other observers (Le Gentil 1759, Pigott 1786, Koch 1802).

Olbers (1816) was the first who tried to mathematically describe the period lengthening (through his first-time application of the least-squares method; the procedure was communicated to him by Gauss in 1803). From a number of well-determined maxima from 1687 to 1815 Olbers (1841) also derived a quadratic ephemeris–though some cycles with deviations amounting to almost four weeks were not considered because Olbers regarded these deviations as irregularities of the period that could not be explained by his formula. The next to make a deep investigation of the period was Argelander (1869), who replaced the second-order term in Olbers' solution by a periodic term. Chandler (1894), then, combined Olbers' and Argelander's formulae by taking into account the quadratic as well as the cyclic term, but a satisfactory formal description for the period changes was never found.

Rosenberg (1906), in an extensive investigation of the star's light variations, confined himself to a representation with a quadratic term only, thus his elements turn out to be very close to those obtained by Olbers.

3. The times of maximum and minimum light

Since it is important to derive the times of minimum or maximum light with a consistent method, we have derived moments of light extremum in the same way as applied by Sterken et al. (1987), viz. by fitting a polynomial of third degree over a time interval that is symmetric with respect to an eye-estimated time of extremum. All $T_{\max}$ were calculated using data in a time interval of 60 days centered on the estimated maximum; the $T_{\min}$ were calculated using a time interval of 120 days centered on the estimated minimum. The results are given in Tables 1 and

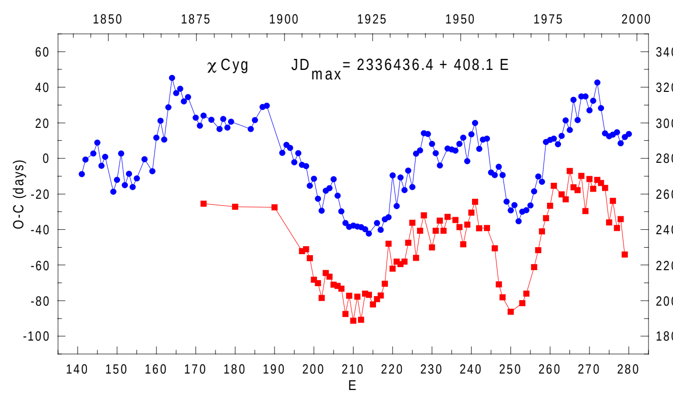

Fig. 3. $O-C$ diagram covering the last 150 years, linear ephemeris. The top curve is for the times of maximum and corresponds to the left Y-axis, the lower curve is for the times of minimum and relates to the Y-axis on the right side (the $O-C_{\min}$ were calculated with the linear ephemeris for the maxima given in the figure).

2. The difference in base time is necessary due to the presence of the hump on the rising branch of the light curve, so both intervals were chosen in such a way as to exclude any features belonging to the hump phase. As such, the number of data points at minimum often exceeds the number available at maximum, though in many cases the maxima are better covered. The number of available magnitudes and their quality (especially during minimum light) determine the accuracy of the calculated time of extremum. We estimate the times of minimum/maximum to be accurate within $0\overset{d}{.}5$–10 d.

4. The $O-C$ diagrams

Fig. 3 illustrates the $O-C$ diagram for $P = 408\overset{d}{.}1$ resulting from applying a linear ephemeris to the $T_{\max}$ over the last 150 years–that is, the time interval with an almost uninterrupted sequence of maxima and minima. The shape of the curve is cyclic, not strictly periodic. The 123 maxima wander between

