The physical pendulum

- In “real pendulums”, mass is distributed; consider torques & moment of inertia
- Newton’s 2nd Law for angles: $\tau = I\alpha = -mg(d\sin \theta)/dt$

Damped SHM

- Real SHM slowly dies away — the effect is called “damping.”
- Caused by friction, air drag, viscous drag etc.
- In simplest case, damping forces have the form $F_d = -bv$ where $v$ is the velocity and $b$ is the damping constant.
- EoM now $\sum F = m(d^2x/dt^2) = -kx - b(dx/dt)$

Critical damping & overdamping

- Envelope decay timescale is $2m/b$; compare with SHM timescale $1/\omega_0 = \sqrt{m/k}$.
- When envelope decay is as fast as the oscillation, we get the condition for critical damping: (e.g. shock absorbers in cars)
  \[ b^2 = 4mk \implies \omega' = 0 \]
- “Overdamped” means damping larger than critical damping. Such systems do not oscillate at all.

The physical pendulum (2)

- Using the small angle approx. again, the EoM is
  \[ \frac{\tau}{I} = \alpha = \frac{d^2 \theta}{dt^2} = - \left( \frac{mgd}{I} \right) \theta \]
- Here $\omega^2 = mgd/I$, so
  \[ T = 2\pi \sqrt{\frac{I}{mgd}} \]
- Physical pendulums can be used to measure $g$.

Energy loss in damping

- Energy is still $E = \frac{1}{2} mv^2 + \frac{1}{2} kx^2$ but decreases with time: how fast?
- Differentiating,
  \[ \frac{dE}{dt} = mv \frac{dv}{dt} + kx \frac{dx}{dt} = (ma + kx)v = (-bv)v = -bv^2 \]
- This is always negative

Effects of damping

- This is called a “ringing curve”
- Amplitude decays exponentially
- Energy decays exponentially
- The angular frequency is smaller than without damping

Example 13.9

Consider a rod of length $L$, pivoted at one end. Find $T$.

Solution: Moment of inertia for a uniform rod about an axis through one end is $1/3mL^2$. So

\[ T = 2\pi \sqrt{\frac{1}{3g} \frac{1}{mgL/2} \frac{L}{2}} = 2\pi \sqrt{\frac{2\times1.00 \text{ m}}{3 \times 9.80 \text{ m/s}^2}} = 1.64 \text{ s} \]

Forced oscillations

- In everyday situations, damping is overcome with an applied driving force
- What if the applied force is also periodic? $F_{ext}(t) = F_{max} \cos(\omega t + \phi)$
- Amplitude given by
  \[ A = \frac{F_{max}}{\sqrt{(k - m\omega)^2 + b^2 \omega^2}} \]
Forced oscillations (2)

• When $\omega_d = \omega'$ the amplitude of the oscillations becomes extremely large — resonance.

• All mechanical structures have natural resonant frequencies $\omega'$.

• Some AC electrical circuits also exhibit resonance.