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High-spin states in boson models with applications to actinide nuclei

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Abstract

We use the $1/N$ expansion formalism in a systematic study of high-spin states in the sd and sdg boson models with emphasis on spin dependence of moment of inertia and E2 transitions. The results are applied to the high-spin states in the actinide nuclei ^{232}Th , $^{234-238}\text{U}$, where the need for g bosons is especially acute but until now, no realistic calculation existed. We find that the d -boson energy plays a crucial role in description of the high-spin data.

One of the early predictions of the the interacting boson model [1] (IBM) is the so called boson cutoff effect. Due to finite number of bosons N in the system, the ground band terminates at spin $L = l_{\text{max}}N$, where l_{max} denotes the maximum boson spin. As a result, yrast $B(E2)$ values are predicted to fall off relative to the rotor values, vanishing at the maximum spin. In the sd -IBM, the cutoff occurs at $L = 2N$ which is low enough to be experimentally accessible and, therefore, has prompted many investigations (see, for example, [2–6] When the predicted falloffs did not materialize in any of the experiments this was interpreted as due to insufficient collectivity of the sd -boson system at high-spins which could be ameliorated by including g bosons [7–9]. Since then, evidence for g bosons from low-spin spectra have continuously grown with concurrent developments in the sdg -IBM calculations (see [10,11] for recent reviews). Ironically, the high-spin data, which have been the harbinger of g bosons, could not be treated in realistic sdg -IBM calculations.

This happens because the basis space for deformed nuclei ($N = 12 - 16$) is too large and has to be severely truncated for diagonalization, limiting their validity to low-lying states. Although the $SU(3)$ limit has been used in discussing some features of high-spin states in the sdg -IBM [7–9], realistic calculations indicate that this limit is rather strongly broken [12]. Hence it can only provide a qualitative picture but otherwise is not very useful in applications to spectroscopy, or in addressing some basic questions on the shortcomings of the sd -IBM raised by Bohr and Mottelson [13], e.g. the spin dependence of moment of inertia. An unfortunate consequence of the lack of sdg -IBM calculations is that the sd -IBM results are still being used in comparisons with high-spin data with obvious negative connotations (see [6,14] for recent examples in experimental and theoretical studies).

The angular momentum projected mean field theory, which leads to a $1/N$ expansion for physical quantities [15], presumably offers the only viable alternative for a realistic description of high-spin states in the sdg -IBM. An obstacle in realizing this goal, namely

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evaluation of matrix elements up to order $(L/N)^6$ which is necessary for an accurate representation of high-spin states, has been recently overcome through the use of computer algebra [16]. The purpose of this letter is to use the extended $1/N$ expansion formulas in a systematic study of high-spin states (before the backbend) in boson models. We address, in particular, the moment of inertia and boson cutoff problems in the sd -IBM, and discuss how far they are resolved with the inclusion of g bosons. Applications are made to the actinide nuclei, ^{232}Th , $^{234-238}\text{U}$, for which the ground band has been followed up to spins $L > 2N$, and hence the need for g bosons is most acute.

The $1/N$ expansion method has previously been discussed in detail [15] and the recent extensions to higher orders are given in [16]. Therefore, we give only a short account of the formalism here. The starting point of the $1/N$ calculations is the boson condensate

$$|N, \mathbf{x}\rangle = (N!)^{-1/2} (b^\dagger)^N |0\rangle, \quad b^\dagger = \sum_l x_l b_{l0}^\dagger, \quad (1)$$

where b_{l0}^\dagger denote the boson creation operators and x_l the associated mean fields. The summation index runs over $l = 0, 2$ in the sd model and $l = 0, 2, 4$ in the sdg model. The axial symmetry assumption in Eq. (1) has been justified in [16] by comparing the $1/N$ expansion results with the exact diagonalization ones. For a given Hamiltonian H , one evaluates the projected energies $E_L = \langle N, \mathbf{x} | H P_{00}^L | N, \mathbf{x} \rangle$ and determines x_l by variation after projection (VAP) [15]. The resulting energy expression is a double expansion in $1/N$ and $\bar{L} = L(L+1)$, and has the generic form

$$E_L = N^2 \sum_{n,m} \frac{e_{nm}}{(aN)^m} \left(\frac{\bar{L}}{a^2 N^2} \right)^n, \quad (2)$$

