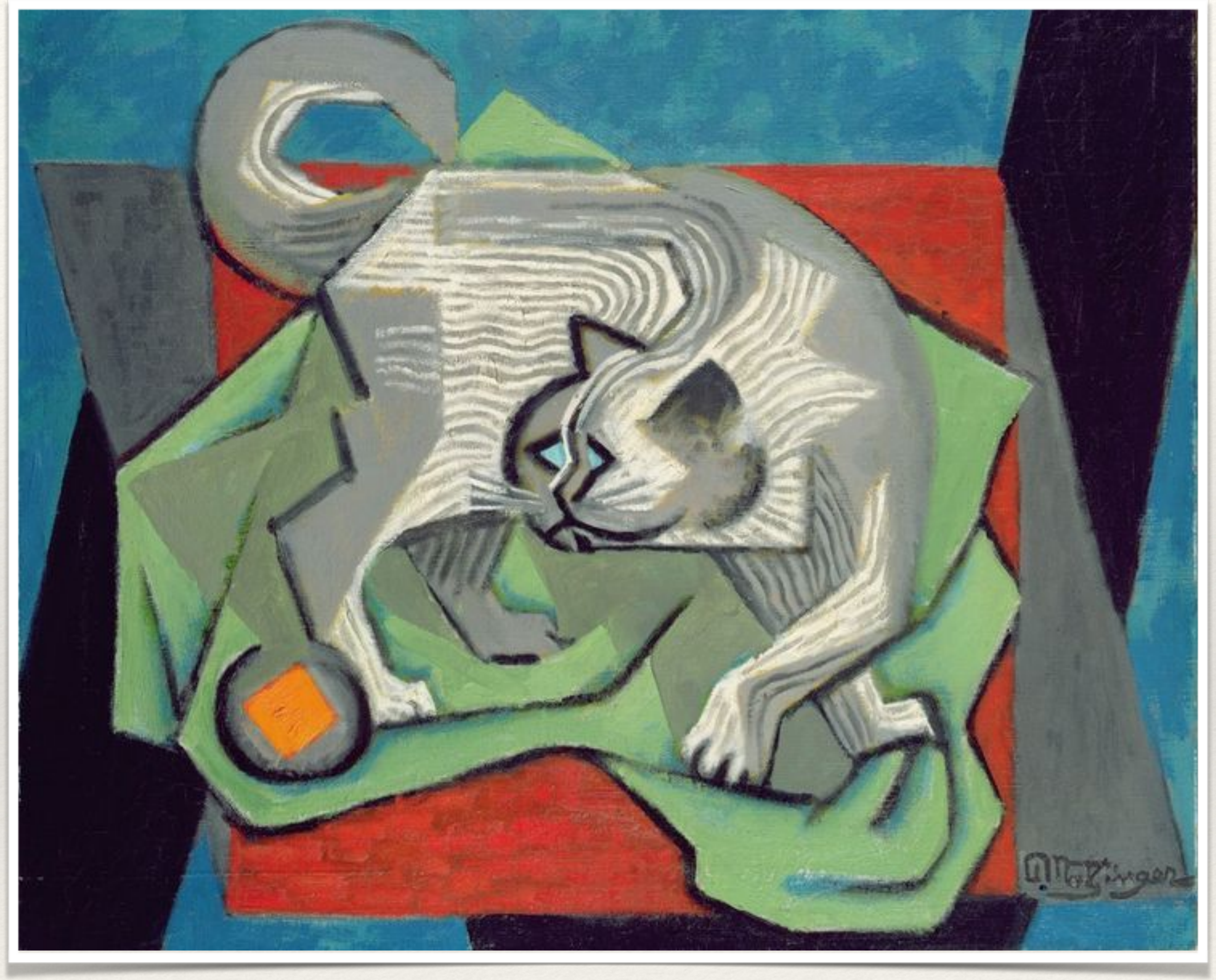
Guest lecture by Dr. Arne Grimsmo

Quantum Mechanics

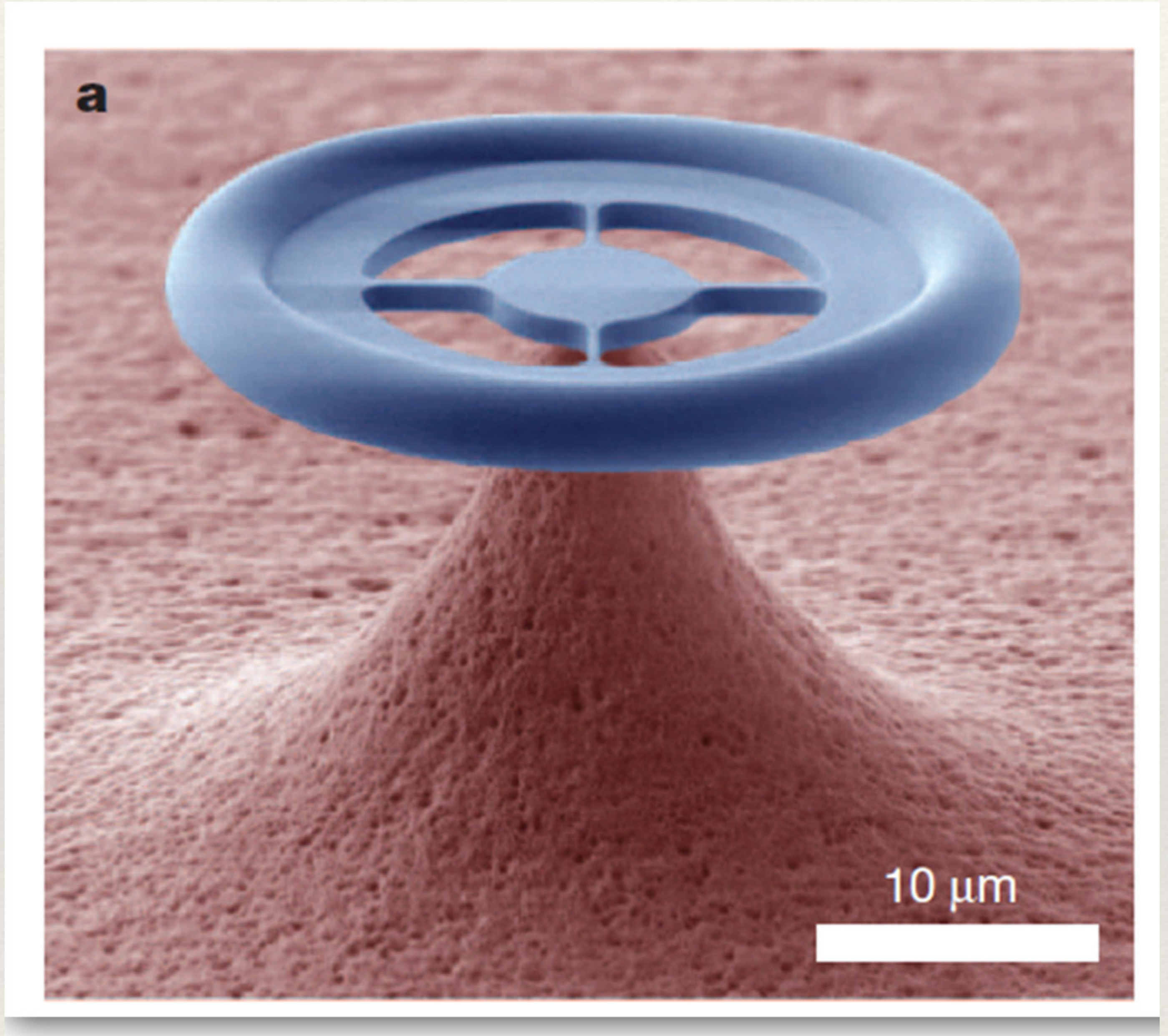
Lecture 19 (non-examinable)

Cat states;
Encoding quantum information in harmonic oscillators.

