Intensity

- Intensity = \( \frac{\text{(average power)}}{\text{(unit area)}} \)
- \( I = \frac{<P>}{A} = \frac{\langle \phi(x,t) \rangle}{A} \)
- But \( \phi(x,t) = BkA \cos(\omega t - kx) \)

So \( I = \frac{BkA^2}{2} \) \( \cos^2(\omega t - kx) \)

or \( I = \frac{\phi_{\text{max}}^2}{2B} \) \( \text{Note: } \langle x \rangle \text{ means average of } x \).

Example 16.6

Find \( I \) from Ex. 16.1 if the air has \( \rho = 1.20 \text{kgm}^{-3} \) (\( \rho_{\text{max}} = 3 \times 10^{-5} \text{Pa} \) from Ex. 16.1)

- Solution:
  - Use \( I = \frac{\rho_{\text{max}}^2}{2\rho V} \)
  - \( = \frac{(3.0 \times 10^{-3} \text{Pa})^2}{(2 \times 1.20 \text{kgm}^{-3} \times 344 \text{ ms}^{-1})} \)
  - \( = 1.1 \times 10^{-6} \text{Wm}^{-2} \)

Example 16.7

Find \( A \) and \( \rho_{\text{max}} \) for a 20 Hz wave with the same intensity as the 1000 Hz wave from Ex. 16.1.

- Solution:
  - For constant \( I \) with \( f, \omega A \) must be constant. i.e. \( 2\pi \times 20 \times A_{\text{20}} = 2\pi \times 1000 \times A_{\text{1000}} \)

So \( A_{\text{20}} = (1000/20)A_{\text{1000}} = 6.0 \times 10^{-7} \text{m} \)

\( = 0.60 \mu \text{m} \) (c.f. 0.012 \( \mu \text{m} @ 1000 \text{Hz} \))

\( \rho_{\text{max}} \) must be the same as in 16.1 (3 \( \times 10^{-5} \text{Pa} \)) since \( I \) is the same.

Example 16.9

We want \( I = 1 \text{Wm}^{-2} \) at 20m from speakers in all directions. What speaker power is needed?

- Solution: "all directions" means hemisphere above ground, with area \( \frac{1}{2} \times 4\pi(20 \text{m})^2 \).

So acoustic power = \( 1 \text{ Wm}^{-2} \times 2513\text{m}^2 = 2.5 \text{kW}; \) actually need more electrical power than this since speakers are inefficient.

Example 16.11

By how many dB does the intensity drop when you move twice as far away from a singing bird?

- Solution:
  - The difference is given by \( B_2 - B_1 = 10 \text{dB} \log(I_2/I_1) \)
  - \( = 10 \text{dB} \log(r_1^2/r_2^2) \)
  - \( = 10 \text{dB} \log(1^2/(2^2)) \)
  - \( = 10 \text{dB} \log(1/4) \)
  - \( = -6.0 \text{dB} \)

Example 16.15

Speakers A & B emit in-phase sinusoidal waves, \( v = 350 \text{ ms}^{-1} \).

Find what frequencies yield a) constructive & b) destructive interference at point P.

- Solution: 1st find difference between path lengths AP & BP.

\( \sqrt{(2^2+4^2)} = 4.47 \text{m}, BP = \sqrt{(1^2+4^2)} = 4.12 \text{m} \)

Path diff \( d = 4.47 - 4.12 = 0.35 \text{m} \)

Interference in Travelling Sound Waves

- If two sources produce identical sound signals in-phase (coherent) then depending on difference in path lengths from sources to listener, constructive (loud sound) or destructive (soft sound) interference may occur.

- If path difference \( d \) is whole number of wavelengths \( d = n\lambda \), then constructive.

  If path difference is half integer \( d = (n+\frac{1}{2})\lambda \), then destructive.

  Note: Also works for 2 light sources e.g. Young’s double slit

Interference in Travelling Sound Waves

- Here we DON’T have standing waves. Both waves travel in roughly the same direction

  Interference of travelling waves from two identical sources: but,

  - Energy flow is channeled

  - Pressure & displacement have the same nodes

  - Nodes where paths differ by \( d = (n+\frac{1}{2})\lambda \) (or \( d = n\lambda \) for odd \( n \)) (destructive interference)

  - Antinodes where paths differ by \( d = n\lambda \) (constructive interference)
Example 16.15 (2)

- Constructive (loud sound) when $d = n\lambda = nv/f$; or $f = nv/d$
  $f = n \times 350/0.35 = 1000, 2000, 3000$ Hz, ....

- Destructive (soft sound) when $d = (n+1/2)\lambda$ = $(n+1/2)v/f$; or $f = (n+1/2)v/d$
  $f = (n+1/2) \times 350/0.35 = 500, 1500, 2500$ Hz, ...

Beats

- “Beats” occur when two sounds have almost the same frequency. “Beats” means periodically varying amplitude
- Mathematically:
  $A_1 \sin \omega_1 t - A_2 \sin \omega_2 t = 2A \sin \left( \frac{\omega_1 - \omega_2}{2} t \right) \cos \left( \frac{\omega_1 + \omega_2}{2} t \right)$
  Beat Envelope

- Result is an amplitude that varies at the beat frequency $f_{\text{beat}} = f_1 - f_2$

Doppler effect (2)

- In front of S: distance travelled in 1 cycle of sound emission is
  $\lambda = \lambda_S - d_S = v/f_S - u_S/f_S = (v-u_S)/f_S$
  $f_L = f_S (v-u_L)/\lambda$

- Behind S: distance travelled in 1 cycle
  $\lambda = \lambda_S + d_S = v/f_S + u_S/f_S = (v+u_S)/f_S$
  Crests arrive at listener at speed $v+u_L$, so listener hears a frequency $f_L = (v+u_L)/\lambda$
  thus...

Doppler effect (3)

- The Doppler equation for a moving Source and moving Listener:
  $f_L = \frac{v + u_L}{v + u_S} f_S$

- Remember! $u$ positive from L to S, negative in opposite direction