where $a = \sum_l \bar{L} x_l^2$ and the expansion coefficients e_{nm} involve various quadratic forms of the mean fields x_l . The coefficients e_{nm} have recently been derived up to the order \bar{L}^3/N^6 [16]. Since the N dependence is not relevant to the discussion, we will suppress it for simplicity and rewrite the energy formula (2) as

$$E_L = e_1 \bar{L} + e_2 \bar{L}^2 + e_3 \bar{L}^3, \quad (3)$$

where the coefficients e_n can be read off from the expressions given in [16]. We note that similar expressions are used in the geometrical model analysis

of deformed nuclei [17]. The difference between the two models is that in the IBM the coefficients e_n follow from an underlying Hamiltonian (which is used in describing other properties) whereas in the geometrical model they are directly extracted from the data. The moment of inertia problem raised in [13] refers to the fact that in the sd -IBM, (i) the e_1 coefficient gets a substantial contribution from the dipole interaction, $L \cdot L$, which has no dynamical content, and (ii) the e_2 coefficient is much smaller than the experimental values. The two problems are in fact interrelated. Although the latter can be resolved by renormalizing the moment of inertia at high-spins (e.g. by modifying $L \cdot L \rightarrow L \cdot L / (1 + fL \cdot L)$ [18]), such modifications are purely kinematical in origin and do not address the dynamical problem.

Another observable of interest in the study of high-spin states is the yrast E2 transitions which have the generic form

$$\langle L+2 \| T(\text{E2}) \| L \rangle = \alpha N \hat{L} \langle L020 | L+20 \rangle [m_1 + m_2 L(L+3)], \quad (4)$$

where α is an effective boson charge, $\hat{L} = [2L+1]^{1/2}$ and the coefficients m_n are given in [16]. The first term in (4) gives the familiar rigid-rotor result. The second term is negative and is responsible for the falloffs predicted in E2 transitions.

In order to compare the sd and sdg model predictions, we first present a brief study of the sd -IBM results. We use the standard Hamiltonian

$$H = -\kappa Q \cdot Q + \kappa' L \cdot L + \varepsilon_d n_d, \quad (5)$$

where L and n_d are the angular momentum and d -boson number operators, and the quadrupole operator is given by

$$Q = [s^\dagger \bar{d} + d^\dagger \bar{s}]^{(2)} + \chi [d^\dagger \bar{d}]^{(2)}. \quad (6)$$

Here brackets denote tensor coupling of the boson operators and $\bar{b}_{lm} = (-1)^m b_{l-m}$. For consistency, the same quadrupole operator is used in the E2 transition operator, $T(\text{E2}) = \alpha Q$ (4) as in the Hamiltonian (5). Since $L \cdot L$ does not play any role in the dynamics of the system, we will not discuss it further (it can be easily restored by changing $e_1 \rightarrow e_1 + \kappa'$). In presenting systematics, we find it convenient to use ratios which eliminate the undesired effects of the scale parameters

