Lecture 12 - Momentum

Linear Momentum of a Body

We define momentum of an object as:

\[ \mathbf{p} = m \mathbf{v} \]

where \( m \) = mass and \( \mathbf{v} \) = velocity.
\( \mathbf{p} \) is a vector and is in the same direction as \( \mathbf{v} \).

(Don’t confuse \( p \) with power or pressure.)

Units: kg.m.s\(^{-1}\)

Momentum and Newton’s 2nd Law

If \( \mathbf{F}_{\text{net}} \) and \( m \) are constant;

\[ \mathbf{F}_{\text{net}} = ma = m \frac{\Delta \mathbf{v}}{\Delta t} \]

\[ = \frac{\Delta (mv)}{\Delta t} = \frac{\Delta \mathbf{p}}{\Delta t} \]

Newton originally expressed his second law in terms of momentum.

If \( m \) or \( \mathbf{F} \) are NOT constant then:

\[ \mathbf{F}_{\text{net}} = \frac{d\mathbf{p}}{dt} \]

Example 7.8 from Hecht

Calculate the recoil speed of a pistol (mass 0.90 kg) given that the bullet has mass 8.0 g and emerges from the pistol with a speed of 352 ms\(^{-1}\). Assume the momentum of the exhaust gas is negligible.

\[ \mathbf{p}_i = \mathbf{p}_f = \mathbf{0} \quad \text{(both pistol and bullet initially @ rest)} \]

Consider horizontal components only

\[ \mathbf{p}_f = m_B \mathbf{v}_B + m_P \mathbf{v}_P = 0 = 0.008 \text{kg} \cdot 352 \text{ms}^{-1} + 0.9 \text{kg} \cdot \mathbf{v}_P \]

\[ \mathbf{v}_P = -0.008 \text{kg} \cdot 352 \text{ms}^{-1} / 0.9 \text{kg} = -3.1 \text{ms}^{-1} \]

(speed = 3.1 ms\(^{-1}\))

Impulse (1)

Impulse \( J \) of a force is defined as;

\[ J = \int \mathbf{F} \, dt \]

or \( J = \Delta \mathbf{p} = \mathbf{F}_{\text{av}} \Delta t \)

However, if \( \mathbf{F} \) is not constant;

Consider a ball hitting a wall:

\[ \mathbf{v}_i \]

\[ \mathbf{v}_f \]

\[ \mathbf{F} \]

acting for time \( \Delta t \)

\[ J = \Delta \mathbf{p} \]

You can minimise the average force \( F_{\text{ave}} \) during an impact, by increasing the “impact time” \( \Delta t \)

e.g. seat belts, driver’s air bag, wicket-keeper’s glove, thick landing mat for high jumper.

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Impulse (2)

Impulse (3)

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