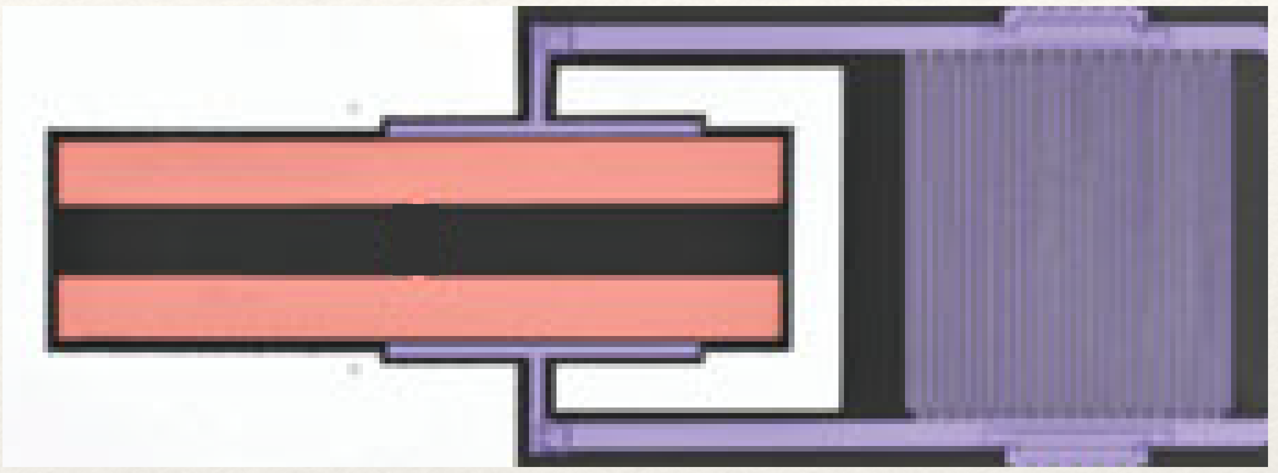


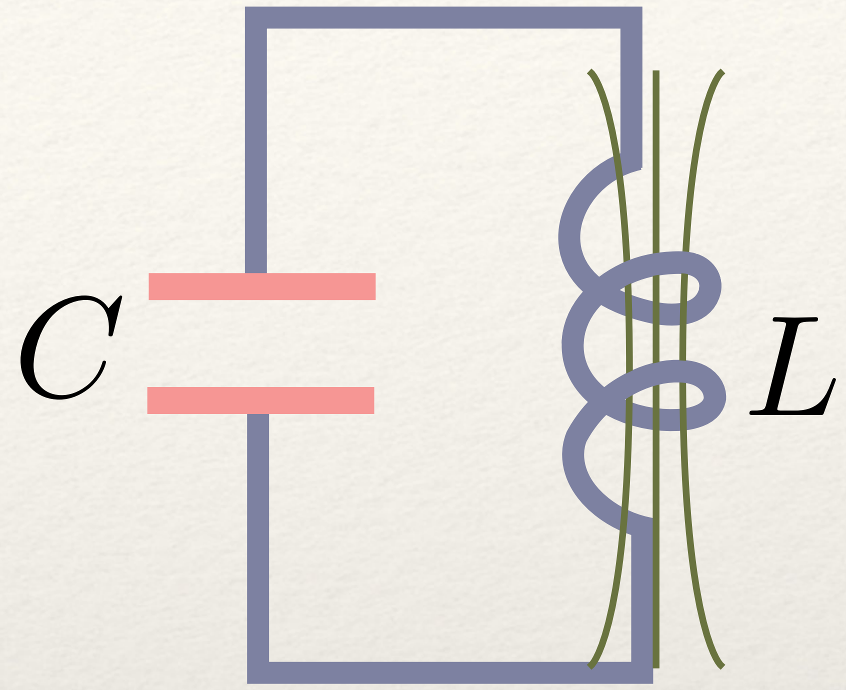
Quantum harmonic oscillators in real life