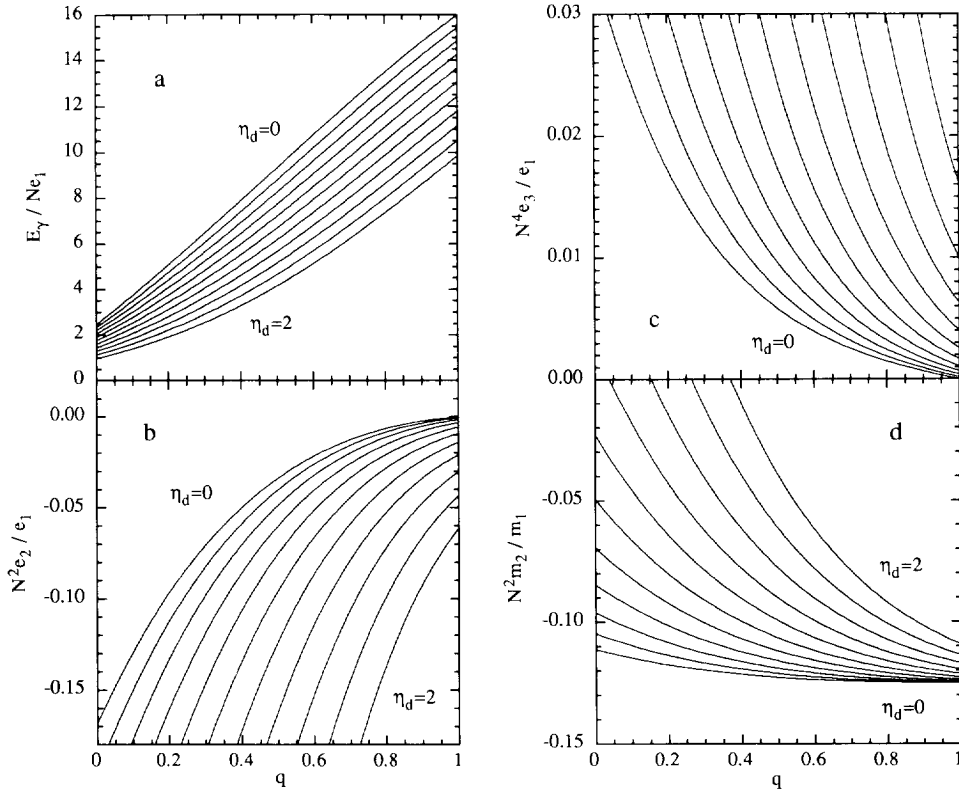


Fig. 1. Systematic study of moment of inertia (a), its spin dependence (b), (c), and the boson cutoff effect in E2 transitions (d) in the *sd*-IBM. The quadrupole parameter q is normalized to 1 in the SU(3) limit, and the d -boson energy parameter $\eta_d = \varepsilon_d/N\kappa$ is varied from 0–2 in 10 equal steps.

κ and N . The energy scale can be fixed, for example, by fitting κ to the excitation energy of the γ band, E_γ . In Fig. 1, we show four such quantities as a function of $q = \chi/\chi_{\text{SU3}}$ for various values of $\eta_d = \varepsilon_d/N\kappa$. We comment on their behaviour and contrast them with the experimental data below.

(a) E_γ/Ne_1 : This ratio relates the energy scales of the γ and ground bands, and its mismatch with experiment has been a source of criticism [13]. It is around 4–5 in the rare-earth region and increases to 8–10 in the actinides. The SU(3) limit ($q = 1, \eta_d = 0$) is seen to give the maximum value which overestimates it by a factor of 2–4. It decreases rapidly with ε_d and q though, and through a judicious use of these parameters, it should be possible to describe this ratio (and hence the the moment of inertia) without using the $L \cdot L$ term.

(b) $N^2 e_2/e_1$: This ratio measures the deviation from the rigid rotor behaviour (SU(3) limit) due

to loss of pairing. It ranges from about -0.2 in the rare-earth region to -0.1 in the actinides. Clearly, it can not be explained by the standard *sd*-IBM Hamiltonians currently in use for deformed nuclei which assume $\eta_d = 0, q \sim 0.4$ – 0.5 . However, it is quite sensitive to η_d values and the experimental range can be easily attained by including the d -boson energy in the Hamiltonian.

(c) $N^4 e_3/e_1$: There is some uncertainty in the extraction of this ratio from data, especially in the rare-earth region. In the actinides, it is about 0.01. It depicts even more sensitivity to both q and η_d , and therefore its description should not pose any problems.

(d) $N^2 m_2/m_1$: As there is no boson cutoff effect, experimentally this ratio is consistent with zero. For $\eta_d = 0$, it remains rather flat at the SU(3) value which gives the maximum possible effect. Introduction of one-body energy, however, reduces it substantially, becoming more in line with experiments.

The d -boson energy has been mostly neglected in studies of deformed nuclei, presumably due to the success of the consistent- Q formalism (variable χ with $\varepsilon_d = 0$) in explaining the energy and E2 transition systematics of low-lying states [12]. In fact, for small values ($\eta_d \sim 1$), its effect on low-lying states is negligible and it is not really needed in their description [19]. The above analysis indicates that breaking of the SU(3) limit by either the pairing interaction [13] or by varying the χ parameter [12] does not lead to a soft enough energy surface which is the main reason for the perceived moment of inertia problem in the sd -IBM. The obvious way towards a softer energy surface is to include the d -boson energy in the Hamiltonian which is seen to vastly improve the description of the spin-dependent terms in the ground energies and E2 transitions.

