Alfvén surface waves in a magnetized dusty plasma

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Surface wave propagation in a dusty magnetized plasma at frequencies below and of the order of the ion-cyclotron frequency, but well above the dust cyclotron frequency, is considered. The dust grains are assumed to be stationary, but to carry a proportion of the negative charge of the plasma. The dispersion relation for surface waves propagating on an interface between a dusty plasma and a vacuum is derived and discussed. The damping of the waves due to Alfvén resonance absorption in a narrow but non-zero width interface is derived. © *1996 American Institute of Physics.* [S1070-664X(96)03512-4]

I. INTRODUCTION

The physical processes in dusty plasmas[1,2] are interesting because of their importance for a number of applications in space plasmas and the earth's environment, as well as in the laboratory, and in several technologies (e.g., hf plasma etching). Dusty plasmas are characterized as a low-temperature ionized gas whose constituents are electrons, ions, and micron-sized dust particulates. The latter are usually negatively charged due to the attachment of the background plasma electrons on the surface of the dust grains via collisions.

The presence of dust particles (grains) changes the plasma parameters and affects the collective processes in such plasma systems. In particular, the charged dust grains can effectively collect electrons and ions from the background plasma. Thus in the state of equilibrium the electron and ion densities are determined by the neutrality condition which is given by

$$-en_e+en_i-Z_d en_d=0, \tag{1}$$

where $n_{e,i,d}$ is the concentration of plasma electrons (with the charge $-e$), ions (for simplicity, we consider singly charged ions), and dust particles, respectively; the charge of the dust particle $-Z_d e$ can vary significantly depending on plasma parameters. For many dusty plasmas, this charge is negative (i.e., $Z_d>0$) and large ($Z_d\sim 10^2-10^3$).[1–3] Thus because of the neutrality condition (1) it is possible to have in such plasmas,

$$n_e \ll n_i . \tag{2}$$

It has been shown in previous papers[4,5] that the dispersion properties of Alfvén and magnetoacoustic waves in a dusty plasma are strongly modified when a proportion of the negative charge resides on the dust grains. The essential reason for the modification is that the ion Hall current is not compensated by the electron Hall current at low frequencies as well as at frequencies comparable with the ion-cyclotron frequency, as can be readily shown from the general dielectric tensor for a homogeneous plasma with unequal ion and electron charge densities,[6] so that ion-cyclotron effects extend to frequencies much less than the ion-cyclotron frequency. With very small charge on the dust grains, the waves have the usual shear and compressional Alfvén wave properties, while for a large charge on the grains the waves are better described as circularly polarized whistler or helicon waves extending to low frequencies. Other work on the effect of dust on Alfvén and magnetoacoustic waves has focussed on the very low frequency waves that are affected by the motion of the dust grains themselves.[7–10]

In the studies of Alfvén and magnetoacoustic waves in a dusty plasma performed so far, the plasma has been assumed to be uniform in space. However in many laboratory and astrophysical situations, the plasma will have a non-uniform density or magnetic field, including quite sharp interfaces between plasmas of differing properties, such as the surfaces of magnetic flux tubes in the solar coronal and other plasmas,[11] and the boundaries between plasmas of different properties in the solar wind and the Earth's magnetosphere. Alfvén and magnetoacoustic waves play roles in the energy balance in dusty interstellar molecular clouds and in the magnetic braking of protostellar clouds,[7] and since such plasmas could be highly structured due to the presence of magnetic flux tubes, it is important to understand the dispersion and damping properties of the waves in highly structured dusty plasmas.

Wave energy can be concentrated in plasma regions of nonuniform density and/or magnetic field, and in the limiting case of density or magnetic field discontinuities, when a well-defined surface is present, wave eigenmodes exist whose amplitudes decay approximately exponentially in each direction away from the surface. These are the Alfvén surface wave eigenmodes, which have been shown by theory and experiment (e.g., Refs. 12 and 13) to play an important role in the Alfvén wave heating process, because they can be easily excited by an antenna in a laboratory fusion plasma which is separated from the vessel walls by a low-density region. Evidence for Alfvén surface waves in the laboratory was first provided in Ref. 14, and the idea of Alfvén resonance heating of fusion plasmas using surface wave eigenmodes was suggested in Ref. 15. Alfvén surface waves are also expected to exist in the astrophysical plasmas with flux tubes and other boundaries mentioned above. The surface waves may be excited by movement of the footpoints of the flux tubes or by instabilities on the plasma boundaries.[16] If the surface is not perfectly sharp, but has a non-zero transition width, Alfvén resonance absorption will occur at the point in the transition where the wave frequency matches the Alfvén frequency, i.e., the frequency of the local shear Alfvén wave. This process is the basis for the Alfvén wave heating mechanism. The modification of the Alfvén resonance absorption mechanism due to the negative charge residing on the grains in a dusty plasma was investigated in Ref. 5, and it was shown that the wave energy propagating at oblique angles to the magnetic field in an increasing density gradient can be very efficiently absorbed at the Alfvén resonance in a dusty plasma.

In this paper, we study linear surface wave propagation in a simplified model of a highly structured dusty plasma, viz., the interface between a uniform dusty magnetized plasma and a vacuum. The analysis is a generalization of that previously carried out for the dust-free case.[17] The wave frequency is supposed to be less than and of the order of the cyclotron frequency of the plasma ions, and the interface is assumed to be a sharp surface or of small width compared with the wavelength. It is shown that if a proportion of the negative charge resides on the dust grains, the dispersion relation of the surface waves on a sharp surface is strongly modified and the Alfvén resonance damping of the waves in the non-zero width surface is increased by the presence of the dust. The paper is organized as follows. In Sec. II, the plasma model and wave equations are derived. In Sec. III the general dispersion relation for surface waves propagating at arbitrary angles with respect to the external magnetic field is obtained. In Sec. IV, we present graphical solutions and demonstrate the modification of the dispersion relations for waves on a sharp surface in the presence of dust. In Sec. V we consider the Alfvén resonance damping of the surface waves for a non-zero width surface, and in Sec. VI we briefly discuss all the results obtained.