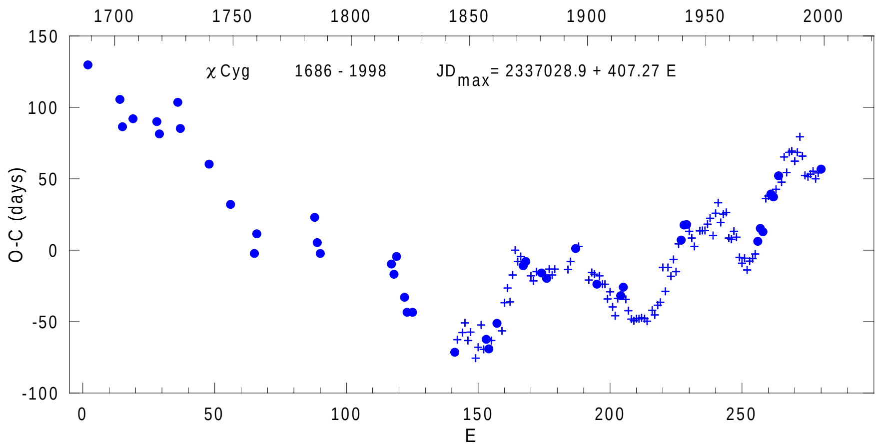

Fig. 4. $O-C$ diagram over three centuries, linear ephemeris based on $T_{\rm max}$. Some data with $E > 140$ have about the same time distribution as the data belonging to the cycle interval 1–140 (filled circles).

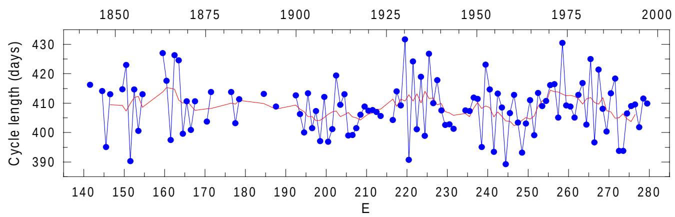

Fig. 5. Cycle lengths derived from 110 successive times of light maximum of χ Cygni for the last 150 years. Average period is $408^{\rm d}.7$, $\sigma = 8^{\rm d}.9$. Successive maxima are connected by straight lines, the continuous curve is the running average (bin length 7 datapoints).

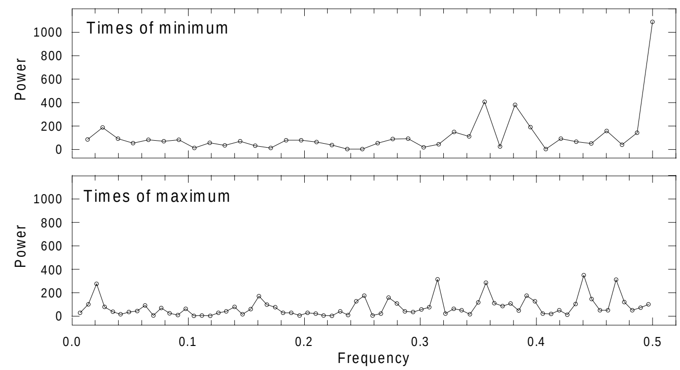

Fig. 6. Periodograms of the mean periods for the times of maximum and minimum.

6 weeks early and about 6 weeks late, in accord with Maraldi's early findings and with Argelander's attempt to fit a cyclic term. The minima follow the maxima by 231 ± 3 days on average. The agreement between the two trends is very good, while short-term differences may be due to measurement errors and cycle-to-cycle jitter in the period.

Fig. 4 illustrates the $O-C$ diagram resulting from applying a linear ephemeris over the time span of three centuries. For the sake of illustration, we have highlighted datapoints with $E > 140$ that follow about the same sampling distribution as the data belonging to the cycle interval 1–140.

Fig. 5 shows cycle lengths derived from 110 successive times of light maximum of χ Cygni for the last 150 years. The average period length is $408^{\rm d}.7$, and the apparent standard deviation is $8^{\rm d}.9$. Successive maxima are connected by straight lines, the continuous curve is the running average (bin length 7 datapoints). The diagram shows sequences of intervals where the cycle length seems to be highly erratic ($\sigma = 16$ d over 7 consecutive cycles: long cycles are typically followed by short ones) or much less fluctuating ($\sigma = 6$ d over 7 consecutive cycles). The moving average also suggests that some real period variation is present.

5. Discussion

5.1. The relevance of the $O-C$ diagrams

Eddington & Plakidis (1929) were the first to infer the existence of cumulative errors in the periods of long-period variables. Sterne (1934a,b) demonstrated that it is not necessarily true that curvatures in the $O-C$ diagram *always* indicate period variations: such deviations can also be caused by the cumulative effect of random errors in the lengths of the cycles of the star itself. In the case of Mira stars, for example, Koen (1992) even showed that white noise could lead to an $O-C$ plot with apparently significant indications of the presence of an entirely spurious period variation. The question thus arises whether the interesting structures revealed in Figs. 3 and 4 do stand for changes of the pulsation period.