Mechanical

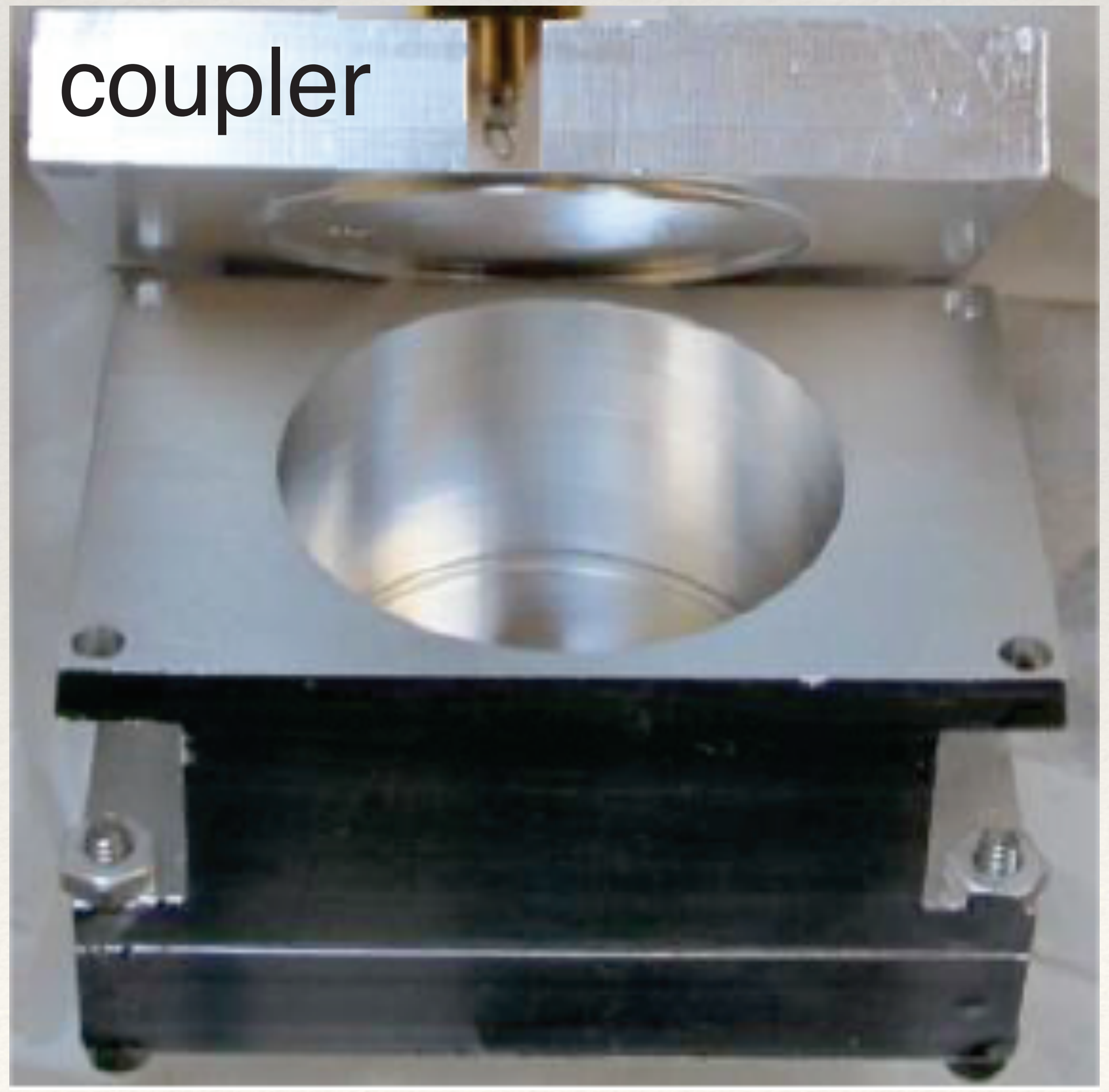


Electrical





Electromagnetic
fields in cavities



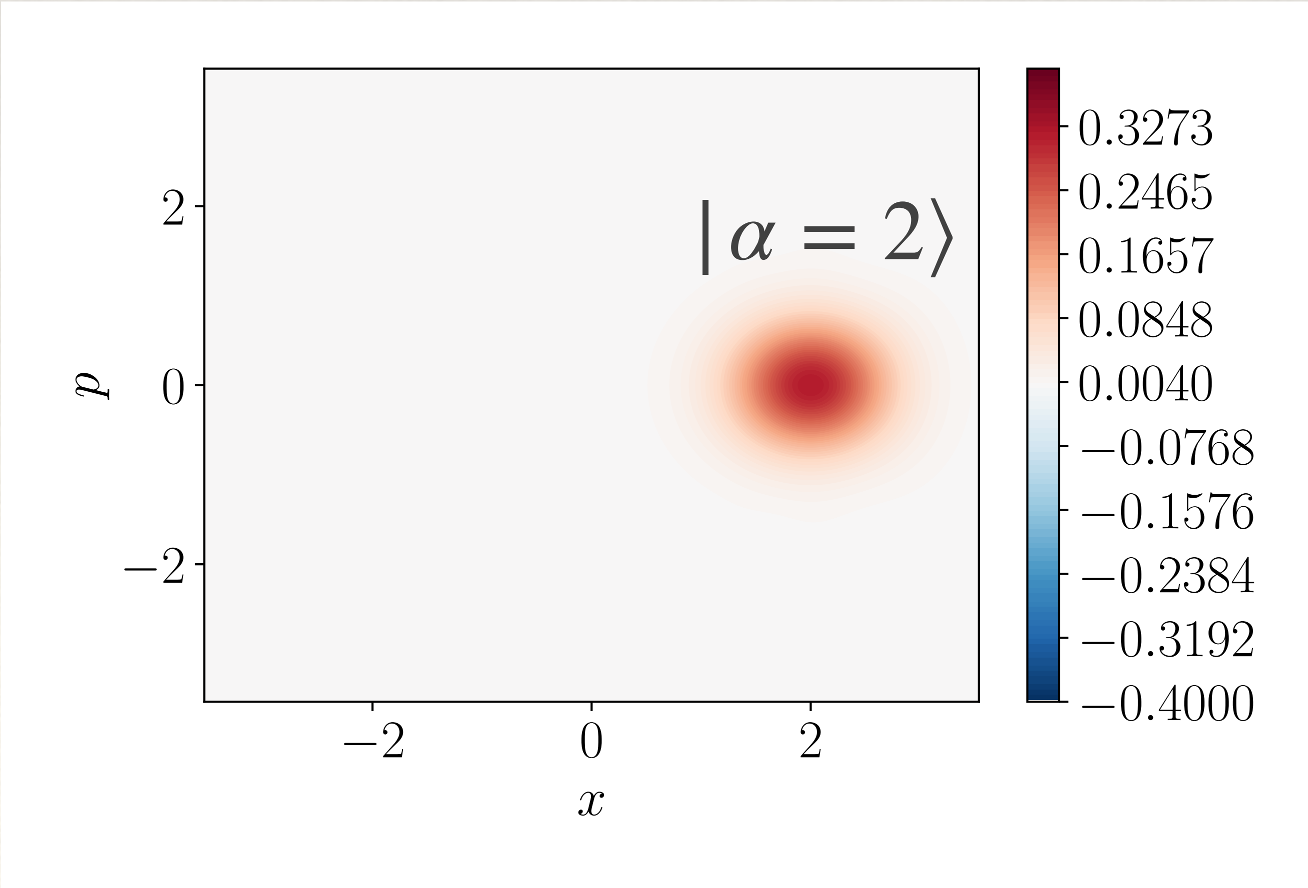
Quantum phase space and Wigner functions

One way to make sense of quantum phase space is with the *Wigner function*. Starting from a wave function ψ, we transform it as follows:

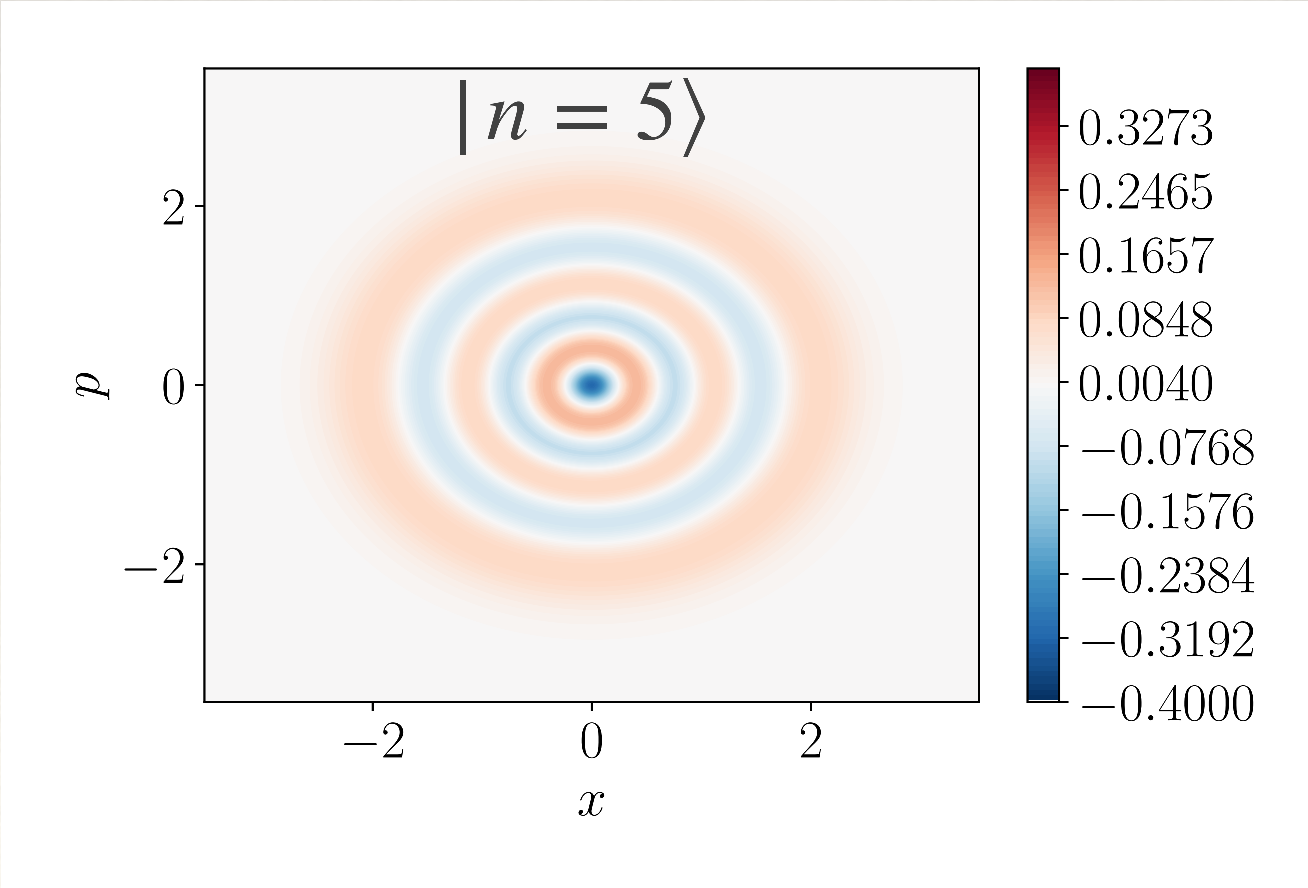
$$W(x,p) = \frac{1}{\pi\hbar}\int_{-\infty}^{\infty} \langle x-y\,|\,\psi\rangle\langle\psi\,|\,x+y\rangle\, \mathrm{e}^{2iyp/\hbar}\,\mathrm{d}y$$

W(x,p) can be used to visualize quantum states

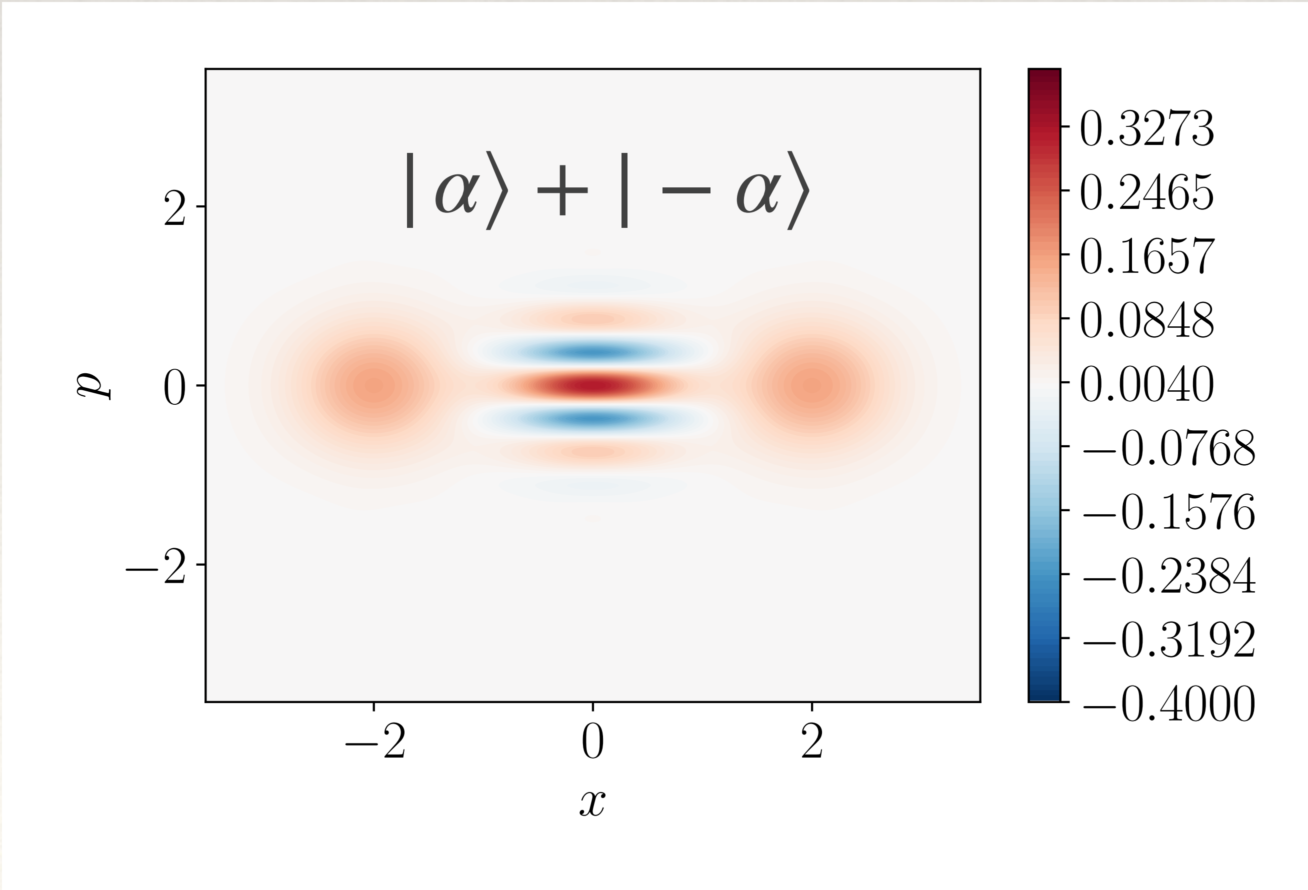
coherent state



number state



"cat state"



Bits and qubits: From classical to quantum information

A single bit takes a binary value {0, 1} and is the "unit" of classical information