II. MODEL AND WAVE EQUATIONS

The standard two-fluid model of the plasma is invoked,[5,15,18] which consists of the fluid momentum equations for plasma ions (singly charged) and electrons, and the ion and electron continuity equations, as well as Maxwell's equations ignoring the displacement current. Electron inertia is neglected, and the ions and electrons are assumed cold. The uniform background magnetic field $\mathbf{B}_0$ is assumed to be in the z-direction, and the electron and ion densities in the uniform dusty plasma are n_{e0} and n_{i0}. The parameter $\delta = n_{e0}/n_{i0}$ measures the charge imbalance in the plasma, with the remainder of the charge residing on the dust particles, so that the total system is charge neutral. We define the direction perpendicular to the interface between the plasma and the vacuum to be the x-axis. We assume a half-space $x<0$ occupied by the dusty plasma, and a vacuum for $x>0$, with the ion density $n_{i0}(x)$ decreasing smoothly from a constant value of n_{i0} for $x \leqslant -a$ to zero at $x=0$. The electron density and the amount of negative charge on the dust grains are assumed to decrease in the same manner as the ion density, so that δ is constant. The y-axis is then in the $\mathbf{B}_0 \times \mathbf{n}$ direction, where $\mathbf{n}=\hat{\mathbf{x}}$ is the unit vector normal to the interface and pointing into the vacuum.

The equations are linearized, so the wave fields can be assumed to vary as

$$f(x)\exp(ik_z z + ik_y y - i\omega t), \tag{3}$$

where k_z and ω are assumed positive. The surface wave only exists if it propagates in the surface at an oblique angle to the magnetic field, i.e., it has a non-zero wavenumber component (positive or negative) in the y-direction. The plasma transition region is assumed to be narrow, so that $|k_y a| \ll 1$ and $|k_z a| \ll 1$. We assume for simplicity that the charge on the dust particles is not affected by the wave, i.e., we neglect the dust charging effects discussed e.g., in Refs. 19,20. The dust is also assumed to remain stationary. Here we stress an important point. In Refs. 15,18 the electric field was assumed to have, as well as a component parallel to the magnetic field, a single component transverse to the field. However, in the case of charge imbalance in the electron and ion fluids as occurs in the presence of dust particles, or in plasmas in solids with unequal electron and hole numbers,[6] both electric field components transverse to the magnetic field are present, even at frequencies much less than the ion cyclotron frequency. Since the plasma is assumed to be cold and the wave frequency is assumed to be much less than the plasma frequency, the component of the electric field parallel to the background magnetic field, E_z, is zero.

Thus the basic linearized equations in the wave variables $\mathbf{v}_i$ and $\mathbf{v}_e$ (the ion and electron perturbed velocities), $\mathbf{E}$ (the wave electric field), $\mathbf{B}$ (the wave magnetic field), n_i and n_e (the ion and electron density perturbations) are

$$m_i \frac{\partial \mathbf{v}_i}{\partial t} = e(\mathbf{E} + \mathbf{v}_i \times \mathbf{B}_0), \tag{4}$$

$$0 = \mathbf{E} + \mathbf{v}_e \times \mathbf{B}_0, \tag{5}$$

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_{\alpha 0} \mathbf{v}_\alpha) = 0, \quad \alpha = i, e, \tag{6}$$

$$\nabla \cdot \mathbf{E} = \frac{e}{\epsilon_0}(n_i - n_e), \tag{7}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{8}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{9}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} = \mu_0 e(n_{i0}\mathbf{v}_i - n_{e0}\mathbf{v}_e). \tag{10}$$

We have neglected the electron inertia in Eq. (5), which is equivalent to assuming that the wave frequencies of interest are much less than the electron cyclotron frequency. We have also neglected the displacement current in Eq. (10), since the phase velocity is assumed much less than the speed of light c.

Define the local Alfvén speed using the plasma ion density, $v_A(x) = \sqrt{B_0^2/\mu_0 \rho_{i0}}$, where $\rho_{i0}(x) = m_i n_{i0}(x)$ and $f = \omega/\Omega_i$, where Ω_i is the plasma ion-cyclotron frequency. Assuming the time dependence (3) for the fields, Eqs. (4) and (5) may be used to express $\mathbf{v}_i$ and $\mathbf{v}_e$ in terms of $\mathbf{E}$, yielding the current density transverse to $\mathbf{B}_0$:

$$\mathbf{j}_\perp = e n_{i0}\left[\left(\frac{1}{1-f^2} - \delta\right)\frac{(\mathbf{E}\times\mathbf{B}_0)_\perp}{B_0^2} - i\frac{f}{1-f^2}\frac{\mathbf{E}_\perp}{B_0}\right]. \tag{11}$$

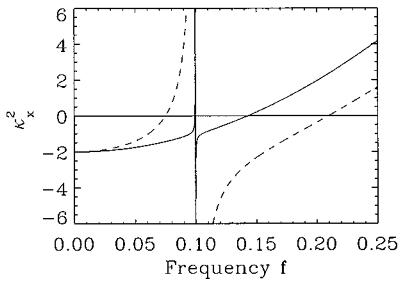

FIG. 1. The normalized square of the perpendicular wavenumber $\kappa_x = k_x/k_z$ plotted against normalized frequency $f = \omega/\Omega_i$, for $\alpha = 0.01$ and $s = 1$. Solid curves for $\delta = 1$, and dashed curves for $\delta = 0.9$.