5.2. Is there a significant period change?

In a first approach we applied Sterne's (1934b) procedure on the almost uninterrupted sequence of $T_{\rm max}$ from $E = 192$ to 280. This results in an estimate of the standard deviation of the measurement error $\sigma_{\rm m} = 5.3$ and standard deviation of the intrinsic period scatter $\sigma_{\rm a} = 7.7$.

The best all-purpose test for changes in the mean period we are aware of is the periodogram-based statistic of Lombard (1998). Fig. 6 shows the periodograms of the mean periods for the times of maximum and minimum. The lower panel ($T_{\rm max}$), especially, shows strong evidence for an increase in power at the lowest frequencies. Unfortunately, Lombard only presents forms of the test that are suitable for either complete datasets, or

Table 1. JD of maximum light $T_{\max}$ and cycle number E.

$T_{\max}$	E	$T_{\max}$	E	$T_{\max}$	E	$T_{\max}$	E	$T_{\max}$	E	$T_{\max}$	E
2337550.6	2	2395619.7	145	2407467.5	174	2420883.3	207	2431111.7	232	2441307.9	257
2342414.6	14	2396014.8	146	2408278.5	176	2421284.8	208	2431937.3	234	2441713.0	258
2342802.9	15	2396427.9	147	2408692.3	177	2421690.9	209	2432344.9	235	2442143.5	259
2344437.7	19	2397224.5	149	2409095.5	178	2422099.7	210	2432752.3	236	2442552.7	260
2348101.9	28	2397639.2	150	2409506.9	179	2422507.2	211	2433164.2	237	2442961.5	261
2348500.5	29	2398062.2	151	2411543.2	184	2422914.9	212	2433575.7	238	2443366.6	262
2351374.2	36	2398452.5	152	2411956.4	185	2423321.9	213	2433970.8	239	2443779.4	263
2351763.0	37	2398867.1	153	2412780.0	187	2423727.5	214	2434393.9	240	2444196.2	264
2356218.8	48	2399267.7	154	2413188.8	188	2424549.6	216	2434808.5	241	2444598.9	265
2359449.5	56	2399680.7	155	2414794.7	192	2424953.9	217	2435202.0	242	2445023.9	266
2363081.1	65	2400507.6	157	2415207.3	193	2425367.9	218	2435615.3	243	2445420.6	267
2363502.2	66	2401317.0	159	2415613.6	194	2425777.2	219	2436023.8	244	2445842.0	268
2372475.2	88	2401744.0	160	2416013.6	195	2426208.9	220	2436413.0	245	2446250.1	269
2372865.0	89	2402161.6	161	2416427.0	196	2426599.6	221	2436819.6	246	2446650.5	270
2373264.5	90	2402559.1	162	2416828.5	197	2427023.8	222	2437232.4	247	2447063.9	271
2384255.4	117	2402985.4	163	2417235.8	198	2427424.9	223	2437635.8	248	2447482.2	272
2384655.8	118	2403410.0	164	2417632.9	199	2427843.9	224	2438029.0	249	2447876.0	273
2385075.2	119	2403809.6	165	2418045.0	200	2428242.8	225	2438432.1	250	2448269.8	274
2386268.7	122	2404220.2	166	2418441.9	201	2428669.6	226	2438843.1	251	2448676.3	275
2386665.6	123	2404621.1	167	2418843.1	202	2429079.6	227	2439242.2	252	2449085.3	276
2387480.2	125	2405031.7	168	2419262.5	203	2429497.4	228	2439655.7	253	2449494.9	277
2393969.7	141	2405836.2	170	2419672.0	204	2429905.0	229	2440064.7	254	2449896.7	278
2394385.9	142	2406239.9	171	2420085.1	205	2430307.6	230	2440475.4	255	2450308.3	279
2395205.6	144	2406653.7	172	2420484.1	206	2430710.4	231	2440891.5	256	2450718.2	280

for incomplete sets with missing values which form a stationary sequence. Because the χ Cyg sequences of times of minimum or maximum are incomplete, a refinement of the method was necessary (see also the discussion in Koen & Lombard 1995). We adopted analogous estimators, and tested their adequacy by simulating 2000 datasets with realistic zero-mean Gaussian measurement errors ($\sigma_m = 5$ d) and intrinsic period scatter ($\sigma_a = 6$ d), using cycle numbers for the the times of maximum and minimum of χ Cyg. Our result establishes that the Lombard statistic is significant at better than the 5% level for the maxima, while the significance level for the minima is between 5% and 10%. We conclude that there is quite strong evidence for a changing mean period in the dataset comprising the times of maximum, while the support for a changing period is somewhat weaker in the other dataset.

