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## Attractive forces between charged particulates in plasmas

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### Abstract

It is shown that charged particulates having the same polarity can attract each other due to collective interactions involving the dust acoustic waves. This might provide the possibility of lattice formation in Coulomb systems.

Some years ago, Ikezi [1] proposed that a Coulomb crystal can be formed in strongly coupled plasmas provided that the ratio of the average Coulomb energy between the micron-sized charged dust particulates to the average thermal energy is greater than unity. This opens new perspectives for creating new materials which can be utilized for advanced schemes of radiation sources, etc.

Recently, several authors [2–6] have experimentally demonstrated the formation of microscopic Coulomb crystals of solid particles as well as particulate coagulation [7,8] in low-temperature radio frequency discharges. In association with the Coulomb crystallization in strongly coupled radio-frequency dusty plasmas, there have been indications of low-frequency fluctuations [3].

In this Letter, we present a novel possibility of charged particle attraction through the collective interactions involving very low-frequency electrostatic

fluctuations in dusty plasmas. Our proposed mechanism is analogous to the Cooper pairing [9] of electrons in superconductors. Thus, the collective effects in multi-species dusty plasmas can contribute to the attraction force between the particulates. A similar scenario has been envisioned by Nambu and Akama [10] for an electron–ion plasma in which the possibility of electron attraction has been pointed out.

The electrostatic potential around the test dust particle is given by [11]

$$\Phi(\mathbf{x}, t) = \int \frac{q_t}{2\pi^2 k^2} \frac{\delta(\omega - \mathbf{k} \cdot \mathbf{v})}{\varepsilon(\mathbf{k}, \omega)} \exp(i\mathbf{k} \cdot \mathbf{r}) d\mathbf{k} d\omega, \quad (1)$$

where  $\mathbf{r} = \mathbf{x} - \mathbf{v}t$ ,  $\mathbf{v}$  is the velocity vector of the test particulate, and  $q_t$  is the charge of the test particle. The dielectric response function of the multi-species dusty plasma in the presence of fluctuations is given by

$$\varepsilon(\mathbf{k}, \omega) = 1 + \sum_{j=e,i,d} \chi_j, \quad (2)$$

where  $\mathbf{k}$  is the wavevector,  $\omega$  is the frequency, the subscript  $j$  equals e for electrons, i for ions, and d for dust particles. The plasma susceptibility  $\chi_j$  reads

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$$\chi_j = \frac{1}{k^2 \lambda_{Dj}^2} \left[ 1 + \frac{\omega}{k v_{Tj}} Z \left( \frac{\omega}{k v_{Tj}} \right) \right], \quad (3)$$

where,  $\lambda_{Dj}$  is the Debye length,  $v_{Tj}$  is the thermal velocity, and  $Z$  is the plasma dispersion function [11].

Let us consider two types of dusty plasma response. First, when the phase velocity of the oscillations is much smaller (larger) than the electron and ion thermal velocities (dust thermal velocity) then the electrons and ions obey the Boltzmann distribution, whereas the dust component is inertial. Correspondingly, the dielectric constant associated with the slow dust-acoustic waves [12] is given by

$$\varepsilon(\mathbf{k}, \omega) = 1 + \frac{1}{k^2 \lambda_D^2} - \frac{\omega_{pd}^2}{\omega^2}, \quad (4)$$

where  $\lambda_D = (\lambda_{De}^{-2} + \lambda_{Di}^{-2})^{-1/2}$  is the effective Debye length of the dusty plasma [13],  $\lambda_{De}$  ( $\lambda_{Di}$ ) is the electron (ion) Debye radius,  $\omega_{pd} = (4\pi q_d^2 n_{d0}/m_d)^{1/2}$  is the dust plasma frequency,  $q_d$  ( $n_{d0}$ ) is the unperturbed dust charge (dust number density), and  $m_d$  is the mass of the particulates.

Second, when the phase velocity of the oscillations is much smaller (larger) than the electron thermal velocity (ion and dust thermal velocities) then the dielectric response function involving fast dust-acoustic waves [14] is given by

$$\varepsilon(\mathbf{k}, \omega) = 1 + \frac{1}{k^2 \lambda_{De}^2} - \frac{\omega_{pi}^2}{\omega^2} \left( 1 + \frac{Z_d n_{d0} m_i}{Z_i n_{i0} m_d} \right), \quad (5)$$

where  $Z_d$  and  $Z_i$  are the number of charges residing on particulates and ions, respectively, and  $m_i$  is the ion mass.