The same result is obtained from the dielectric tensor for a cold dusty plasma, e.g., Ref. 4. By taking the components of Eqs. (9) and (10), and using (11) and (3), we find the following two differential equations in x for the field components E_y and B_z:

$$\frac{dE_y}{dx} - \frac{k_y D}{A} E_y = i\omega \frac{A - k_y^2}{A} B_z, \tag{12}$$

$$\frac{dB_z}{dx} + \frac{k_y D}{A} B_z = \frac{i}{\omega} \frac{A^2 - D^2}{A} E_y, \tag{13}$$

where A and D are defined as

$$A = \frac{\omega^2}{v_A^2(1 - f^2)} - k_z^2, \tag{14}$$

$$D = \frac{\omega \Omega_i}{v_A^2} \left(\frac{1}{1 - f^2} - \delta \right), \tag{15}$$

and are functions of x through their dependence on v_A.

In a uniform plasma the solutions of Eqs. (12) and (13) are of the form $E_y, B_z \propto \exp(ik_x x)$, where the wavenumber k_x is given by

$$k_x^2 = -k_y^2 + \left(\frac{A^2 - D^2}{A} \right), \tag{16}$$

which can be written in terms of dimensionless parameters f, δ, $\kappa_x = k_x/k_z$, $s = |k_z/k_y|$, and $\alpha = v_A k_z/\Omega_i$,

$$\kappa_x^2 = -1 - 1/s^2 + \frac{f^2}{\alpha^2} \left[\frac{\delta^2 f^2 - \alpha^2 - (1-\delta)^2}{f^2 - \alpha^2(1 - f^2)} \right]. \tag{17}$$

The dependence of κ_x^2 on the normalized frequency f, for given values of s and α, and two values of δ, one being the dust-free case $\delta = 1$, is indicated in Fig. 1. For given s, α, and δ, there are two cutoff frequencies, f_1 and f_3, where $\kappa_x^2 = 0$, and a resonance frequency, f_2, where $\kappa_x^2 \to \infty$. A body wave, referred to as the ion-cyclotron wave in the dust-free case,[17] propagates in the x-direction when $f_1 < f < f_2$, while a fast body wave propagates when $f > f_3$.

In the case of $k_y = 0$, the cutoff frequencies f_1 and f_3 are given by

$$f_{1,3} = \left[\left(\frac{1 - \delta + \alpha^2}{2\delta} \right)^2 + \frac{\alpha^2}{\delta} \right]^{1/2} \mp \left(\frac{1 - \delta + \alpha^2}{2\delta} \right). \tag{18}$$

These are the frequencies of the two wave modes that propagate parallel to the magnetic field in a uniform dusty plasma. For oblique propagation ($k_y \neq 0$), the higher frequency f_3 of the two modes is given by, for $|k_{y,z}| v_A \ll \Omega_i$,

$$f_3 = \frac{1 - \delta}{\delta} \left[1 + \frac{(1 + 2s^2)\alpha^2}{2(1-\delta)^2 s^2} \right]. \tag{19}$$

Thus the cutoff frequency as $k_{z,y} \to 0$ is given by

$$\omega = \Omega_m = \Omega_i \frac{1 - \delta}{\delta}. \tag{20}$$

The characteristic frequency f_1 of the lower frequency mode under the assumptions $|k_{y,z}| v_A \ll \Omega_i$ is

$$f_1 = \frac{(1 + s^2)^{1/2} \alpha^2}{(1 - \delta) s} \left[1 - \frac{(1 + 2s^2)\alpha^2}{2(1-\delta)^2 s^2} \right]. \tag{21}$$

Therefore, as has been shown in Ref. 5, the effect of dust in the plasma is to decrease the lower cutoff-frequency and increase the upper cutoff-frequency, as indicated in Fig. 1; this has been shown in Ref. 5 to lead to more efficient resonant absorption of wave energy at the Alfvén resonance.

For arbitrary k_y, the resonance frequency f_2 is given by $A = 0$, i.e.,

$$f_2 = \frac{\alpha}{(1 + \alpha^2)^{1/2}}. \tag{22}$$

The resonance frequency corresponds to the Alfvén resonance generalized for finite ion-cyclotron frequency, which is referred to as the perpendicular ion cyclotron resonance in Ref. 21.

III. DISPERSION RELATION FOR SURFACE WAVES

We consider in this section the solutions of Eqs. (12) and (13) for the case of a narrow width surface separating the plasma and vacuum. The dispersion relation is found by requiring that the tangential components of the electric and magnetic field, E_y and B_z, are both continuous across the boundaries $x = 0$ and $x = -a$, i.e., that the homogeneous dusty plasma and vacuum solutions be matched with the solution inside the non-zero width surface. In the following section we derive the dispersion relation for the special case of a sharp surface.