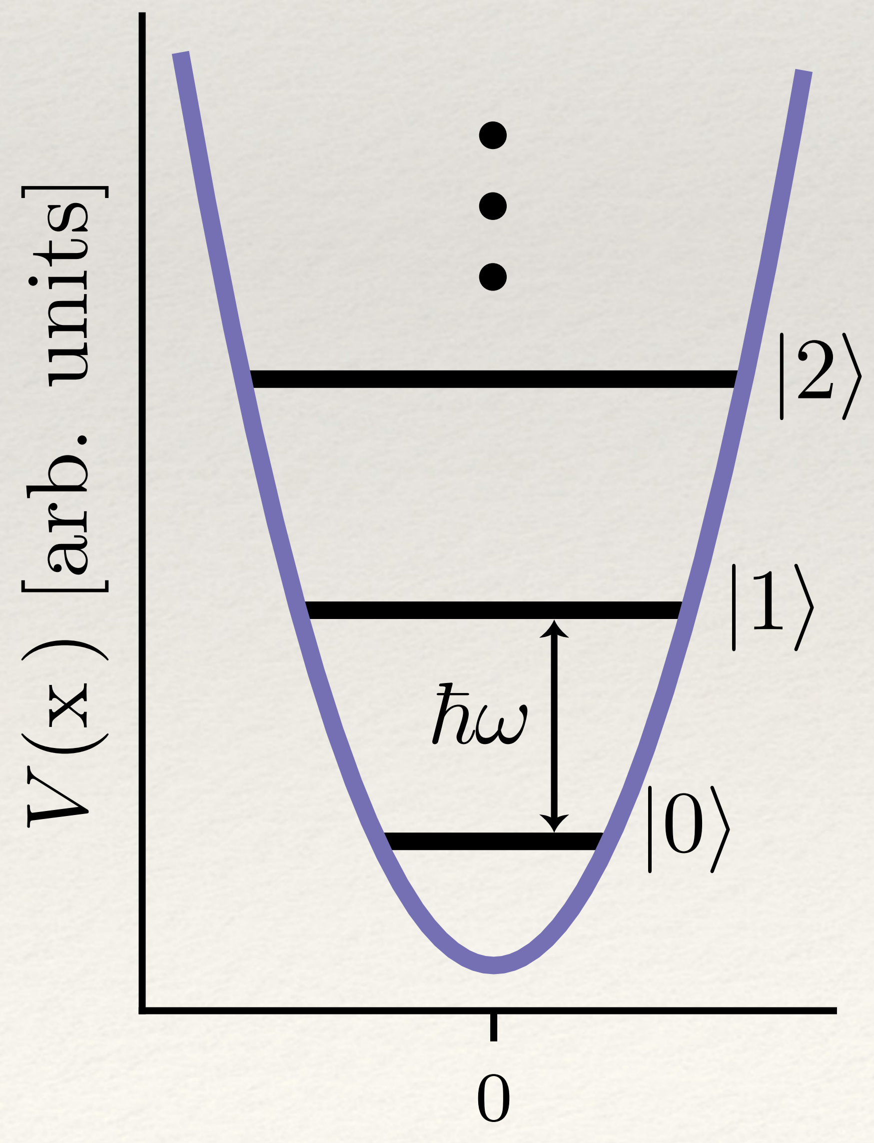
Is there a quantum analog, a "unit" of quantum information?

The "qubit": $\{|0\rangle, |1\rangle\}$ $\qquad c_0|0\rangle + c_1|1\rangle$

In principle any pair of quantum states will do

For example, the two lowest states of a quantum harmonic oscillator

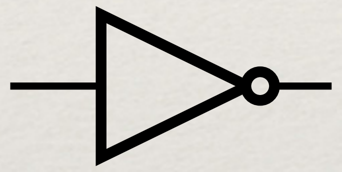
$$|n=0\rangle, |n=1\rangle$$



Manipulating classical information

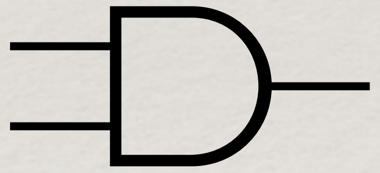
Any classical computation can be generated using only a very small set of "logic gates" acting on 1 or 2 bits at a time, e.g.

NOT



A	not A
0	1
1	0

AND



A	B	A and B
0	0	0
0	1	0
1	0	0
1	1	1

Manipulating quantum information

Quantum "logic gates" are unitary *operators* acting on a state

One-qubit gate: $U|\psi\rangle = U\left(c_0|0\rangle + c_1|1\rangle\right)$

(2x2 matrix)

Two-qubit gate: $U|\psi\rangle = U\left(c_{00}|0\rangle|0\rangle + c_{01}|0\rangle|1\rangle + c_{10}|1\rangle|0\rangle + c_{11}|1\rangle|1\rangle\right)$

(4x4 matrix)

Example: $\mathrm{NOT} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \hat{\sigma}_x$

A	not A
$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$

What are quantum computers good for?

Quantum computers can (in theory) solve in a matter of days/hours/weeks some computational problems that would take a conventional computer longer than the lifetime of the universe. The potential is vast…

Chemistry and material simulations (drug discovery, new materials etc.)