We next present a similar study in the sdg -IBM. A minimal extension of the sd -IBM to the sdg model can be achieved by including the g -boson energy term, $\varepsilon_g n_g$ in the Hamiltonian (5), and modifying the quadrupole operator (6) to

$$Q = [s^\dagger \tilde{d} + d^\dagger \tilde{s}]^{(2)} + q_{22}[d^\dagger \tilde{d}]^{(2)} + q_{24}[d^\dagger \tilde{g} + g^\dagger \tilde{d}]^{(2)} + q_{44}[g^\dagger \tilde{g}]^{(2)}. \quad (7)$$

The quadrupole parameters q_{24} and q_{44} play an important role in the description of the hexadecapole bands but otherwise, the ground band properties are not very sensitive to their variations. In order to limit the number of parameters, we scale the three quadrupole parameters q_{22} , q_{24} , q_{44} from their SU(3) values with a single factor q as suggested by microscopics [20]. In Fig. 2, we show the ratios in Fig. 1 as a function of q for various values of $\eta_g = \varepsilon_g/N\kappa$ with $\eta_d = 0$. Before commenting on specific ratios, we point out some general features. For large η_g , the g bosons decouple and the results converge to those of the sd model shown in Fig. 1. This convergence is apparent from the overlap of lines in Figs. 2a and 2b but requires even larger values of η_g in the case of Figs. 2c and 2d. It is harder to pin down realistic values for η_g due to lack of data, nevertheless, we quote the literature values for comparison which range from 3–6.

(a) E_γ/Ne_1 : Inclusion of g bosons increases this ratio which is contrary to the experimental trend. However, for realistic η_g values, this adverse change is too small to worry about.

Table 1
Parameters used in the sdg -IBM calculations

Nucleus	N	κ	q	η_d	η_g	α
^{232}Th	12	14.3	0.7	1.85	4.2	0.210
^{234}U	13	15.6	0.7	1.62	3.1	0.195
^{236}U	14	16.2	0.7	1.62	3.1	0.195
^{238}U	15	16.3	0.7	1.67	3.1	0.195

κ is in keV and α in eb.

(b) $N^2 e_2/e_1$: This ratio also increases (in absolute value) which is good but again too small for realistic η_g values to have any impact.

(c) $N^4 e_3/e_1$: This ratio shows some sensitivity to g bosons, however, it is nowhere near the effect of η_d in Fig. 1c, and therefore not likely to have much relevance.

(d) $N^2 m_2/m_1$: The boson cutoff was the original reason for the introduction of g bosons and it is clear from this figure why. In the SU(3) limit, this ratio is reduced by a factor of 4 compared to the sd -IBM. Its q and η_g dependence, however, is opposite to that of Fig. 1d, and things get worse away from the SU(3) limit. For realistic parameters, the reduction from the sd -IBM result (with $\eta_d = 0$) is less than 40% which is certainly not enough, and one needs the d -boson energy to reduce it further.

The somewhat surprising conclusion of the above systematic study is that introduction of the g bosons, though necessary to describe states with $L > 2N$, hardly improves the dynamics of the boson system. The problems attributed to the sd -IBM are, in fact, due to not having a soft enough energy surface and can only be resolved by including the d -boson energy in the Hamiltonian (and not by introduction of g bosons alone).

In the light of the systematic trends discussed above, we carry out fits to the actinide nuclei, ^{232}Th , $^{234-238}\text{U}$, for which the ground band has been followed up to spins $L > 2N$, and hence the need for g bosons is most acute but no realistic IBM calculation yet exists. The parameters used in the fits are shown in Table 1. The ground band energies (normalized with $L(L+1)$ for a fairer representation of all spins) are shown in Fig. 3. A good description of the ground band energies is obtained for all four nuclei with relative errors of about 1%. It is worth emphasizing that the moment of inertia and its spin dependence are explained