The surface wave solutions that we seek will be in the frequency range in Fig. 1 corresponding to $k_x^2 < 0$ in the uniform plasma region $x < -a$, i.e., the wave fields there vary as $\exp(k_p x)$, where $k_p = |k_x|$. In the vacuum ($x > 0$) they vary as $\exp(-k_v x)$ where, since the phase velocity is assumed $\ll c$, we have to a good approximation

$$k_v^2 = k_y^2 + k_z^2. \tag{23}$$

An approximate solution of Eqs. (12) and (13) is obtained within the narrow plasma transition region by the perturbation technique of Refs. 17, 22 and 23. The wavelength in the plane of the surface is assumed much larger than the width of the surface transition, so that the wave fields E_y and B_z may be expanded in series using the small parameter $\epsilon = k_z a$;

$$E_y = E_{y0} + \epsilon E_{y1} + \ldots \quad \text{and}$$
$$B_z = k_z(\psi_0 + \epsilon\psi_1 + \ldots), \tag{24}$$

where E_{y0} and $B_{z0} = k_z\psi_0$ are the wave fields at the interface in the sharp interface case, and the terms of order ϵ are the corrections to the wave fields due to the finite width of the interface transition.

Changing to a new space variable $\overline{x} = x/a$ (of order 1 within the transition), substituting (24) in Eqs. (12) and (13) and collecting terms of the same order in ϵ gives, up to first order in ϵ,

$$\frac{dE_{y0}}{d\overline{x}} = 0, \quad \frac{d\psi_0}{d\overline{x}} = 0, \tag{25}$$

$$\frac{dE_{y1}}{d\overline{x}} - \frac{k_y D}{k_z A} E_{y0} = i\omega \frac{A - k_y^2}{A}\psi_0, \tag{26}$$

$$\frac{d\psi_1}{d\overline{x}} + \frac{k_y D}{k_z A}\psi_0 = \frac{i}{\omega}\frac{A^2 - D^2}{k_z^2 A} E_{y0}. \tag{27}$$

If the solution for E_y in the vacuum region is $E_y = E_v \exp(-k_v x)$, then $E_{y0} = E_v$ and, from the vacuum field relations, $\psi_0 = (i/\omega)(k_z/k_v)E_v$.

Integrating Eqs. (26) and (27) gives

$$E_{y1}(\overline{x}) = E_v \int_0^{\overline{x}} \left[\frac{k_y D}{k_z A} - \frac{k_z}{k_v}\frac{(A - k_y^2)}{A}\right] d\overline{x'}, \tag{28}$$

$$\psi_1(\overline{x}) = -\frac{i}{\omega} E_v \int_0^{\overline{x}} \left[\frac{k_y D}{k_v A} - \frac{(A^2 - D^2)}{k_z^2 A}\right] d\overline{x'}. \tag{29}$$

The fields in the region $x \leqslant -a$ are given by

$$E_y = E_p \exp(k_p x), \tag{30}$$

$$B_z = B_p \exp(k_p x) = \frac{i}{\omega}\frac{A_p k_p - k_y D_p}{k_y^2 - A_p} E_p \exp(k_p x), \tag{31}$$

where A_p and D_p are A and D evaluated at $x = -a$.

Continuity at $x = -a$ then gives, to first order in ϵ,

$$E_p \exp(-k_p a) = E_{y0} + \epsilon E_{y1}(-1),$$
$$B_p \exp(-k_p a) = k_z(\psi_0 + \epsilon\psi_1(-1)). \tag{32}$$

Substituting Eqs. (28) and (29) into Eq. (32), and using Eqs. (30) and (31), gives the dispersion equation:

$$\mathscr{D}(\tilde{\omega}) = \mathscr{D}_0(\tilde{\omega}) + \epsilon\mathscr{D}_1(\tilde{\omega}) = 0, \tag{33}$$

with $\tilde{\omega} = \omega - i\gamma$, the damping rate γ being of order $\epsilon\omega$, and

$$\mathscr{D}_0(\tilde{\omega}) = \frac{k_p A_p - k_y D_p}{A_p - k_y^2} + \frac{k_z^2}{k_v}, \tag{34}$$

and $\mathscr{D}_1$ is the correction:

$$\mathscr{D}_1(\tilde{\omega}) = \frac{k_z}{a}\int_0^{-a}\frac{1}{A}\left[-2\frac{k_y}{k_v}D + \frac{A^2 - D^2}{k_z^2} + \frac{k_z^2}{k_v^2}(A - k_y^2)\right]dx. \tag{35}$$

The integrands in (28), (29), and (35) may contain poles where $A = 0$, i.e., at the Alfvén resonance. The integrations may be carried out using standard techniques described by Ginzburg[23] in the context of the similar problem of the resonance absorption of radio waves in the ionosphere: a small imaginary part is introduced in A due to small dissipation, the path of integration is distorted in the complex x-plane to avoid the singularity, and the dissipation then approaches zero. The result is that E_{y1}, ψ_1, and $\mathscr{D}_1$ acquire imaginary parts due to the residues of the integrands at the pole $A = 0$ which are independent of the size of the small dissipation. This leads to Alfvén resonance absorption, which is discussed further in Sec. V. It is to be noted that E_y and B_z remain finite through the transition, ensuring the validity of the expansion (24), however the fields E_x and B_y become singular at the resonance, as shown in Refs. 21,22.

IV. THE SURFACE WAVE AT A SHARP BOUNDARY

The dispersion relation for the surface wave on a sharp dusty plasma–vacuum boundary, as $\epsilon \to 0$, is given by $\mathscr{D}_0(\omega) = 0$, and is investigated in this section. The same result can of course be obtained directly by matching the homogeneous dusty plasma and vacuum solutions across the sharp boundary. Such a direct approach would be more suitable than attempting to solve the wave equations in the non-zero width transition, if the more general problem including such effects as high frequencies, non-zero plasma pressure, resistive or viscous effects, or other kinetic effects were to be considered. However, we are restricting the present investigation to a cold dissipationless dusty plasma at frequencies less than the ion-cyclotron frequency but well above the dust cyclotron frequency, so that Eqs. (12) and (13) are a good description.