5.3. *What is the form of the period change?*

A variety of continuous time state space models as described by Koen (1996) were fitted to the two datasets. The Akaike and Bayes Information Criteria (see e.g. Koen & Lombard 1993) were used to compare the models. The best results, according to these criteria, were a deterministic, linear period change for the maxima, and no period change in the case of the minima. Both the optimal models also required non-zero intrinsic period scatter.

The particularly simple solutions selected by the state space modelling technique imply that we can apply ordinary regression methods to the problem, provided that account is taken of the correlation between successive observations. We used the formulation in Koen (1996), Sect. 2, to find

$$P_E = 405.273\,(0.80) \qquad \sigma_m = 4.5 \qquad \sigma_a = 6.6$$
$$dP/dt = 3.19 \times 10^{-5}\ (1.2 \times 10^{-5})$$

for the maxima (P_E being the period at the start of the observations). The model fit was assessed by checking the transformed residuals, as described by Koen (1996): the residuals were uncorrelated, mean-stationary and Gaussian. The best model for the minima had

$$P_E = 407.8776\,(1.45) \qquad \sigma_m = 5.0 \qquad \sigma_a = 7.4$$

and the residuals again satisfied the relevant tests for randomness, stationarity and normality.

Further models in which the intrinsic period scatter standard deviation was constrained to be the same as for the maxima (i.e. $\sigma_a = 6.6$), were also fitted to the χ Cyg minima. The best such model, again, had a constant period

$$P_E = 407.8935\,(1.31) \qquad \sigma_m = 5.8$$

and the residuals' properties were satisfactory.

A simple quadratic fit $T_{\max} = T_0 + P_E E + 0.5\beta E^2$, where $\beta = PdP/dt$ over the complete range of available data yields $\dot{P} = 0.013\ \mathrm{d\,y^{-1}}$, implying that χ Cygni's pulsation period has increased by about 4 days or 1% since the discovery date three centuries ago.

Table 2. JD of minimum $T_{\rm min}$ and cycle number E.

$T_{\rm min}$	E	$T_{\rm min}$	E	$T_{\rm min}$	E	$T_{\rm min}$	E	$T_{\rm min}$	E	$T_{\rm min}$	E
2406884.2	172	2421119.8	207	2426436.4	220	2432178.9	234	2439884.3	253	2446077.3	268
2410147.2	180	2421513.7	208	2426848.4	221	2432993.3	236	2440297.7	254	2446465.6	269
2414227.9	190	2421932.1	209	2427255.2	222	2433397.5	237	2441128.9	256	2446891.7	270
2417059.9	197	2422326.1	210	2427664.7	223	2433795.9	238	2441546.5	257	2447294.4	271
2417469.1	198	2422747.7	211	2428083.3	224	2434215.0	239	2441965.1	258	2447707.5	272
2417872.1	199	2423142.9	212	2428502.6	225	2434629.9	240	2442380.7	259	2448113.8	273
2418268.2	200	2423565.7	213	2428891.1	226	2435044.0	241	2442795.7	260	2448519.2	274
2418674.3	201	2423973.1	214	2429314.3	227	2435437.2	242	2443215.0	261	2448907.8	275
2419074.1	202	2424375.8	215	2429731.1	228	2436253.7	244	2444026.4	263	2449328.0	276
2419496.3	203	2424786.9	216	2430529.3	230	2437058.4	246	2444431.8	264	2449721.0	277
2419902.2	204	2425197.0	217	2430946.8	231	2437446.3	247	2444855.8	265	2450134.1	278
2420305.8	205	2425611.6	218	2431360.6	232	2437847.1	248	2445254.7	266	2450522.2	279
2420713.2	206	2426042.3	219	2431763.0	233	2438655.1	250	2445661.2	267		

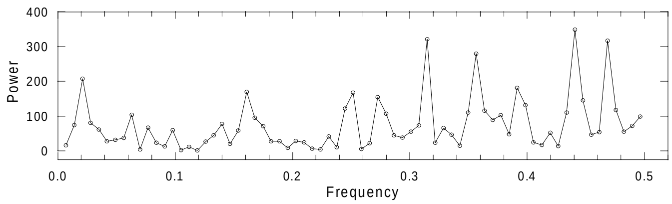

Fig. 7. Periodogram of the residuals after subtraction of a linear period trend (times of maximum).