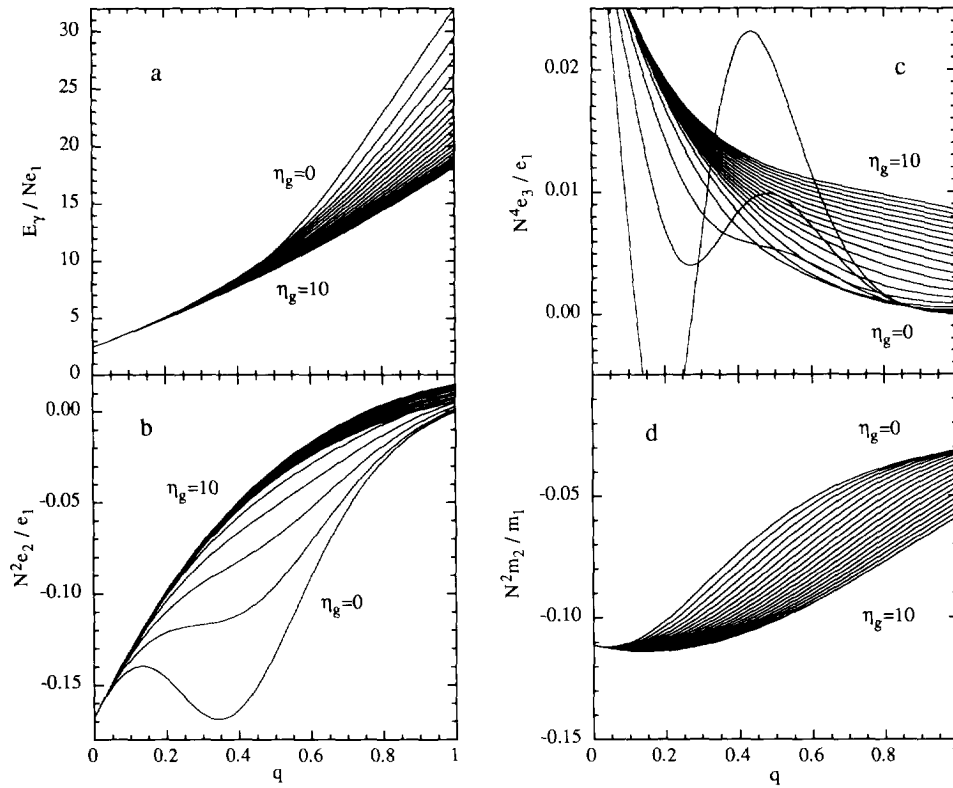


Fig. 2. Same as Fig. 1 but in the sdg -IBM. The quadrupole parameters q_{22}, q_{24}, q_{44} are scaled from their $SU(3)$ values with a single factor q , $\eta_d = 0$ and the g -boson energy parameter $\eta_g = \epsilon_g / N\kappa$ is varied from 0–10 in 20 equal steps.

Table 2
Comparison of the β and γ band energies (in keV) and E2 transitions (in eb) with the sdg -IBM calculations in the actinide region

Nucleus	E_β		E_γ		$\langle 2_\beta T(E2) 0_g \rangle$		$\langle 2_\gamma T(E2) 0_g \rangle$	
	Cal.	Exp.	Cal.	Exp.	Cal.	Exp.	Cal.	Exp.
^{232}Th	731	730	786	785	0.27	0.33 ± 0.07	0.40	0.37 ± 0.07
^{234}U	804	809	872	927	0.21	< 0.25	0.35	0.36 ± 0.08
^{236}U	903	919	978	958	0.22	–	0.37	–
^{238}U	949	993	1040	1060	0.22	0.24 ± 0.05	0.37	0.37 ± 0.04

The data are from [21].