Setting $\mathscr{D}_0(\omega) = 0$ in Eq. (34) leads to an expression for $k_p = k_p^{(1)}$ which must be consistent with the expression for $k_p = k_p^{(2)}$ which is derived from Eq. (16), in which $k_x^2 = -k_p^2$. Thus we find that

$$(k_p^{(2)})^2 - (k_p^{(1)})^2 = \frac{(A_p - k_y^2)}{A_p^2}\left[\frac{k_z^4}{k_v^2}(A_p - k_y^2) - 2\frac{k_z^2 k_y}{k_v}D_p + A_p^2 - D_p^2\right]. \tag{36}$$

Setting Eq. (36) to zero gives solutions for the surface wave frequency ω, provided care is taken to exclude spurious solutions introduced by the squaring procedure. We choose to solve this equation for the phase velocity in the magnetic field direction, relative to the Alfvén speed, i.e., $V = \omega/v_A k_z$. It is straightforward to show that the first factor on the right hand side of Eq. (36) must be non-zero. Equating the second factor to zero leads to the following cubic equation for V:

$$-\delta^2 V^3 + \frac{2\sigma\delta\alpha}{(1+s^2)^{1/2}}V^2 + \left[\frac{2+s^2}{1+s^2} + \frac{(1-\delta)^2}{\alpha^2}\right]V + \frac{2\sigma(1-\delta)}{\alpha(1+s^2)^{1/2}} = 0, \tag{37}$$

where $\sigma = \text{sign}(k_y)$.

In terms of V and the other dimensionless parameters, the expression for k_p obtained from Eq. (34) by setting $\mathscr{D}_0(\omega)=0$ may be written as

$$\frac{k_p}{k_z}=\frac{1}{V^2(1+\alpha^2)-1}\left[\frac{\sigma V}{s\alpha}(1-\delta+\delta\alpha^2V^2)+\frac{(1+s^2)(1-\alpha^2V^2)-s^2V^2}{s(1+s^2)^{1/2}}\right]. \tag{38}$$

For a valid surface wave solution, the condition $k_p>0$ must be satisfied, and this leads to the existence of cutoff frequencies for the solutions where $k_p=0$.

In the dust-free case ($\delta=1$), there are two possible positive phase velocity solutions[17] depending on the sign of the wavenumber component in the $\mathbf{B}_0\times\mathbf{n}$ or y direction, a fast wave (V^+) for positive k_y, and a slow wave (V^-) for negative k_y:

$$V^{\pm}=\frac{(2+s^2+\alpha^2)^{1/2}\pm\alpha}{(1+s^2)^{1/2}}. \tag{39}$$

When $\delta=1$ and $\alpha\ll1$, we obtain the low-frequency Alfvén surface wave on a plasma–vacuum interface,[24] whose frequency is given by

$$\omega=v_A k_z\left(\frac{k_z^2+2k_y^2}{k_z^2+k_y^2}\right)^{1/2}. \tag{40}$$

We now discuss the effects of dust on the dispersion relation for the surface waves. Again there is a single valid solution of Eq. (37) for each sign of k_y and for given s, α, and δ.

A. Positive k_y

Figure 2 shows the fast wave dispersion relations, i.e., the solutions for V for positive k_y, plotted as a function of s, for three values of α and for three values of δ. Each curve ends at a cutoff value of s, where $k_p=0$, just as was found for the dust-free case in Ref. 17. It is seen from Fig. 2 that as δ decreases from 1, the major effect is on the small α dispersion relation, with the $\alpha=0.1$ phase velocity becoming greater than the $\alpha=0.5$ phase velocity at $\delta=0.9$. There is also a general increase in the phase velocity for all α, as well as a small decrease in the cutoff value of s. The behaviour of the small α dispersion relation is illustrated more clearly in Fig. 3, where the phase velocity is plotted against α for given $s=1$ and for the same three values of δ as in Fig. 2. For $\delta=1$ the phase velocity decreases monotonically as α decreases, whereas for $\delta<1$ there is a critical value of α below which V increases rapidly. This behaviour is verified by the approximate solution of Eq. (37) for small α:

$$V\approx\frac{(1-\delta)}{\delta\alpha}\left[1+\frac{(1+\sqrt{1+s^2})^2\alpha^2}{2(1-\delta)^2(1+s^2)}\right]. \tag{41}$$

The limiting frequency of the surface wave as $\alpha\to0$ is therefore $\omega=\Omega_m$, where Ω_m is defined in Eq. (20), i.e., the surface wave for $k_y>0$ has the same cutoff frequency for $k_z\to0$ as the higher frequency mode that propagates parallel to the magnetic field in the uniform dusty plasma, and is therefore closely related to that mode.