5.4. *The apparent cycles in the O-C diagram*

The above suggests that there is a systematic linear period change which is only detectable in the times of maximum, with its longer baseline. Nonetheless, the pattern of cycles in the $O-C$ diagrams is seductive, and it must be checked whether there is any evidence for residual period changes once the overall linear change has been prewhitened from the times of maximum. We therefore studied the *untransformed*, (i.e. raw) residuals after subtraction of the linear trend. A little thought shows that Lombard's theory as described above applies also to these residuals. The periodogram of the residuals is shown in Fig. 7: there still appears to be an excess of low-frequency power. However, the residuals' statistics attain significance levels of around 10%, which is not entirely convincing: it may be, for example, that the global period change is slightly nonlinear, so that prewhitening by a linear trend is not quite appropriate.

5.5. *The interpretation of the period changes*

Mira stars are highly-evolved stars with strongly-reduced supplies of nuclear fuel and dense carbon/oxygen cores surrounded by a growing thin layer of helium. They represent a very short-lived phase in stellar evolution, and in the H-R diagram they are located at the top of the AGB. Contrary to popular belief, there is no evidence that Miras systematically evolve to longer periods as they age (Whitelock 1996). When helium-burning begins, the helium shell expands till most of the helium is consumed, the shell shrinks and the star returns to hydrogen burning, and the building up of a new helium shell. This process repeats a number of times, shell ignition and burning being called the "shell flash". Continuous period variation are expected via surface luminosity and radius changes: indeed, period changes consistent with such a flash have been described by Wood & Zarro (1981, herefater WZ), see also Gál & Szatmáry (1995) for the Mira stars R Hya, R Aql, W Dra and T UMa. WZ compared rates of period change with rates of changes predicted by theoretical flash calculations. One of their stars, W Dra ($P = 264^{\rm d}\!.9$), shows an $O-C$ diagram with similar characteristics as seen in Fig. 4, though with a much stronger parabolic character and with relatively less-pronounced short-term fluctuations (note that the time base over which W Dra was observed was only about 100 cycles, or ~ 70 years). WZ place W Dra between points B and C in their Fig. 1, which shows the behaviour of the surface luminosity during a typical fully-developed flash cycle. Barthès & Tuchman (1994) identified the dominant pulsation mode of χ Cygni with the first overtone, and derived a mass of $3.4\,M\odot$, which is in the range of the masses used by WZ. We used this mass estimate in WZ's formulae with their parameter values for first overtone pulsation, and calculated that the luminosity variation corresponding to the period variation $dlog(L/L\odot)$ amounts to 0.00002 y^{-1}, about 20 times less than WZ's result for W Dra. The three-centuries time span of the χ Cyg data, and the low gradient of the luminosity change is compatible with a location of this star after point E in WZ's Fig. 1–that is, during the rise in luminosity when the H shell takes over as the main energy producer. We then calculated $L/L\odot$ as a function of time for a sequence of core masses $M_{\rm c}/M\odot = 0.6$ to 0.9, and find that this rate of luminosity change occurs at about 0.15–0.20 of the total flash cycle length after point E. We stress that the calculated luminosity-change rate and the derived epoch within the flash cycle are not critically sensitive to numerical values of the parameters entering the equations. Indeed, using the extreme values of the various coefficients in WZ and the lowest quoted mass ($0.9\,M\odot$, Barthès 1998) will result in only a small variation of $dlog(L/L\odot)$.

The interpretation outlined above, evidently, cannot account for any alleged short-term variations seen in the $O-C$ diagram.

6. Conclusions

There is quite strong evidence for a linearly increasing period in the dataset of maximum times, the evidence for a period change in the dataset of minima is weak. There is substantial cycle-to-cycle variation in the pulsations (standard error ~ 6.5 d). There is no compelling evidence that the apparent cyclical period changes are real; the residuals left after fitting a linear period increase plus intrinsic period scatter to the maxima (or intrinsic period scatter only, to the minima) have properties consistent with being white noise. The Lombard frequency domain test for a changing period applied to the residuals, also gives unconvincing significance levels. The low gradient of the luminosity change with period is compatible with the star undergoing a rise in luminosity when H shell-burning takes over.

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