Optimization problems

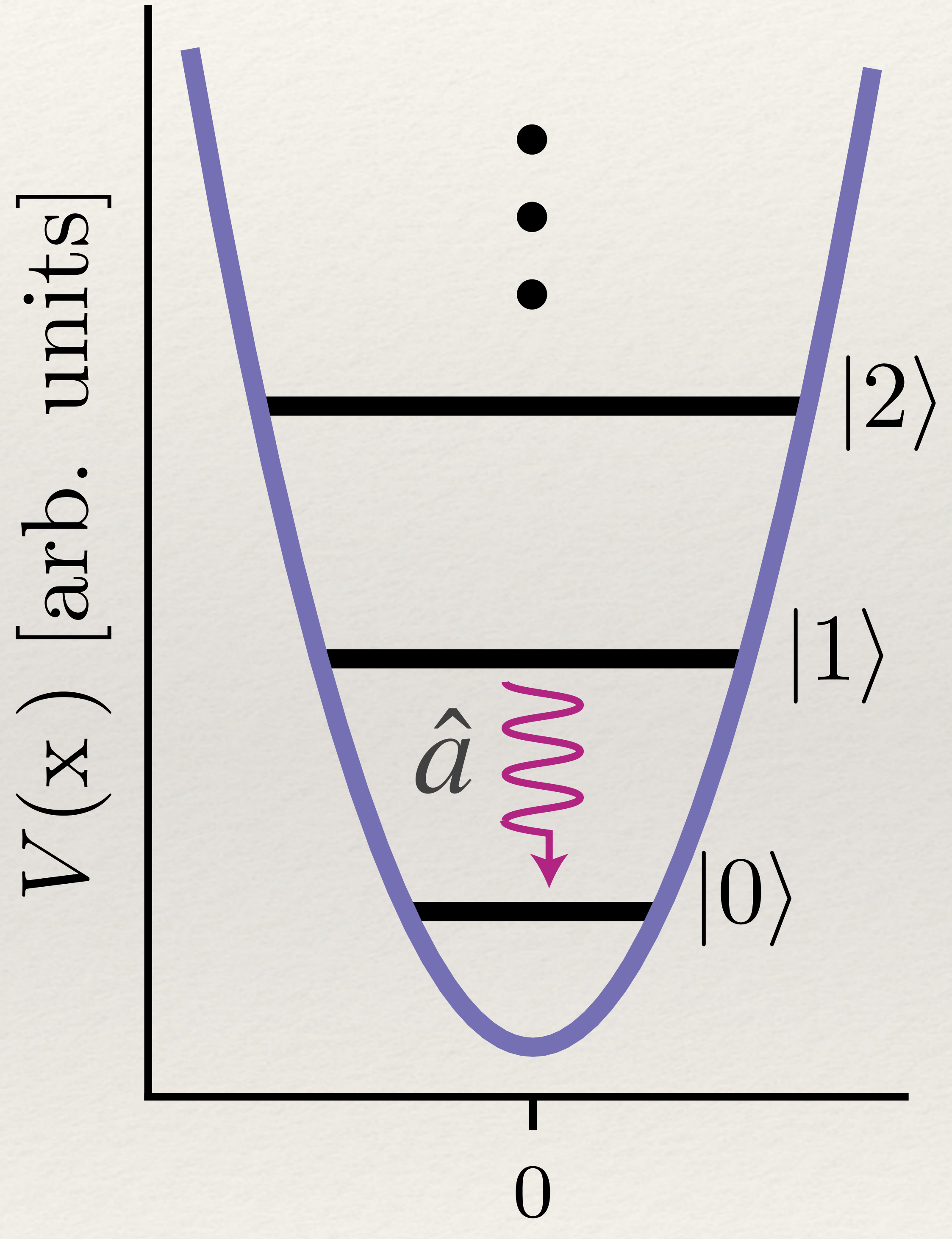
Machine learning

Save the environment, cure cancer, end poverty, …

Why don't we have (useful) quantum computers yet?

The fundamental problem: qubits are extremely fragile

Energy loss $\hat{a}(c_0|0\rangle + c_1|1\rangle) = c_1|0\rangle$

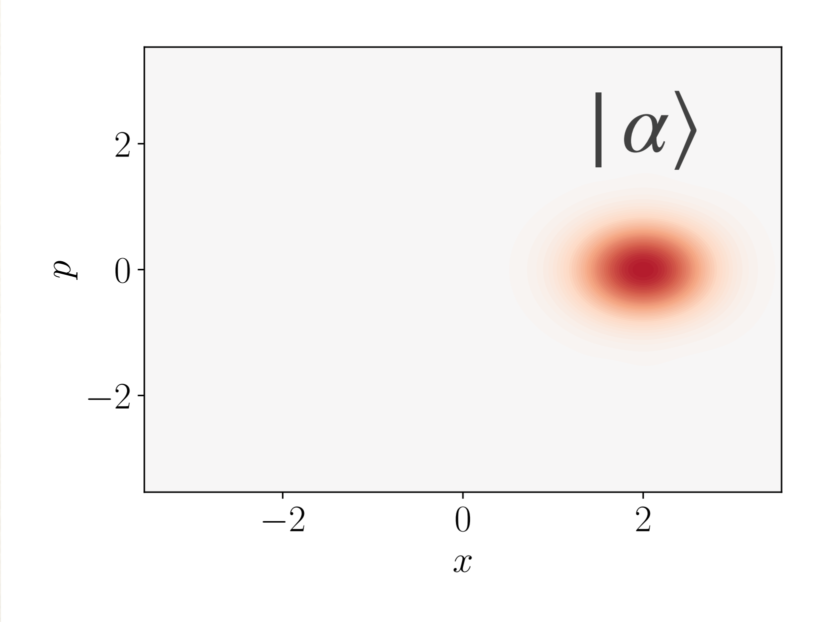


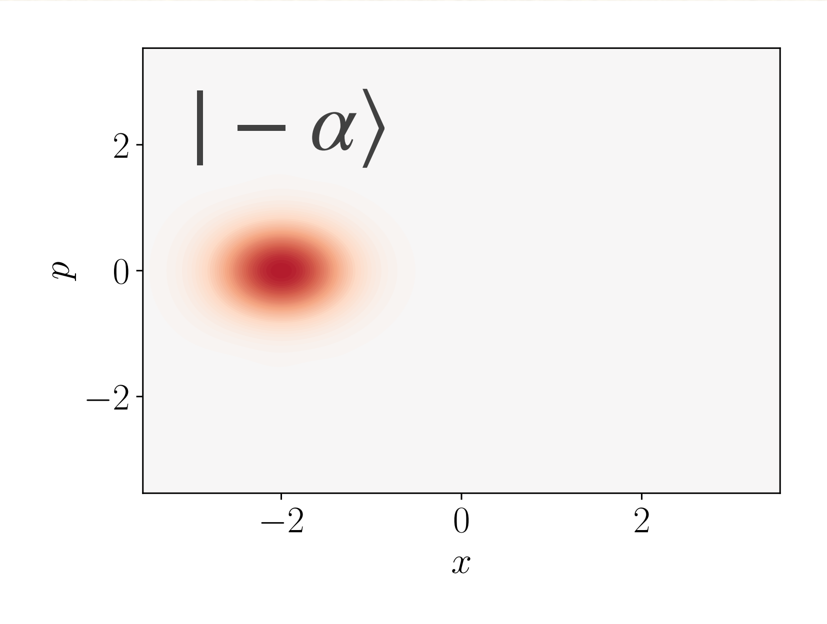
Encoded information is lost!

In practice, many other sources of errors, but energy loss is often the dominant cause of faults.

Encoding quantum information robustly

Idea: Use coherent states as logical states





$$|0\rangle \to |\alpha\rangle, |1\rangle \to |-\alpha\rangle$$

States not destroyed: $\hat{a}|\pm\alpha\rangle \propto \pm|\alpha\rangle$ Good!

But superpositions not preserved:

$$\hat{a}(c_0|\alpha\rangle + c_1|-\alpha\rangle) \propto c_0|\alpha\rangle - c_1|-\alpha\rangle$$

$c_1 \to -c_1$

this is called a "phase error"

Bad!

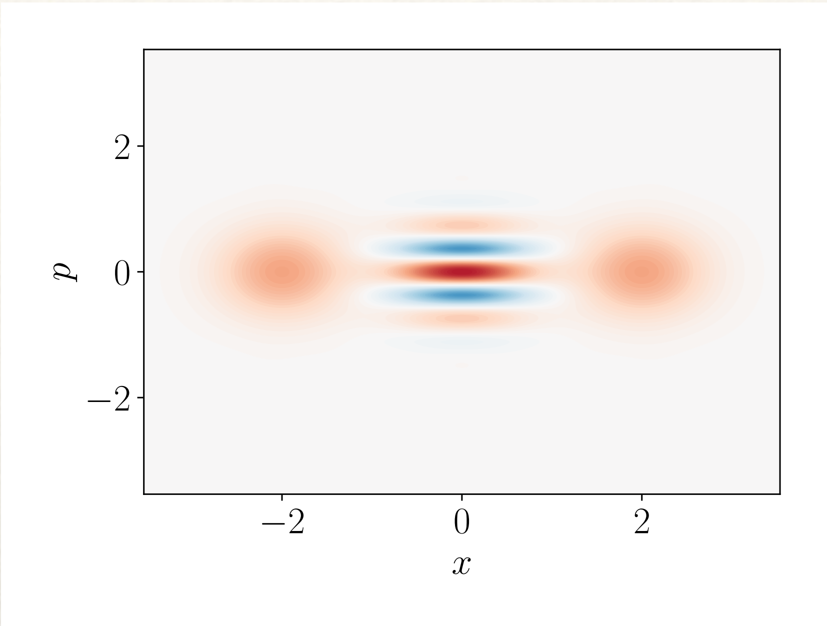
So this did not quite work… can we fix it?