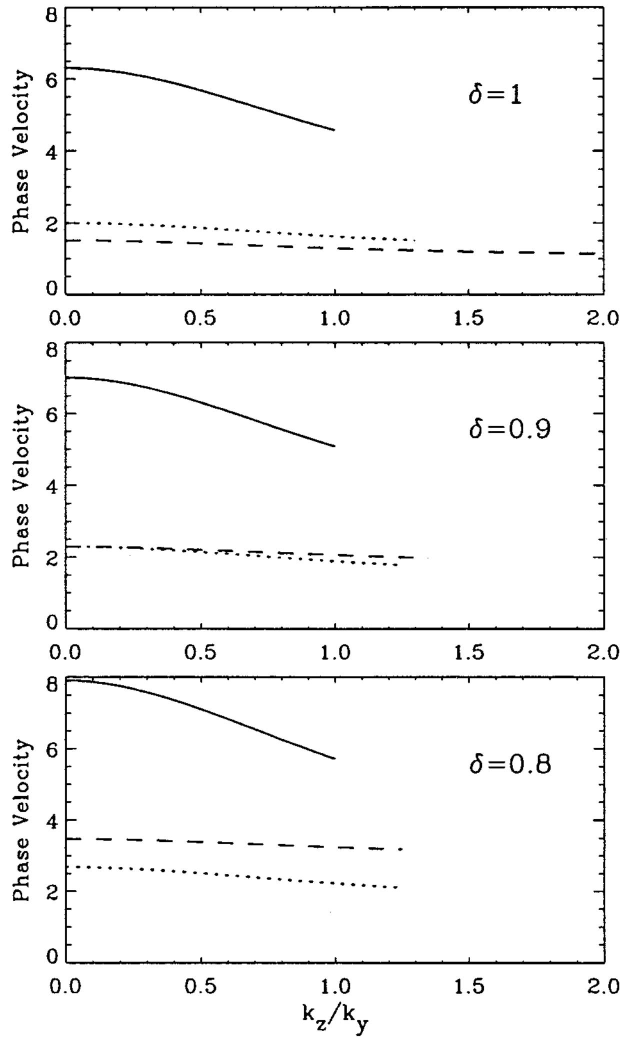

FIG. 2. The normalized phase velocity V, for positive k_y, as a function of $s=k_z/k_y$, for $\alpha=0.1$ (dashed curves), $\alpha=0.5$ (dotted curves), and $\alpha=3$ (solid curves), and for $\delta=1$, $\delta=0.9$, and $\delta=0.8$.

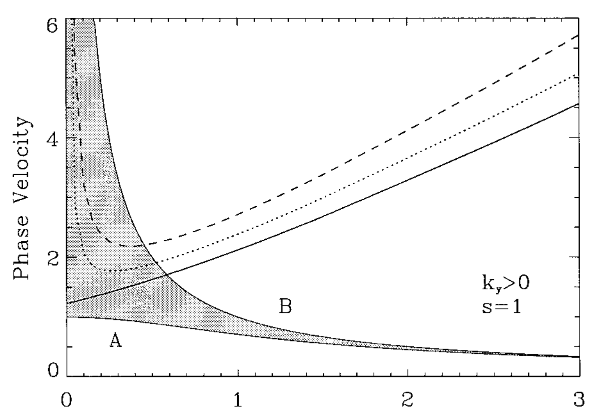

FIG. 3. The normalized phase velocity V, for positive k_y, as a function of α, for $s=1$, and for $\delta=1$ (solid curve), $\delta=0.9$ (dotted curve), and $\delta=0.8$ (dashed curve). The curves A and B enclose the shaded region where Alfvén resonance damping occurs.

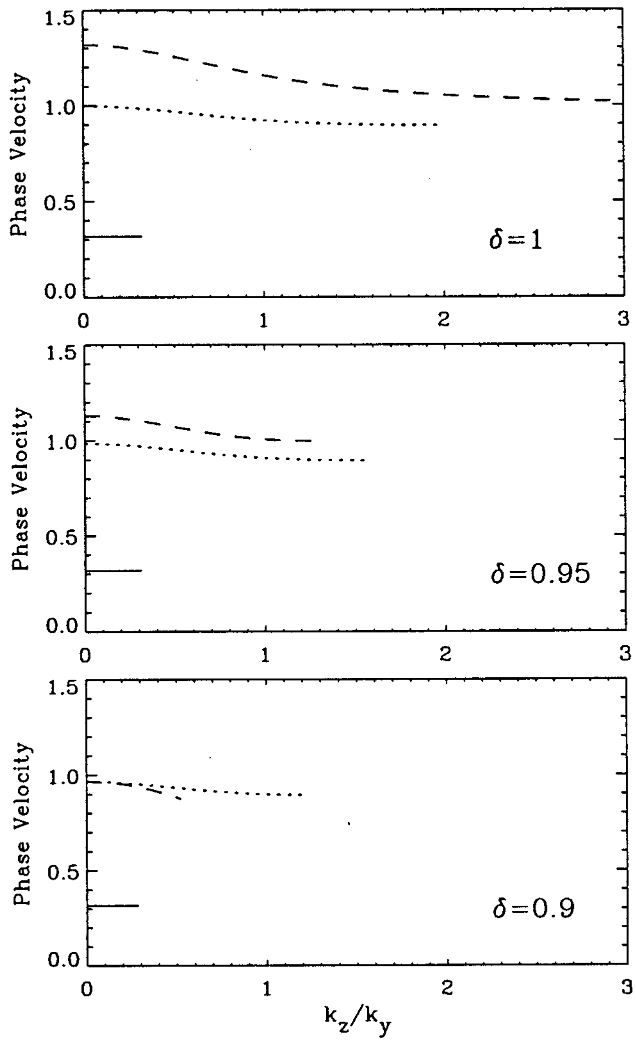

FIG. 4. The normalized phase velocity V, for negative k_y, as a function of $s=k_z/k_y$, for $\alpha=0.1$ (dashed curves), $\alpha=0.5$ (dotted curves), and $\alpha=3$ (solid curves), and for $\delta=1$, $\delta=0.95$, and $\delta=0.9$.

