

Quantum plasmadynamics

I. Unmagnetized plasmas

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Preface

The idea of synthesizing quantum electrodynamics (QED) and the kinetic theory of plasmas first occurred to me in the early 1970s. The project to do so has been carried out bit by bit over the subsequent years. The name “quantum plasmadynamics” (QPD) is my own jargon for the synthesized theory.

Both QED and the kinetic theory of plasmas are concerned with the same underlying physics: the interaction between charged particles and the electromagnetic field. Differences between the two theories include: (1) QED is a quantum mechanical theory and is usually formulated in terms of quantized fields, whereas the kinetic theory of plasmas is a purely classical. (2) QED is essentially a single-particle theory in which the electrons are assumed to have no effect on the dispersion of the photons, whereas the kinetic theory of plasmas is a collective-medium theory in which the response of the medium causes the natural wave modes to have properties that can be quite different from those of electromagnetic waves in vacuo. (3) QED is conventionally formulated as a covariant theory with the field described in terms of the 4-potential whose gauge is arbitrary, whereas the kinetic theory of plasmas is conventionally formulated in terms of 3-vectors and 3-tensors and in terms of fields rather than potentials.

The synthesis of these two theories involves four basis steps. The first is to formulate the classical kinetic theory in a covariant manner. The next step involves generalizing QED to allow for the dispersive properties of an ambient medium. This requires separating the total system of particles and electromagnetic field into background and wave subsystems. The quantization of the wave subsystems is then trivial. The third step involves extending QED to include Feynman diagrams that describe additional processes that are possible in a medium but not in a vacuum. Formally, these terms arise from the interaction terms in the re-arrangement of Lagrangians of the free particle and free field systems into the background and wave subsystems in the medium. The final step is to use QED to calculate the response tensor of the medium.

An important generalization is the inclusion of a static magnetic field, and this generalization is needed in both the classical covariant formalism

and in QED. This separates the development of the theory into four parts: classical covariant and QED theories for unmagnetized and magnetized systems. In separating the material into two volumes I needed to decide whether to keep have the classical covariant parts in one volume and the QED parts in another, or two have one volume on unmagnetized systems and the other on magnetized systems. After first trying the former, I finally opted for the latter.

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