Assignments & quiz

- Two assignments for this module
  (see module handout)
- Due dates:     Friday 28th May
  Friday 11th June
- Two questions out of four will be marked.
- There will be a quiz (not assessed) in Lecture 6
  (27th May).

Oscillations

- Oscillations are periodic (repetitive) motion.
- They involve motion relative to a position of equilibrium.
- When displaced from equilibrium, a restoring force tends to try and restore the system to equilibrium.
- “Inertia” ensures that the oscillating object overshoots equilibrium position, so cycle begins again.
- Period \( T \): time (in s) for one complete cycle of motion.

Relation between \( T, f, \omega \)

- Frequency \( f = 1/T \, s^{-1} \) or \( 1/T \) Hz. The frequency is the number of oscillations per second.
  (units: hertz or Hz, also “cycles per sec”)
- The angular frequency \( \omega \) has units \( \text{rads}^{-1} \):
  \[
  \omega = \frac{2\pi}{T} = 2\pi f
  \]
- It corresponds to the angular speed of the representative point on the reference circle.

Example 13.1

An ultrasonic generator oscillates at \( f = 6.7 \, \text{MHz} \). What is the period and angular frequency?

**Solution:**

\[
T = \frac{1}{f} = 1/(6.7 \times 10^9 \, \text{Hz})
= 1.5 \times 10^{-9} \, \text{s} = 0.15 \, \mu\text{s}.
\]

\[
\omega = 2\pi f = (2\pi \, \text{rad/cycle}) (6.7 \times 10^9 \, \text{cycles/s})
= 4.2 \times 10^9 \, \text{rad/s}.
\]

Simple harmonic motion

- Many mechanical systems vibrate in a particularly simple manner.
- If acceleration is proportional \( - \) (displacement):
  \[
a = \frac{d^2x}{dt^2} = -Cx, \quad (C \text{ is } +ve)
\]
  (This is the equation of motion - EoM.)
- Then we call this special kind of oscillation “simple harmonic motion” (SHM).
- Since \( F = ma \), restoring force also proportional to \(-x \).
Typical examples of SHM

- Mass on spring
- Pendulum (for small oscillations)
- Tuning fork
- Quartz crystal in your watch

Simple harmonic motion (3)

- $x(t) = A \cos(\omega t + \phi)$ is the amplitude of the oscillation, which represents the maximum displacement from equilibrium.

- $x(t) = A \cos(\omega t + \phi)$ is the phase of the oscillation, or the exact point in the cycle.

- The amplitude and initial phase $\phi$ are initial conditions not determined by the equation of motion.

Simple harmonic motion (2)

- Many kinds of periodic motion are not SHM, but can often be approximated by SHM especially for small amplitude oscillation.

- SHM can be described by a “harmonic” function (sine or cosine):
  $x(t) = A \cos(2\pi \nu T + \phi)$

SHM: Velocity & acceleration

- Proof that harmonic functions satisfy the EoM:
  Differentiating with respect to time,
  $\dot{x}(t) = -\omega A \sin(\omega t + \phi)$
  $\ddot{x}(t) = -\omega^2 A \cos(\omega t + \phi)$

- Thus we get the proportionality constant in the definition of SHM: $a = -\omega^2 x$.

SHM & circular motion

- An object moves with uniform angular velocity $\omega$ in a circle.

- The projection of the motion onto the $x$ axis is
  $x(t) = A \cos(\omega t + \phi)$

- The projected velocity & acceleration also agree with SHM.

- Every kind of SHM can be related to a motion around an equivalent reference circle.