The value of k_p corresponding to the small α value of V given by Eq. (41) is given from Eq. (38) as

$$k_p \approx k_z \frac{(1+s^2)^{1/2}-s^2}{s(1+s^2)^{1/2}}, \tag{42}$$

which is positive, corresponding to a valid solution, for $s<1.27$, in agreement with the end points of the $\alpha=0.1$ curves in Fig. 2. Note that when $s^2=(1+s^2)^{1/2}$ (i.e., $k_p=0$), the surface wave frequency $f=V\alpha$, as derived from (41), exactly coincides with frequency (19) of the volume mode as it should, because for zero attenuation coefficient k_p the surface mode loses its near surface character, and is indistinguishable from the volume wave.

B. Negative k_y

Figure 4 shows the slow wave dispersion relations, i.e., the solutions for V for negative k_y, plotted as a function of s, for three values of α and for three values of δ. Again, as δ decreases from 1, the major effect is on the small α dispersion relation, with the $\alpha=0.1$ phase velocity in this case becoming smaller than the $\alpha=0.5$ phase velocity at $\delta=0.9$.

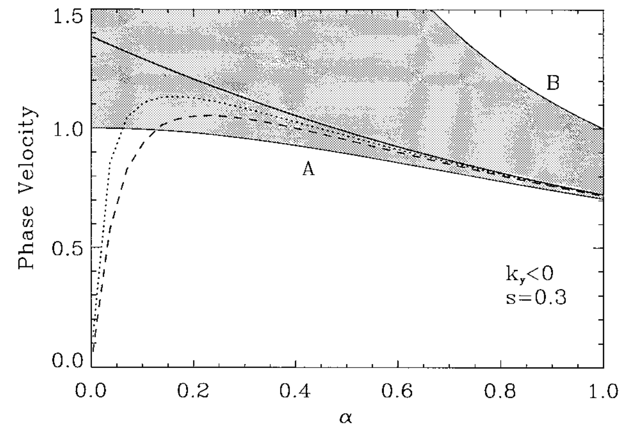

FIG. 5. The normalized phase velocity V, for negative k_y, as a function of α, for $s=0.3$, and for $\delta=1$ (solid curve), $\delta=0.95$ (dotted curve), and $\delta=0.9$ (dashed curve). The curves A and B enclose the shaded region where Alfvén resonance damping occurs.

The cutoff value of s for $\alpha=0.1$ decreases rapidly as δ decreases, and less rapidly for larger values of α. The behaviour of the small α dispersion relation is illustrated more clearly in Fig. 5, where the phase velocity is plotted against α for given $s=0.3$ and for the same three values of δ as in Fig. 4. For $\delta=1$ the phase velocity increases monotonically as α decreases, whereas for $\delta<1$ there is a critical value of α below which V decreases rapidly. The approximate solution of Eq. (37) for small α is in this case:

$$V \approx \frac{2\alpha}{(1-\delta)(1+s^2)^{1/2}}\left[1-\frac{(2+s^2)\alpha^2}{(1-\delta)^2(1+s^2)}\right]. \tag{43}$$

The limiting frequency of the $k_y<0$ surface wave as $s\to 0$ is therefore

$$\omega = \frac{2v_A^2 k_z^2}{(1-\delta)\Omega_i}, \tag{44}$$

which is twice the frequency, for small k_z, of the lower frequency mode propagating parallel to the magnetic field in the uniform dusty plasma, given by f_1 in Eq. (18) as $\alpha\to 0$. It is interesting to note that the curves for $\alpha=0.5$ and $\alpha=3$ in Fig. 4 end at a value of s where $k_p\to\infty$, i.e., where the Alfvén resonance condition (22) holds, in accord with the result found for the dust-free $k_y<0$ case in Ref. 17. This is also true for the $\alpha=0.1$ curves for $\delta=1$ and $\delta=0.95$, but for $\delta=0.9$ the curve ends where $k_p=0$, i.e., if the end point lies in the low-α region of Fig. 5 where the phase velocity is decreasing with decreasing α, it has a cutoff rather than a resonance value of k_p. This is shown explicitly by the value of k_p obtained from Eq. (38) by using V given by Eq. (43):

$$k_p \approx k_z \frac{1-s^2}{s(1+s^2)^{1/2}}. \tag{45}$$

Thus the $k_y<0$ surface wave phase velocity for small α has a cutoff when $k_p=0$, for $s\approx 1$, in agreement with the numerical results. Again, we note that under the condition $k_p=0$ the surface wave frequency $f=V\alpha$ derived from (43) is the same as the frequency (21) of the corresponding volume mode.

V. DAMPING OF THE SURFACE WAVE

The damping rate, to first order in ϵ, of the surface wave due to the non-zero width of the surface is obtained from Eq. (33) as

$$\gamma = \epsilon \frac{\text{Im } \mathscr{D}_1(\omega)}{\partial \mathscr{D}_0 / \partial \omega}\bigg|_{\omega=\omega_s}, \tag{46}$$

where ω_s is the surface wave frequency obtained in the previous section.