The inverse of the dielectric response function associated with the slow dust-acoustic oscillations, obtained from (4), is

$$\frac{1}{\varepsilon(\mathbf{k}, \omega)} = \frac{k^2 \lambda_D^2}{1 + k^2 \lambda_D^2} \left( 1 + \frac{\omega_{sd}^2}{\omega^2 - \omega_{sd}^2} \right), \quad (6)$$

where  $\omega_{sd} = k \lambda_D \omega_{pd} / (1 + k^2 \lambda_D^2)^{1/2}$  is the frequency of the slow dust-acoustic wave. Similarly, from (5) we readily obtain

$$\frac{1}{\varepsilon(\mathbf{k}, \omega)} = \frac{k^2 \lambda_{De}^2}{1 + k^2 \lambda_{De}^2} \left( 1 + \frac{\omega_{fd}^2}{\omega^2 - \omega_{fd}^2} \right), \quad (7)$$

where  $\omega_{fd} = k \lambda_{De} \omega_{pi} \alpha / (1 + k^2 \lambda_{De}^2)^{1/2}$  and  $\alpha = (1 + Z_d^2 n_{d0} m_i / Z_i^2 n_{i0} m_d)^{1/2}$ .

If we substitute Eqs. (6) and (7) into (1), then besides the well-known Debye screening potential of the dusty plasma [13], viz.

$$\Phi_D = \frac{q_t}{r} \exp(-r/\lambda_D), \quad (8)$$

there appears an additional potential involving collective effects caused by the slow (fast) dust-acoustic oscillations, namely

$$\Phi_C = \int \frac{q_t}{2\pi^2 k^2} F(\mathbf{k}, \omega) \times \exp(i\mathbf{k} \cdot \mathbf{r}) \delta(\omega - \mathbf{k} \cdot \mathbf{v}) d\mathbf{k} d\omega, \quad (9)$$

where  $F(\mathbf{k}, \omega) = k^2 \lambda_D^2 \omega_{sd}^2 / (1 + k^2 \lambda_D^2) (\omega^2 - \omega_{sd}^2)$  for the slow dust-acoustic oscillations, and  $F(\mathbf{k}, \omega) = k^2 \lambda_{De}^2 \omega_{fd}^2 / (1 + k^2 \lambda_{De}^2) (\omega^2 - \omega_{fd}^2)$  for the fast dust-acoustic oscillations.

It follows from (9) that the potential changes its sign due to the overscreening depending on whether  $\omega$  is larger or smaller than  $\omega_{sd}$  or  $\omega_{fd}$ . However, if  $\omega$  is close to one of these frequencies then there appears strong resonant interaction between the oscillations and the test particulate. When the latter moves with a velocity slightly larger than the phase velocity of the slow or fast dust-acoustic waves, then the potential behind the test particulate oscillates as a wake field. Thus, the formation of quasi-lattice structures is, in principle, possible because there are regions of attractive and repulsive forces between the particulates of the same polarity.

The wake potential arises from the residues at the poles at  $\omega = \pm\omega_{sd}, \pm\omega_{fd}$ . Accordingly, integrating Eq. (9) following the standard procedure [10], we obtain the non-Coulombian part of the wake potential. For long wavelength (in comparison with the Debye radius) slow dust-acoustic oscillations with  $k_\perp \rho \ll 1$  and  $|z - vt| > \lambda_D$ , where  $z$  and  $\rho$  are the cylindrical coordinates of the field point, and  $k_\perp$  is the wavenumber component perpendicular to the  $z$ -axis, the wake potential is

$$\Phi_C(\rho = 0, z, t) \approx \frac{2q_t}{|z - vt|} \cos \left( \frac{|z - vt|}{L_{sd}} \right), \quad (10)$$

where  $L_{sd} = \lambda_D (v^2 - c_{sd}^2)^{1/2} / c_{sd}$  is the effective length and  $c_{sd} = \omega_{pd} \lambda_D$  is the slow dust-acoustic speed, which is supposed to be smaller than the velocity ( $v$ ) of the test particulate along the  $z$ -axis. For the fast dust-

acoustic mode  $\lambda_D$  and  $c_{sd}$  should be replaced by  $\lambda_{De}$  and  $\sqrt{\alpha}\omega_{pi}\lambda_{De}$ , respectively.

It follows from (10) that the wake potential is attractive for  $\cos(|z - vt|/L_{sd}) < 0$ . The attractive potential can dominate over the repulsive (Debye screening) potential because of the rapid decrease of the latter beyond the shielding cloud.

Thus, in order for the present attractive forces to be operative it is required that one has continuous injection of the test light particulates in a dusty plasma whose constituents are electrons, ion, and massive highly charged grains which participate in the dynamics of the dust-acoustic oscillations. In such a situation, the phase velocity of the latter could be in resonance with the test particulate, forming the oscillating wake potential which serves the purpose of attracting particles of the same polarity. The test particulates can attract each other as well as the massive grains that are immersed in the background plasma.

In conclusion, we have presented a novel scheme for particulate attraction in dusty plasmas involving collective interaction between the dust-acoustic oscillations and the test particulate. Although the dust-acoustic oscillations are described by the susceptibilities which hold for weakly coupled plasmas, it is anticipated that these oscillations would also survive in strongly coupled plasmas in which some modification of the dust-acoustic velocity can occur owing to strong correlations arising from the thermal agitation and the modification of the transport properties.

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