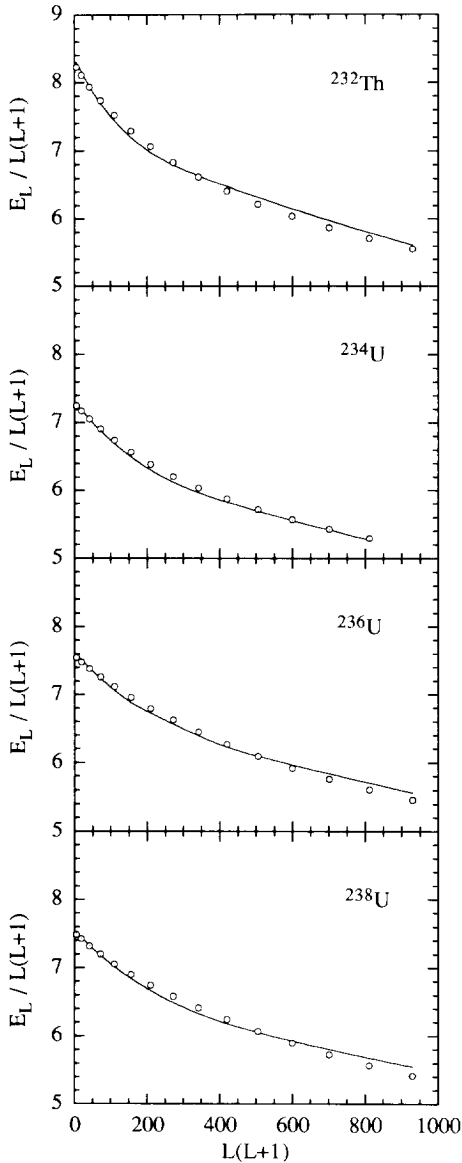


Fig. 3. Comparison of the experimental (circles) and calculated (solid lines) ground band energies $E_L/L(L+1)$ (in keV) in the actinide nuclei. The data are from [21].

without appealing to the $L \cdot L$ term or its phenomenological variations. To appreciate the importance of the d -boson energy in the fits, we note from Fig. 2b that with $\eta_d = 0$, the slopes of the curves in Fig. 3 would be an order of magnitude smaller, leading to almost flat lines like in a rigid rotor. In Fig. 4, the calculated E2 transitions along the ground band are compared to

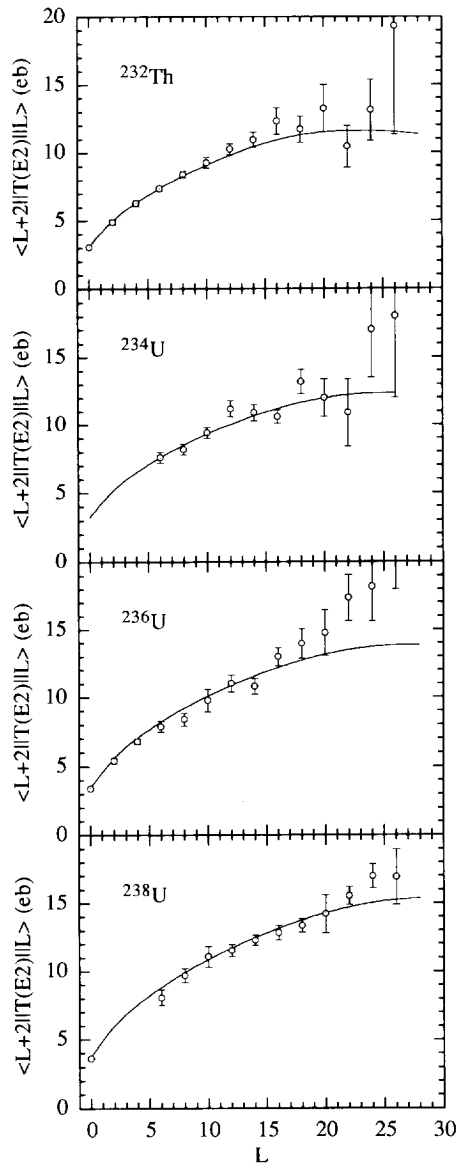


Fig. 4. Comparison of the experimental (circles) and calculated (solid lines) yrast E2 transitions in the actinide nuclei. The data are from [3,21].

experiment. Again a reasonably good description of the data is obtained. Finally, to show that the ground band results are not achieved at the expense of other bands, we compare in Table 2 the predictions for the β , γ band head energies and $E2(2_{\beta,\gamma} \rightarrow 0_g)$ matrix elements with experiment. Note that these calculations are the leading order $1/N$ expansion results and hence

are correct to order $1/N$. The good agreement obtained in Table 2 verifies that the inclusion of the d -boson energy does not detract from the usual quality of the β and γ band systematics achieved in the IBM calculations.

In conclusion, we reiterate that the perceived problems with the sd -IBM in its description of spin dependent quantities is not due to lack of higher spin bosons but rather due to the energy surface not being soft enough. Inclusion of the d -boson energy, together with the extension to sdg space, can successfully resolve these problems as demonstrated in the fits to the actinide nuclei. The g bosons are necessary for extending the model space but otherwise they play a marginal role in the dynamics of the ground band and can not resolve alone the problems mentioned above. Details of the present work, including applications to the rare-earth nuclei, will be presented in a longer paper [22].

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