As discussed in Sec. III, the only imaginary contribution to $\mathscr{D}_1$ comes from the pole of the integrand in Eq. (35), i.e., at that point in the surface transition where $A=0$, which corresponds to the Alfvén resonance condition. Thus the damping is due to Alfvén resonance absorption of the surface wave, although in this dissipationless model there is no indication of how the damped energy is finally dissipated. More realistic models of resonance absorption show that the energy is absorbed via mode conversion into a short wavelength mode which is subsequently damped by collisional or collisionless processes. The damping rate is found, by evaluating the contribution of the residue at the pole $A=0$ in the integral (35), to be

$$\begin{aligned}\gamma &= -\epsilon\pi \frac{(D+k_z^2 k_y/k_v)^2}{k_z(\partial \mathscr{D}_0/\partial\omega)(\overline{dA/dx})} \\ &= -\epsilon\pi k_z^3 \frac{[(1+s^2)^{-1/2}+\sigma(\delta f+(1-\delta)/f)]^2}{(\partial \mathscr{D}_0/\partial\omega)(\overline{dA/dx})},\end{aligned} \tag{47}$$

where $\partial \mathscr{D}_0/\partial\omega$ is evaluated using $\omega=\omega_s$, and D and $\overline{dA/dx}$ are evaluated at $x=x_r$, with x_r the resonant point where $A=0$.

When evaluating the damping of the surface wave, an important point to determine is whether or not the Alfvén resonance is encountered in the surface layer, where the plasma density is assumed to decrease from its value at $x=-a$ to zero at $x=0$, and the local Alfvén speed correspondingly increases. If the resonance is not encountered, no Alfvén resonance damping occurs, and the only damping will be due to resistive, viscous or other collisional processes,[25] or collisionless Landau damping.[26] The resonance condition (22) is equivalent to the following condition on the ion density n_{ir} at the resonant point:

$$\frac{n_{ir}}{n_{i0}} = \frac{1-\alpha^2 V^2}{V^2}, \tag{48}$$

where n_{i0} is the ion density in the uniform dusty plasma at $x<-a$. The conditions $0<n_{ir}/n_{i0}<1$ are then equivalent to

$$\frac{1}{\alpha^2} > V^2 > \frac{1}{1+\alpha^2}, \tag{49}$$

i.e., if the surface wave phase velocities for given α and s found in the previous section do not satisfy (49), the surface wave is not resonance damped. Note that the left inequality in (49) is equivalent to the condition $\omega<\Omega_i$; otherwise the function A is always negative and so cannot be zero in the surface layer.

We have plotted in Figs. 3 and 5 the curves $V^2\alpha^2=1$ (curve A) and $V^2(1+\alpha^2)=1$ (curve B), so that solutions in the shaded regions lying between curves A and B will satisfy the inequalities in (49) and so will be resonance damped. Thus the fast surface waves obtained for $k_y>0$ (Fig. 3), for which $V>1$, will always satisfy the right inequality in Eq. (49). However above a critical value of α, given by the intercept of the dispersion curves in Fig. 3 with curve B, the surface wave will not experience resonance damping. This critical value of α decreases as the proportion of dust increases. The presence of dust has been shown in the previous section to not have a large effect on the large α surface wave dispersion relation, and the effect of dust similarly does not greatly modify the resonant damping rate calculated for the dust-free case and reported in Ref. 17. In the low-α limit, where the dust has a large effect on the dispersion relation, and we can use the dispersion relation Eq. (41), the left inequality in (49) is satisfied provided $0.5<\delta<1$. The damping rate in that case, as a fraction of the limiting frequency of the surface wave Ω_m, is given by

$$\frac{\gamma}{\Omega_m} \approx \pi\epsilon s[1+(1+s^2)^{-1/2}]^2, \tag{50}$$

if we assume A to vary linearly with x in the transition region.

In the case of $k_y<0$ (Fig. 5), resonance damping will occur for solutions with large α, as shown by the region between the curves A and B, with a small effect of dust on the dust-free results reported in Ref. 17. However for small α where the effect of dust is to strongly reduce the phase velocity below the Alfvén speed in the uniform plasma, the Alfvén resonance condition, viz. that the phase velocity becomes equal to the local Alfvén speed in the surface layer, cannot be satisfied and the $k_y<0$ surface waves will propagate in the dusty plasma with no resonance damping.

VI. CONCLUSIONS

To conclude, we have shown that the presence of dust particles which acquire a proportion of the negative charge in a plasma can strongly modify the dispersion properties of Alfvén surface waves. We have considered the ideal case of a narrow plane interface between a uniform magnetized dusty plasma and a vacuum, and have derived the dispersion relation of surface waves on a sharp interface. We have shown that the most important effects of the dust on the surface wave dispersion relation occur at low wavenumber k_z parallel to the magnetic field. For propagation of the wave in the surface in a direction intermediate between the magnetic field ($\mathbf{B}_0$) direction and the $\mathbf{B}_0\times\mathbf{n}$ direction, where $\mathbf{n}$ is the unit vector normal to the surface and pointing to the vacuum, (i.e., $k_z>0$ and $k_y>0$), the surface wave frequency approaches a cutoff frequency as $k_z\to 0$. The corresponding wave in the dust-free plasma has no such cutoff. On the other hand, for propagation of the wave in a direction intermediate between the $\mathbf{B}_0$ direction and the negative $\mathbf{B}_0\times\mathbf{n}$ direction (i.e., $k_z>0$ and $k_y<0$), the surface wave phase velocity $V\to 0$ as $k_z\to 0$, in contrast to the corresponding wave in the dust-free plasma, which has a phase velocity greater than the Alfvén speed in this limit.

We have also calculated the Alfvén resonance damping of the surface wave for a non-zero width interface, and have shown that the $k_y<0$ wave can propagate for low k_z in the dusty plasma with no resonant damping, in contrast to the corresponding wave in the dust-free plasma. However, a damping due to the dust charging process may still occur, which would be of interest for further studies.

The results we have obtained should be useful for many laboratory and astrophysical impurity-containing plasmas where strong density gradients and/or sharp interfaces between plasmas of different properties appear, such as in edge regions in tokamaks or in the solar wind and the Earth's environment.

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