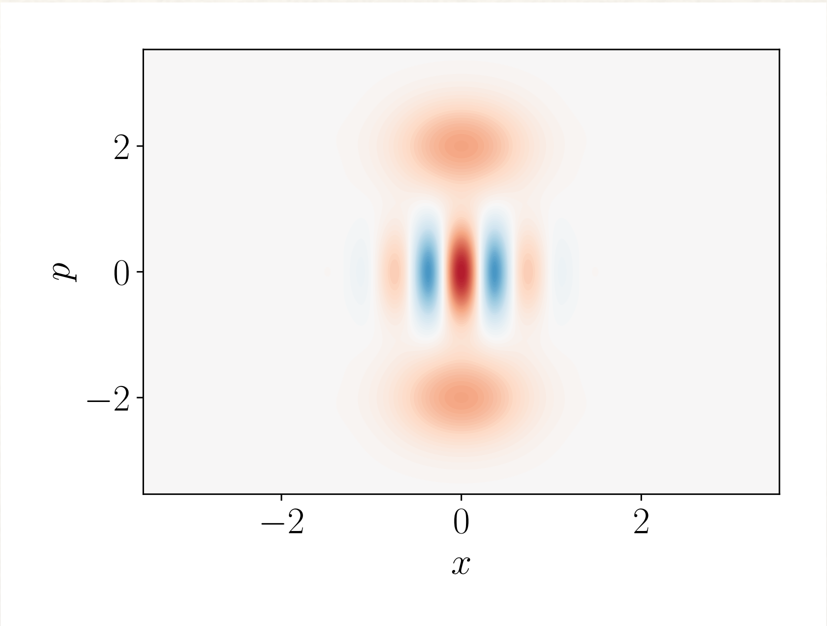
Cat-state qubits

New idea: Use *superpositions* of coherent states as logical states

$$|0\rangle \to |0_L\rangle = |\alpha\rangle + |-\alpha\rangle$$

$$|1\rangle \to |1_L\rangle = |i\alpha\rangle + |-i\alpha\rangle$$





States not destroyed, but they do change:

$$\hat{a}|0_L\rangle \propto |\alpha\rangle - |-\alpha\rangle =: |0_L'\rangle$$

$$\hat{a}|1_L\rangle \propto |i\alpha\rangle - |-i\alpha\rangle =: |1_L'\rangle$$

$$\hat{a}(c_0|0_L\rangle + c_1|1_L\rangle) \propto (c_0|0_L'\rangle + c_1|1_L'\rangle)$$

Quantum information in principle preserved, even though the states changed!

Cat-state qubits

$$|0\rangle \rightarrow |0_L\rangle = |\alpha\rangle + |-\alpha\rangle \qquad\qquad |1\rangle \rightarrow |1_L\rangle = |i\alpha\rangle + |-i\alpha\rangle$$

$$\hat{a}(c_0|0_L\rangle + c_1|1_L\rangle) \propto (c_0|0'_L\rangle + c_1|1'_L\rangle)$$

If we can find a way to detect that the error has happened, we can simply update the "encoding" $|0_L\rangle, |1_L\rangle \rightarrow |0'_L\rangle, |1'_L\rangle$

But how do we do this without destroying the encoded information?

What does the measurement need to distinguish, and what must it *not* distinguish?

Detecting errors

Let's have a look at what the states look like in the number basis

$$|0_L\rangle = |\alpha\rangle + |-\alpha\rangle = C_\alpha \sum_{n=0}^{\infty} \frac{\alpha^n + (-\alpha)^n}{\sqrt{n!}} |n\rangle = 2C_\alpha \sum_{n=0}^{\infty} \frac{\alpha^{2n}}{\sqrt{(2n)!}} |2n\rangle$$

$$|0_L'\rangle = |\alpha\rangle - |-\alpha\rangle = C_\alpha \sum_{n=0}^{\infty} \frac{\alpha^n - (-\alpha)^n}{\sqrt{n!}} |n\rangle = 2C_\alpha \sum_{n=0}^{\infty} \frac{\alpha^{2n+1}}{\sqrt{(2n+1)!}} |2n+1\rangle$$

Recall: $$|\alpha\rangle = C_\alpha \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

Detecting errors

Similarly

$$|0_L\rangle = 2C_\alpha \sum_{n=0}^{\infty} \frac{\alpha^{2n}}{\sqrt{(2n)!}} |2n\rangle \qquad |0'_L\rangle = 2C_\alpha \sum_{n=0}^{\infty} \frac{\alpha^{2n+1}}{\sqrt{(2n+1)!}} |2n+1\rangle$$

$$|1_L\rangle = 2C_\alpha \sum_{n=0}^{\infty} \frac{(i\alpha)^{2n}}{\sqrt{(2n)!}} |2n\rangle \qquad |1'_L\rangle = 2C_\alpha \sum_{n=0}^{\infty} \frac{(i\alpha)^{2n+1}}{\sqrt{(2n+1)!}} |2n+1\rangle$$

Even number parity

Odd number parity

Measure number parity to detect error!

Is there an observable for number parity? $\hat{\Pi} = (-1)^{\hat{n}} = (-1)^{\hat{a}^\dagger \hat{a}}$

A protocol for storing quantum information robustly

1. Encode

$$|0\rangle \rightarrow |0_L\rangle = |\alpha\rangle + |-\alpha\rangle \qquad |1\rangle \rightarrow |1_L\rangle = |i\alpha\rangle + |-i\alpha\rangle$$

