The SHM force law

- Hooke’s law for springs:
  \[ F = -kx \]
  (where \( k \) is the “force constant”, \( x \) is displacement from equilibrium).
- Then, \( a = \frac{F}{m} = \frac{dx}{dt} = -k \frac{x}{m} \)
- The equation of motion results in SHM
- The angular frequency \( \omega \) depends on \( m \) and \( k \).
  From before, \( \omega^2 = \frac{k}{m} \), or
  \[ \omega = \sqrt{\frac{k}{m}} \]

**Example 13.2**

A horizontal spring is fixed at one end, then stretched 3.0 cm by a force of 6.0 N. Afterwards, a 0.50 kg body is attached, pulled 2.0 cm from its rest position, and released. Find \( k \), \( \omega \), \( f \), \( T \).

**Solutions:**
- \( k = \frac{|F|}{x} = 6.0 \text{ N}/0.030 \text{ m} = 200 \text{ N/m} \)
- \( \omega = \sqrt{\frac{k}{m}} = \sqrt{(200 \text{ kg}.\text{s}^{-2}/0.50 \text{ kg})} = 20 \text{ rad/s} \)
- \( f = \frac{\omega}{2\pi} = 20 \text{ rad.s}^{-1}/2\pi \text{ rad} = 3.2 \text{ Hz} \)
- \( T = \frac{1}{f} = 0.31 \text{ s} \)

**Example 13.3**

In the system of example 13.2
- \( k = 200 \text{ N/m}, \ m = 0.50 \text{ kg} \)
- If \( x_0 = +1.5 \text{ cm} \) and \( v_0 = +0.40 \text{ m/s} \), find \( T, A, \phi \).
- Write equations for \( x(t), v(t), a(t) \)

**Solution:**
- \( T \) is the same as before, 0.31 s, since it doesn’t depend on \( x_0 \) or \( v_0 \).
- \( \omega = \sqrt{\frac{k}{m}} \) & \( \omega = 2\pi/T \)
- \( A = \sqrt{(0.015 \text{ m})^2 + (0.40 \text{ m/s})^2 / (20 \text{ rad/s})} = 0.025 \text{ m} \)

**Example 13.3 (2)**

\( \phi = \tan^{-1} \left( \frac{-0.40 \text{ m/s}}{0.015 \text{ m} / 20 \text{ rad/s}} \right) = \tan^{-1}(-\frac{1}{4}) = -53^\circ = -0.93 \text{ rad} \)

\( x(t) = 0.025 \text{ m} \cos[20 \text{ rad/s}(t - 0.93 \text{ rad})] \)

\( v(t) = -0.50 \text{ m/s} \sin[20 \text{ rad/s}(t - 0.93 \text{ rad})] \)

\( a(t) = -10 \text{ m/s}^2 \cos[20 \text{ rad/s}(t - 0.93 \text{ rad})] \)

Check that we recover \( x_0, v_0 \) at \( t = 0 \! \)!

**Example 13.4**

In the system of example 13.2
- \( k = 200 \text{ N/m}, \ m = 0.50 \text{ kg} & A = 2.0 \text{ cm} \)

Find \( v_{\text{max}}, a_{\text{max}}, \ \nu, \ a, \ E, \ K \) when \( x = 1.0 \text{ cm} = 0.5A \).

**Solution:**
- \( v_{\text{max}} = \omega A = A \sqrt{k/m} = 0.40 \text{ m/s} \), and
- \( a_{\text{max}} = \omega^2 A = \frac{k}{m} A = 8.0 \text{ m/s}^2 \).

Because \( ME \) is constant, \( \frac{1}{2} mv^2 + \frac{1}{2} kx^2 = \frac{1}{2} kA^2 \)

SO...

**Example 13.4 (2)**

- \( |v| = \omega \sqrt{A^2 - x^2} = \omega A \sqrt{1 - (x/A)^2} = 0.35 \text{ m/s} \)
- \( |a| = \omega^2 x = (20 \text{ rad/s})^2 (0.010 \text{ m}) = 4.0 \text{ m/s}^2 \)
- \( U = \frac{1}{2} kx^2 = 0.5 \times 200 \text{ N/m} \times (0.010 \text{ m})^2 = 0.10 \text{ J} \)
- \( K = \frac{1}{2} m v^2 = 0.5 \times 0.50 \text{ kg} \times (0.35 \text{ m/s})^2 = 0.030 \text{ J} \)
- \( E = K + U = 0.040 \text{ J} \)
Example 13.6

A 980 N person gets into a 1000 kg car and it sinks 2.8 cm. The shock absorbers are “shot”; when they hit a bump, they oscillate up and down in SHM. Find $T$, $f$.

Solution:

First find $k$: $k = \frac{F}{\Delta h} = \frac{980 \text{ N}}{0.028 \text{ m}} = 3.5 \times 10^4 \text{ kg/s}^2$

Then $T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{1000 \text{ kg} + \frac{980 \text{ N}}{9.80 \text{ m/s}^2}}{3.5 \times 10^4 \text{ kg/s}^2}} = 1.11 \text{ s}$. $f = 0.90 \text{ Hz}$

Example 13.8

Find $T$ and $f$ of a simple pendulum 1.000 m long, where $g = 9.800 \text{ m/s}^2$.

Solution: $T = 2\pi \sqrt{\frac{L}{g}}$

$T = 2\pi \sqrt{\frac{1.000 \text{ m}}{9.800 \text{ m/s}^2}} \approx 2.007 \text{ s}$

$f = \frac{1}{T} = 0.4983 \text{ Hz}$

The simple pendulum

- Force law: $F_\theta = -mg \sin \theta$
- For small angles, $\sin \theta \approx \theta$, so $F_\theta = -mg \theta = -mgx/L = -m\omega^2 x$

Thus $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{mg}{L \cdot m}} = \sqrt{\frac{g}{L}}$

Period is (very nearly) independent of mass & amplitude $T = 2\pi \sqrt{\frac{L}{g}}$

The simple pendulum (2)

- Result is strictly true only for small amplitudes & “simple” pendulum.

\[ F = -mg \theta \]
\[ F = -mg \sin \theta \]