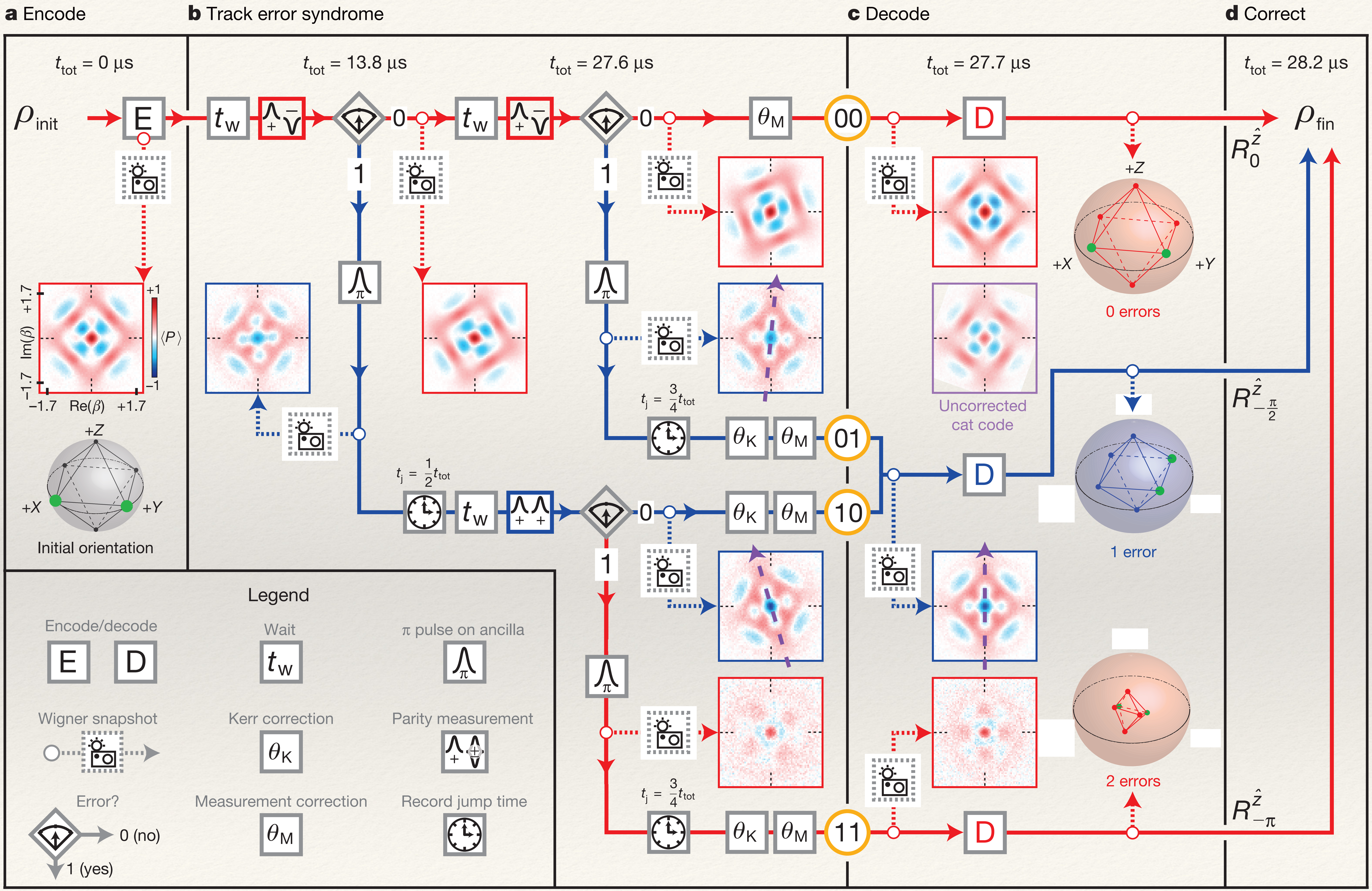
2. Check for errors by measuring number parity

$$\hat{\Pi} = (-1)^{\hat{n}} = (-1)^{\hat{a}^\dagger \hat{a}}$$

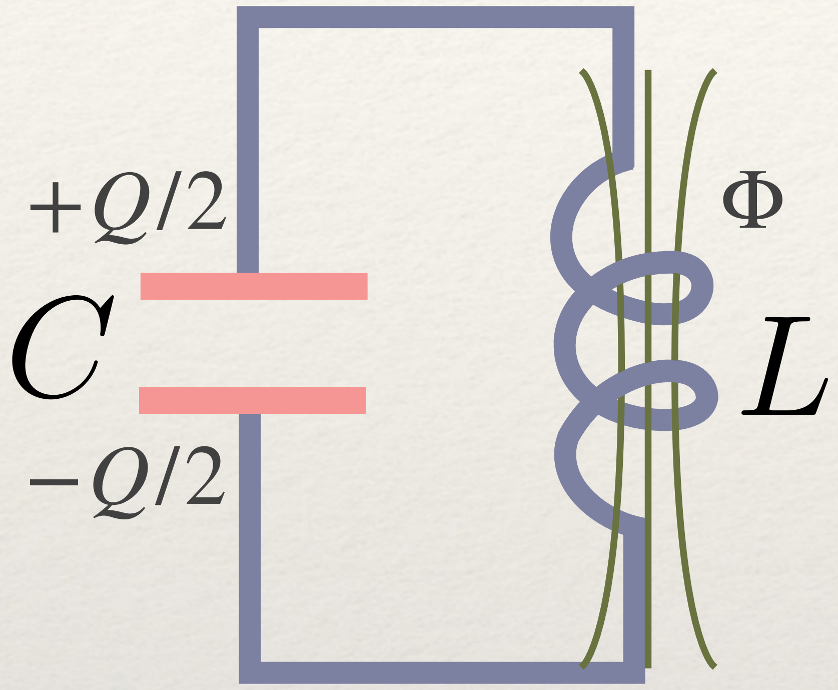
3. Re-define encoding as needed if parity has changed

$$|0_L\rangle \rightarrow |0'_L\rangle \qquad |1_L\rangle \rightarrow |1'_L\rangle$$

4. Repeat 2. & 3. for as long as we need to store the information


a Encode
b Track error syndrome
c Decode
d Correct
t_{tot} = 0 µs
t_{tot} = 13.8 µs
t_{tot} = 27.6 µs
t_{tot} = 27.7 µs
t_{tot} = 28.2 µs
ρ_{init}
ρ_{fin}
$R_0^{\hat{z}}$
$R_{-\frac{\pi}{2}}^{\hat{z}}$
$R_{-\pi}^{\hat{z}}$
$t_j = \frac{1}{2}t_{tot}$
$t_j = \frac{3}{4}t_{tot}$
00
01
10
11
Im(β)
Re(β)
−1.7
+1.7
⟨P⟩
+Z
+X
+Y
Initial orientation
Uncorrected cat code
0 errors
1 error
2 errors
Legend
Encode/decode
Wait
π pulse on ancilla
Wigner snapshot
Kerr correction
Parity measurement
Error?
0 (no)
1 (yes)
Measurement correction
Record jump time

Extra slide: The quantum LC oscillator



Classical energy

$$H = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$$

Q = charge
Φ = magnetic flux

Rewrite by defining $\omega = 1/\sqrt{LC}$

$$H = \frac{Q^2}{2C} + \frac{1}{2}C\omega^2\Phi^2$$

Harmonic oscillator with “mass” C, “position” Φ and “momentum” Q

$$\hat{H} = \frac{\hat{Q}^2}{2C} + \frac{1}{2}C\omega^2\hat{\Phi}^2 \qquad [\hat{\Phi}, \hat{Q}] = i\hbar$$